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FINITE TEMPERATURE EFFECTS ON SPIN POLARIZATION OF NEUTRON MATTER IN A STRONG MAGNETIC FIELD

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ABSTRACT

Magnetars are neutron stars possessing a magnetic field of about 10^{14}-10^{15} G at the surface. Thermodynamic properties of neutron star matter, approximated by pure neutron matter, are considered at finite temperature in strong magnetic fields up to 10^{18} G which could be relevant for the inner regions of magnetars. In the model with the Skyrme effective interaction, it is shown that a thermodynamically stable branch of solutions for the spin polarization parameter corresponds to the case when the majority of neutron spins are oriented opposite to the direction of the magnetic field (i.e. negative spin polarization). Moreover, starting from some threshold density, the self-consistent equations have also two other branches of solutions, corresponding to positive spin polarization. The influence of finite temperatures on spin polarization remains moderate in the Skyrme model up to temperatures relevant for protoneutron stars. In particular, the scenario with the metastable state characterized by positive spin polarization, considered at zero temperature in Phys. Rev. C **80**, 065801 (2009), is preserved at finite temperatures as well. It is shown that, above certain density, the entropy for various branches of spin polarization in neutron matter with the Skyrme interaction in a strong magnetic field shows the unusual behavior, being larger than that of the nonpolarized state. By providing the corresponding low-temperature analysis, we prove that this unexpected behavior should be related to the dependence of the entropy of a spin polarized state on the effective masses of neutrons with spin up and spin down, and to a certain constraint on them which is violated in the respective density range.

Key words : Neutron star models — magnetar — neutron matter — Skyrme interaction — strong magnetic field — spin polarization — finite temperature

1. INTRODUCTION

Magnetars are strongly magnetized neutron stars (Duncan & Thompson 1992) with the magnetic field strength at the surface of about 10^{14}-10^{15} G (Duncan & Thompson 1996; Ibrahim et al. 2002). Such a huge magnetic field can be inferred from observations of magnetar periods and spin-down rates, or from hydrogen spectral lines. Among various classes of neutron stars, soft gamma-ray repeaters and anomalous X-ray pulsars are believed to be most probable candidates for these ultrastrong magnetized astrophysical bodies (Woods & Thompson 2006). Magnetars are relatively frequent objects in the Universe, and comprise about 10% of the whole population of neutron stars (Kouveliotou et al. 1998). In the interior of a magnetar, the magnetic field strength may be even larger, reaching values of about 10^{18} G (Chakrabarty, Bandyopadhyay, & Pal 1997; Broderick, Prakash, & Lattimer 2000). Therefore, magnetars provide a unique playground for studying the properties of neutron star matter under extreme conditions of density and magnetic field strength (Chakrabarty, Bandyopadhyay, & Pal 1997; Broderick, Prakash, & Lattimer 2000; Cardall, Prakash, & Lattimer 2001; Perez-Garcia 2008; Isayev & Yang 2009), which are inaccessible in the terrestrial laboratories.

In a recent study by Perez-Garcia (2008), neutron star matter was approximated by pure neutron matter, in a model with the effective nuclear forces. In the high density region, and in the presence of a strong magnetic field, it has been shown that the behavior of the neutron matter spin polarization depends crucially on whether neutron matter develops a spontaneous spin polarization (in the absence of a magnetic field) at several times nuclear matter saturation density, or the appearance of a spontaneous polarization is not allowed at the considered densities (or delayed to much higher densities). The first case is usual for the Skyrme forces (Rice 1969; Silverstein 1969; Østgaard 1970; Viduarre, Navarro, & Bernabeu 1984; Reddy et

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al. 1999; Akhiezer, Laskin, & Peleminsky 1996; Marcos et al 1991; Maruyama & Tatsumi 2001; Beraudo et al. 2004; Kutschera & Wojcik 1994; Isayev 2003; Isayev & Yang 2004a; Rios, Polls, & Vidaña 2005; Isayev 2006), while the second one is characteristic for the realistic nucleon-nucleon (NN) interaction (Pandharipande, Garde, & Srivastava 1972; Bäckmann & Källman 1973; Haensel 1975; Vidaña, Polls, & Ramos 2002; Fantoni, Sarsa, & Schmidt 2001; Sammarruca & Krastev 2007; Bordbar & Bigdeli 2007). In the former case, a ferromagnetic transition to a totally spin polarized state occurs, while in the latter case a ferromagnetic transition is excluded at all relevant densities and the spin polarization remains quite low, even in the high density region. If a spontaneous ferromagnetic transition is allowed, in a subsequent model consideration with the Skyrme effective forces (Isayev & Yang 2009) it has been shown that the self-consistent equations for the spin polarization parameter, at nonzero magnetic field, have not only solutions corresponding to negative spin polarization (with the majority of neutron spins oriented opposite to the direction of the magnetic field), but also solutions with positive spin polarization (because of the strong spin-dependent medium correlations in the high-density region). In the last case, the formation of a metastable state with the majority of neutron spins oriented along the magnetic field is possible in the high-density interior of a neutron star.

In this study, we extend our previous consideration of neutron matter with Skyrme forces in a strong magnetic field, given at zero temperature (Isayev & Yang 2009), to finite temperatures up to a few tens of MeV being relevant for protoneutron stars. We use a Fermi liquid approach for the description of nuclear matter (Akhiezer et al. 1994, 1997; Isayev & Yang 2006) as a framework for consideration. One of the goals of the research is to study the impact of finite temperatures on spin polarized states in neutron matter in a strong magnetic field, and, in particular, to clarify whether or not the formation of a metastable state with positive spin polarization is possible in the high-density region of neutron matter. Moreover, we provide a fully self-consistent finite temperature calculation of the basic thermodynamic functions of spin polarized neutron matter in a strong magnetic field. It will be shown that, starting from some density, the entropy for various branches of spin polarization in neutron matter in a strong magnetic field shows an unusual behavior: it is larger than that of nonpolarized neutron matter. We relate this unexpected result to the dependence of the entropy on the effective masses of neutrons with spin up and spin down, and to the violation of a certain constraint on them in the corresponding density range.

Note that we consider thermodynamic properties of spin polarized states in neutron matter in a strong magnetic field up to the high density region relevant for astrophysics. However, we take into account the nucleon degrees of freedom only, although other degrees of freedom, such as pions, hyperons, kaons, or quarks could be important at high densities.

2. BASIC EQUATIONS

In what follows, we only present the basic equations useful in this work; more details concerning a Fermi-liquid approach to neutron matter in a strong magnetic field can be found in our earlier article (Isayev & Yang 2009). The normal (nonsuperfluid) states of neutron matter are described by the normal distribution function of neutrons $f_{\kappa_1\kappa_2} = \mathrm{Tr}\,\varrho a^+_{\kappa_2} a_{\kappa_1}$, where $\kappa \equiv (\mathbf{p}, \sigma)$, $\mathbf{p}$ is the momentum, σ is the projection of spin on the third axis, and ϱ is the density matrix of the system. Later on we will assume that the third axis is directed along the external magnetic field $\mathbf{H}$. Given the possibility for alignment of neutron spins along or opposite to the magnetic field $\mathbf{H}$, the normal distribution function of neutrons and single particle energy can be expanded in Pauli matrices σ_i in spin space as

$$\begin{aligned} f(\mathbf{p}) &= f_0(\mathbf{p})\sigma_0 + f_3(\mathbf{p})\sigma_3, \\ \varepsilon(\mathbf{p}) &= \varepsilon_0(\mathbf{p})\sigma_0 + \varepsilon_3(\mathbf{p})\sigma_3. \end{aligned} \tag{1}$$

Expressions for the distribution functions f_0, f_3 in terms of the quantities ε are (Isayev 2003; Isayev & Yang 2004a)

$$\begin{aligned} f_0 &= \frac{1}{2}\{n(\omega_+) + n(\omega_-)\}, \\ f_3 &= \frac{1}{2}\{n(\omega_+) - n(\omega_-)\}. \end{aligned} \tag{2}$$

Here $n(\omega) = \{\exp(Y_0\omega) + 1\}^{-1}$ and

$$\begin{aligned} \omega_\pm &= \xi_0 \pm \xi_3, \\ \xi_0 &= \varepsilon_0 - \mu_0,\ \xi_3 = \varepsilon_3, \end{aligned} \tag{3}$$

μ_0 being the chemical potential of neutrons. The branches $\omega_\pm$ of the quasiparticle spectrum correspond to neutrons with spin up and spin down.

The distribution functions f should satisfy the normalization conditions

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_0(\mathbf{p}) = \varrho, \tag{4}$$

$$\frac{2}{\mathcal{V}}\sum_{\mathbf{p}} f_3(\mathbf{p}) = \varrho_\uparrow - \varrho_\downarrow \equiv \Delta\varrho, \tag{5}$$

where $\varrho = \varrho_\uparrow + \varrho_\downarrow$ is the total density of neutron matter, $\varrho_\uparrow$ and $\varrho_\downarrow$ are the neutron number densities with spin up and spin down, respectively. The quantity $\Delta\varrho$ may be regarded as the neutron spin order parameter. It determines the magnetization of the system $M = \mu_n \Delta\varrho$, μ_n being the neutron magnetic moment. The magnetization may contribute to the internal magnetic field $B = H + 4\pi M$. However, as we discussed earlier (Isayev & Yang 2009), and, as in Perez-Garcia

(2008) and Broderick, Prakash, & Lattimer (2000), we will assume that the contribution of the magnetization to the magnetic field B remains small for all the relevant densities and magnetic field strengths; hence, $B \approx H$.

The self-consistent equations for the components of the single-particle energy have the form (Isayev & Yang 2009)

$$\xi_0(\mathbf{p}) = \underline{\varepsilon}_0(\mathbf{p}) + \tilde{\varepsilon}_0(\mathbf{p}) - \mu_0, \quad (6)$$

$$\xi_3(\mathbf{p}) = -\mu_n H + \tilde{\varepsilon}_3(\mathbf{p}). \quad (7)$$

Here $\underline{\varepsilon}_0(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_0}$ is the free single particle spectrum, m_0 is the bare mass of a neutron, and $\tilde{\varepsilon}_0, \tilde{\varepsilon}_3$ are the Fermi liquid (FL) corrections to the free single particle spectrum; these are related to the normal FL amplitudes $U_0^n(\mathbf{k}), U_1^n(\mathbf{k})$ by

$$\tilde{\varepsilon}_0(\mathbf{p}) = \frac{1}{2\mathcal{V}} \sum_{\mathbf{q}} U_0^n(\mathbf{k}) f_0(\mathbf{q}),\ \mathbf{k} = \frac{\mathbf{p}-\mathbf{q}}{2}, \quad (8)$$

$$\tilde{\varepsilon}_3(\mathbf{p}) = \frac{1}{2\mathcal{V}} \sum_{\mathbf{q}} U_1^n(\mathbf{k}) f_3(\mathbf{q}). \quad (9)$$

To obtain numerical results, we utilize the effective Skyrme interaction (Vautherin & Brink 1972). Expressions for the normal FL amplitudes in terms of the Skyrme force parameters were written by Akhiezer et al. (1997) and Isayev & Yang (2006). Thus, using expressions (2) for the distribution functions f, we obtain the self-consistent equations (6), (7) for the components of the single-particle energy $\xi_0(\mathbf{p})$ and $\xi_3(\mathbf{p})$, which should be solved together with the normalization conditions (4), (5).

Note that the spin ordering in neutron matter can be characterized by the neutron spin polarization parameter

$$\Pi = \frac{\varrho_\uparrow - \varrho_\downarrow}{\varrho} \equiv \frac{\Delta\varrho}{\varrho}.$$

The number densities of neutrons with spin up and spin down are related to the spin polarization parameter Π by

$$\varrho_\uparrow = \frac{\varrho}{2}(1+\Pi),\ \varrho_\downarrow = \frac{\varrho}{2}(1-\Pi). \quad (10)$$

To examine the thermodynamic stability of different solutions of the self-consistent equations, it is necessary to compare the corresponding free energies $F = E - TS$, where the energy functional E is characterized by two FL amplitudes U_0^n, U_1^n (Isayev & Yang 2006) and the entropy is given by

$$S = -\sum_{\mathbf{p}} \sum_{\sigma=+,-} \{n(\omega_\sigma)\ln n(\omega_\sigma) + \bar{n}(\omega_\sigma)\ln \bar{n}(\omega_\sigma)\},\ \bar{n}(\omega) = 1 - n(\omega). \quad (11)$$

3. SOLUTIONS OF SELF-CONSISTENT EQUATIONS AT FINITE T. THERMODYNAMIC FUNCTIONS

In the model with SLy4 and SLy7 Skyrme effective forces (Chabanat et al. 1998), the self-consistent equations were analyzed at zero temperature by Isayev & Yang (2009) for the magnetic field strengths up to $H_{max} \sim 10^{18}$ G, allowed by a scalar virial theorem (Lai & Shapiro 1991). These Skyrme parametrizations were originally constrained to reproduce the results of microscopic neutron matter calculations (pressure-versus-density curve) and give neutron star models in broad agreement with observables (such as the minimum rotation period, gravitational mass-radius relation, the binding energy, released in supernova collapse, etc., see Rikovska Stone et al. (2003)). It was shown that a thermodynamically stable branch of solutions for the spin-polarization parameter, as a function of density, corresponds to a negative spin polarization when the majority of neutron spins are oriented opposite to the direction of the magnetic field. Moreover, starting from some threshold density which depends on the magnetic field strength, the state with positive spin polarization can be realized as a metastable state in the high-density region of neutron matter, in the model with the Skyrme effective forces.

In this section, we study the impact of finite temperatures on spin polarization of neutron matter in a strong magnetic field. In particular, we are interested in whether or not the metastable state with positive spin polarization will survive at temperatures about several tens of MeV typical of protoneutron stars. To this end, we directly find solutions of the self-consistent equations at nonzero temperature. Since the results of calculations with SLy4 and SLy7 Skyrme forces are very similar, here we present only the obtained dependences for the SLy7 Skyrme interaction.

Fig. 1 shows the neutron spin polarization parameter as a function of density, normalized to the nuclear saturation density ϱ_0 (for SLy7 force, $\varrho_0 = 0.158\,\mathrm{fm}^{-3}$), for a set of fixed values of the magnetic field strength at the temperature $T = 20$ MeV. The branches of spontaneous polarization Π_0^-, Π_0^+, corresponding to the vanishing magnetic field, are shown by the solid curves. As one can see, the obtained dependences are qualitatively similar to those obtained at zero temperature (Isayev & Yang 2009). The branches of spontaneous polarization are modified differently by the magnetic field: the branch $\Pi_0^-(\varrho)$ turns to the branch $\Pi_1(\varrho)$ with negative spin polarization, while the branch $\Pi_0^+(\varrho)$ splits into two branches, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, corresponding to positive spin polarization. For the lower branch $\Pi_1(\varrho)$, there are three characteristic density domains. At low densities $\varrho \lesssim 0.5\varrho_0$, the magnitude of the spin polarization parameter increases with decreasing density. At intermediate densities $0.5\varrho_0 \lesssim \varrho \lesssim 3\varrho_0$, there is a plateau in the $\Pi_1(\varrho)$ dependence, whose characteristic

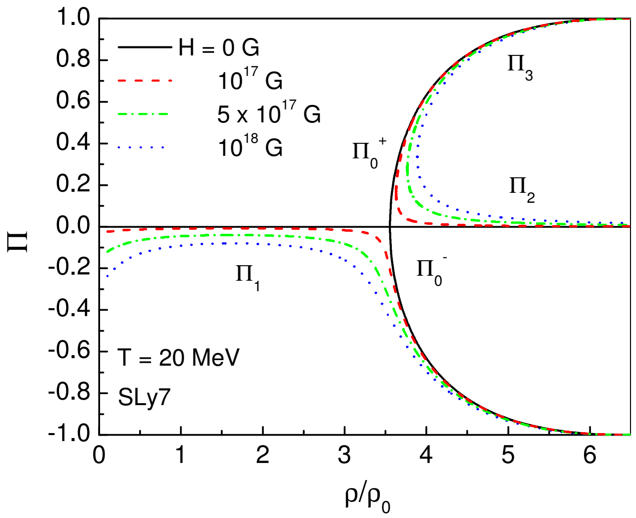

Fig. 1.— (Color online) Neutron spin polarization parameter as a function of density at $T = 20$ MeV, for different magnetic field strengths (SLy7 interaction). The branches of spontaneous polarization Π_0^-, Π_0^+ are shown by the solid curves.

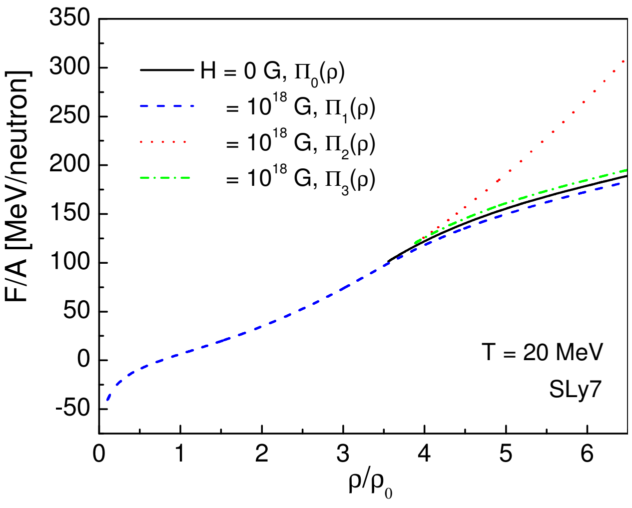

Fig. 2.— (Color online) Free energy per neutron as a function of density at $T = 20$ MeV. Different branches $\Pi_1(\varrho)$-$\Pi_3(\varrho)$ of solutions of the self-consistent equations are considered at $H = 10^{18}$ G for the SLy7 interaction, including a spontaneously polarized state.

value depends on H (at the given temperature). At densities $\varrho \gtrsim 3\varrho_0$, the absolute value of the spin polarization parameter increases with density and tends to unity. Note that, in the low-density domain, the possibility of the appearance of a nuclear magnetic pasta and its impact on the neutrino opacities in the protoneutron star early cooling stage should be explored in a more elaborated analysis as discussed in detail by Perez-Garcia (2008).

The upper branches $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, corresponding to a positive spin polarization, appear stepwise at the same threshold density $\varrho_{\rm th}$ dependent on the magnetic field (at the given temperature), and being larger than the critical density of spontaneous spin instability in neutron matter. For the branch $\Pi_2(\varrho)$, the spin polarization parameter decreases with density and tends to zero, while for the branch $\Pi_3(\varrho)$ it increases with density and approaches unity. Because of the negative value of the neutron magnetic moment, the magnetic field tends to orient the neutron spins opposite to the magnetic field direction. As a result, the spin polarization parameter for the branches $\Pi_2(\varrho)$, $\Pi_3(\varrho)$ with positive spin polarization is smaller than that for the branch of spontaneous polarization Π_0^+; vice versa, the magnitude of the spin polarization parameter for the branch $\Pi_1(\varrho)$ with negative spin polarization is larger than the corresponding value for the branch of spontaneous polarization Π_0^-.

Thus, at densities larger than $\varrho_{\rm th}$, we have three branches of solutions: one, $\Pi_1(\varrho)$, with negative spin polarization and the remaining two, $\Pi_2(\varrho)$ and $\Pi_3(\varrho)$, with positive polarization. In order to establish which branch is thermodynamically preferable, we should compare the corresponding free energies. Fig. 2 shows the free energy per neutron as a function of density at $T = 20$ MeV and $H = 10^{18}$ G for these three branches, compared with the free energy per neutron for a spontaneously polarized state [the branches $\Pi_0^\pm(\varrho)$]. The state with the majority of neutron spins, oriented opposite to the direction of the magnetic field [the branch $\Pi_1(\varrho)$], has a lowest free energy. This result is intuitive, since the magnetic field tends to direct the neutron spins opposite to $\mathbf{H}$, as mentioned earlier. However, the state described by the branch $\Pi_3(\varrho)$ with positive spin polarization has the free energy very close to that of the thermodynamically stable state. This means that, despite the presence of a strong magnetic field $H \sim 10^{18}$ G, the state with the majority of neutron spins directed along the magnetic field can be realized as a metastable state in the dense core of a neutron star, in the model with the Skyrme effective interaction. In this scenario, since such states exist only at densities $\varrho \geq \varrho_{\rm th}$, under decreasing density (from the inner to the outer regions of a magnetar) a metastable state with positive spin polarization at the threshold density $\varrho_{\rm th}$ changes to a thermodynamically stable state with negative spin polarization.

Note that the possibility of the appearance of a metastable state, with the majority of neutron spins oriented along the magnetic field, was missed in the finite temperature calculations by Perez-Garcia (2008). This is because only one branch of solutions for the spin polarization parameter, corresponding to the thermodynamically stable state, was considered in that study for neutron matter in a strong magnetic field, with the

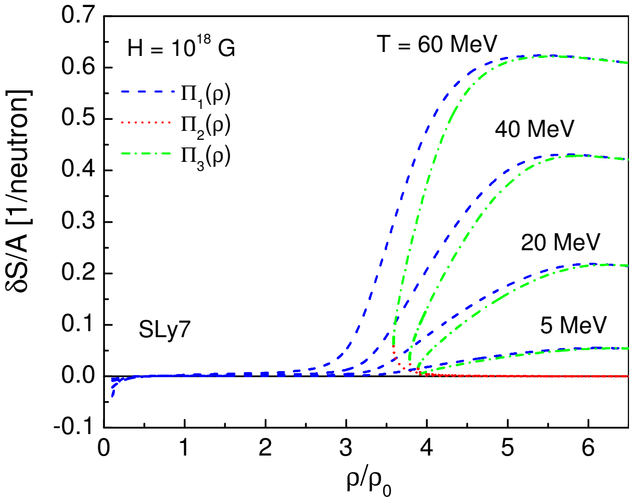

Fig. 3.— (Color online) The entropy per neutron measured from its value in the nonpolarized state for the Π_1–Π_3 branches of spin polarization as a function of density (SLy7 interaction). The magnetic field strength is $H = 10^{18}$ G, and different temperatures are considered.

same Skyrme interaction.

An unexpected moment appears, if we consider separately the entropy for various branches of spin polarization. Fig. 3 shows the difference between the entropy per neutron for the branches Π_1–Π_3, and that of the nonpolarized state (with $\Pi = 0$ at $H = 0$) as a function of density for different temperatures. The strength of the magnetic field is fixed to be $H = 10^{18}$ G. Contrary to the expected behavior, the entropy for the branch Π_1, starting from a certain density weakly dependent on temperature (at the given magnetic field strength), is larger than the entropy of the nonpolarized state. Besides, the entropy for the branches Π_2 and Π_3 is larger than that for the nonpolarized state for all relevant densities. It appears that the spin polarized states described by the Π_1–Π_3 branches of spin polarization are less ordered than the nonpolarized state, at the corresponding densities.

In order to understand qualitatively this behavior, let us consider the low-temperature expansion for the entropy in terms of the effective masses of spin-up and spin-down neutrons. The density of entropy (11) can be written in the form

$$\begin{aligned} s &= \frac{1}{\pi^2\hbar^3}\sum_{\sigma=+,-}\sqrt{\frac{m_\sigma^3 T^3}{2}} \\ &\quad\times\left\{\frac{5}{3}J_{\frac{3}{2}}(\eta_\sigma)-\eta_\sigma J_{\frac{1}{2}}(\eta_\sigma)\right\},\ \eta_\sigma\equiv\frac{\mu_\sigma}{T}, \end{aligned} \tag{12}$$

where

$$J_\nu(\eta)=\int_0^\infty \frac{x^\nu}{e^{x-\eta}+1}dx$$

is the Fermi-Dirac integral of the order ν, μ_σ is the effective chemical potential of neutrons with spin up ($\sigma = +$) and spin down($\sigma = -$), whose explicit expression was derived by Isayev & Yang (2009). In the low-temperature limit $\eta_\sigma \gg 1$, and using the corresponding expansion for Fermi-Dirac integrals in Eq. (12), we obtain

$$s=\sum_{\sigma=+,-}s_\sigma,\quad s_\sigma=\frac{\pi^2}{2\varepsilon_{F\sigma}}T. \tag{13}$$

In the previous equation $\varepsilon_\sigma = \frac{\hbar^2 k_{F\sigma}^2}{2m_\sigma}$ is the Fermi energy of neutrons with spin up and spin down, and $k_\sigma = (6\pi^2\varrho_\sigma)^{1/3}$ is the respective Fermi momentum. The low-temperature expansion (13) is valid till $T/\varepsilon_{F\sigma} \ll 1$. Then, requiring the difference between the entropies of spin polarized and nonpolarized states to be negative, we derive the constraint on the effective masses $m_{n\uparrow}$ and $m_{n\downarrow}$ of neutrons with spin up and spin down in a spin polarized state:

$$D\equiv\frac{m_{n\uparrow}}{m_n}(1+\Pi)^{\frac{1}{3}}+\frac{m_{n\downarrow}}{m_n}(1-\Pi)^{\frac{1}{3}}-2<0. \tag{14}$$

Here m_n is the effective mass of a neutron in nonpolarized neutron matter (Isayev & Yang 2004a),

$$\frac{\hbar^2}{2m_n}=\frac{\hbar^2}{2m_0}+\frac{\varrho}{8}[t_1(1-x_1)+3t_2(1+x_2)], \tag{15}$$

and the effective masses $m_{n\uparrow}, m_{n\downarrow}$ are given by (Isayev & Yang 2009)

$$\begin{aligned} \frac{\hbar^2}{2m_{\uparrow(\downarrow)}} &= \frac{\hbar^2}{2m_0}+\frac{\varrho_{\uparrow(\downarrow)}}{2}t_2(1+x_2) \\ &\quad+\frac{\varrho_{\downarrow(\uparrow)}}{4}[t_1(1-x_1)+t_2(1+x_2)]. \end{aligned} \tag{16}$$

In Eqs. (15), (16), t_i and x_i are phenomenological parameters, which specify a given parametrization of the Skyrme interaction. According to Eq. (10), the number densities of neutrons with spin up and spin down are determined by the spin polarization parameter Π, which, after the self-consistent determination, is a function of the thermodynamic parameters ϱ, T, H. Hence, (14) determines the region in the domain of admissible values of the thermodynamic parameters, where the entropy of a spin polarized state exhibits the expected behavior.

Fig. 4 shows the left-hand side D of constraint (14) for the branches Π_1–Π_3 of spin polarization, as a function of density at $H = 10^{18}$ G and temperature $T = 5$ MeV, when the condition $\varepsilon_{F\sigma}/T \gg 1$ holds true. For the branch Π_1, the difference D is negative up to the density $\varrho_s \approx 0.41\varrho_0$ (see the insert in Fig. 4), where it changes sign and remains positive for all larger densities. Besides, for the branches Π_2 and Π_3, the difference D is positive for all densities at which the corresponding solutions of the self-consistent equations

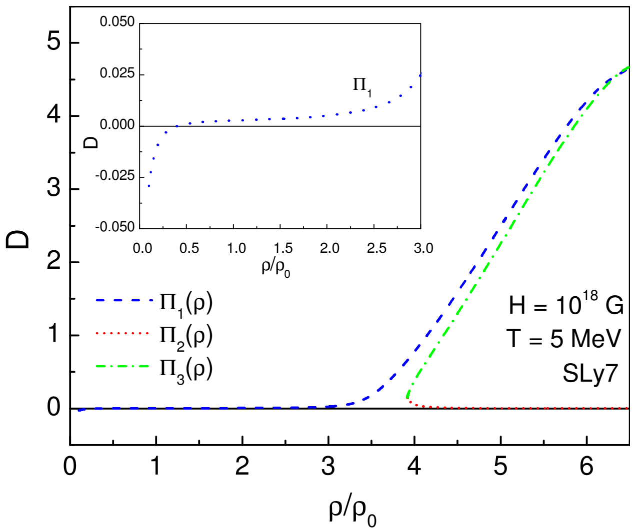

Fig. 4.— (Color online) The difference D in constraint (14) for various branches Π_1–Π_3 of spin polarization as a function of density at $H = 10^{18}$ G and $T = 5$ MeV for the SLy7 interaction.

exist. These features explain the above mentioned unusual behavior of the entropy for various branches of spin polarization in neutron matter with the Skyrme interaction under the presence of a strong magnetic field. Note that such an unexpected behavior of the entropy was also found for the states with spontaneous spin polarization (in the absence of the magnetic field) in neutron matter with the Skyrme interaction (Rios, Polls, & Vidaña 2005), and for antiferromagnetically spin ordered states in symmetric nuclear matter with the Gogny D1S interaction (Isayev & Yang 2004b; Isayev 2005, 2007).

4. CONCLUSIONS

We have studied the impact of finite temperatures on the spin structure in the magnetar interior. We approximated the neutron star matter by pure neutron matter, and considered the Skyrme effective interaction as a potential of NN interaction (SLy7 parametrization). According to the scalar virial theorem, strong magnetic fields up to 10^{18} G can be relevant for the inner regions of magnetars. We shown that, with the thermodynamically stable branch of solutions for the spin polarization parameter corresponding to the case when the majority of neutron spins are oriented opposite to the direction of the magnetic field (negative spin polarization), the self-consistent equations have also two other branches of solutions corresponding to positive spin polarization. The influence of finite temperatures on spin polarization remains moderate in the Skyrme model, at least up to temperatures relevant for protoneutron stars. A thermodynamic analysis, based on the calculation of the free energy for different branches of spin polarization, shows that the scenario with the metastable state characterized by positive spin polarization, considered at zero temperature (Isayev & Yang 2009), is preserved at finite temperatures as well. This is one of the important conclusions of the present work. The possible existence of a metastable state with positive spin polarization will affect the neutrino opacities of a neutron star matter in a strong magnetic field; hence, it will influence the cooling history of a neutron star (Reddy et al. 1999).

We have also shown that, above a density threshold, the entropy for various branches of spin polarization in neutron matter with the Skyrme interaction in a strong magnetic field exhibits an unusual behavior: it is larger than that of the nonpolarized state. To clarify this point, we provided the corresponding low-temperature analysis. This unexpected behavior should be related to the dependence of the entropy of a spin polarized state on the effective masses of spin-up and spin-down neutrons, and to a certain inequality constraint on them which is violated in the respective density range.

It is worth noticing that we approximated neutron star matter with pure neutron matter. This should be considered as a first step towards a more realistic description of neutron stars, taking into account a finite fraction of protons with the charge neutrality and beta equilibrium conditions. In particular, some admixture of protons can affect the onset densities of enhanced polarization in a neutron star matter with the Skyrme interaction (Isayev & Yang 2004a). Nevertheless, at such strong magnetic fields one can expect that the proton fraction is relatively small, and even can completely disappear in the dense interior of a magnetar (Mao et al. 2003).

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