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소형 위그선 선저판의 구조안전성 평가에 관한 연구

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Structural Analysis of the Bottom Plate of Small WIG Craft

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Abstract

A WIG(Wing-In-Ground effect) craft flies close to the water surface by utilizing a cushion of relatively high pressurized air between its wing and water surface. This implies that when one designs such craft it is important to have lightweight structures with adequate strength to resist external loads with some margins. To investigate this requirement, this paper deals with the structural analysis of the bottom plate of small WIG craft having a design landing weight of 1.2-ton. As building materials for the WIG craft, pre-preg carbon/epoxy composites are considered. The strength information of the bottom plate is obtained using the first-ply-failure analysis in conjunction with a mid-plane symmetric laminated plate theory. As a result, the first-ply-failure location, load and deflection of the bottom plate are obtained. The calculated strength information is compared with the water reaction load for the bottom plate of seaplanes considered when they land on the water surface –the same fluid-structure interaction mechanism as that of WIG craft. In the calculation of seaplane water reaction load information, the rules shown in FAR(Federal Aviation Regulations) Part 25 are used. Through the comparison, the structural integrity of the bottom plate for the WIG craft is checked.

Keywords : WIG craft(위그선), Composites(복합재료), Bottom plate(선저판), Lamination scheme(적층계획), The first-ply-failure (적층판의 초기파괴), Structural integrity(구조 건전성)

1. Introduction

A WIG(Wing-In-Ground effect) craft flies close to the water surface by utilizing a cushion of relatively high pressurized air between its wing and the water surface. This implies that when one designs such craft, it is important to have light-weight structures with adequate strength to resist external loads with some margins.

A WIG craft considered in this paper has a design landing weight of 1.2-ton. Materials used for this WIG structure is pre-preg carbon/epoxy composites to achieve excellent strength to weight ratio. When this WIG craft operates, its structural members will be subjected to various external loads. Among them, wave impact pressure load acting on the bottom plate of the WIG craft arising when it lands on the water surface will be significant in magnitude and a primary load case for the design of such plate. Thus, this paper deals with the structural analysis of the bottom plate of the WIG craft under wave impact pressure load. The plate exemplified in this paper is taken from the middle of the WIG craft close to its centre of gravity: the plate is surrounded by two trans-ring type frames.

Four different lamination schemes are considered for the bottom plate and it is assumed that the plate is simply supported around all its edges. Design of the bottom plate is examined by obtaining strength characteristics such as failure load/deflection and failed ply number. Calculations are performed within the first-ply-failure analysis regime in conjunction with a mid-plane symmetric laminated plate theory. In the strength assessment, various anisotropic failure criteria for the laminated plate are used and they are maximum stress, maximum strain, Hoffman's, Tsai-Hill's and Tsai-Wu's.

Finally, the calculated strength information is compared with the water reaction load information for the bottom plate

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of seaplanes considered when they land on the water surface – the same solid-fluid interaction mechanism as that of the WIG craft. In the calculation of seaplanes' water reaction load information, the rules shown in FAR(Federal Aviation Regulations) Part 25 are used. Through the comparison, the structural integrity of the bottom plate for the WIG craft is checked.

2. A Mid-plane Symmetric Laminated Plate Theory

Consider a mid-plane symmetric laminated plate, shown in Fig. 1, subjected to the transverse load $q=q(x)$.

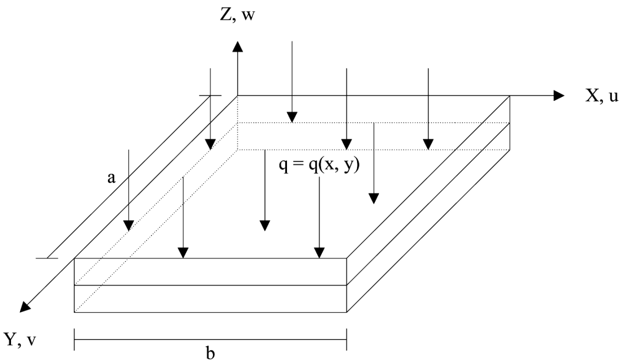

Fig. 1 Plate geometry

Since the plate contains orthotropic layers in which the principal material axes are not parallel to the plate axes, it is characterized by non-vanishing bending-twisting coupling terms, $D_{16}=D_{26}\neq 0$, and vanishing bending-extensional coupling terms, $B_{ij}=0$. The equations of motion in terms of displacements for this laminated plate can be expressed as follows (Whitney, 1987),

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}=q \tag{1}$$

where, $D_{ij}\,(i,\ j,=1,\ 2,\ 6)$ are the flexural stiffness matrix terms. The transverse load q can be represented by the double Fourier series.

$$q=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{2}$$

For a simply-supported plate around the all edges, the following boundary conditions should be satisfied.

At $x=0$ and a

$$w=M_x=-D_{11}\frac{\partial^2 w}{\partial x^2}-2D_{16}\frac{\partial^2 w}{\partial x\partial y}-D_{12}\frac{\partial^2 w}{\partial y^2}=0 \tag{3}$$

At $y=0$ and b,

$$w=M_y=-D_{11}\frac{\partial^2 w}{\partial x^2}-2D_{16}\frac{\partial^2 w}{\partial x\partial y}-D_{22}\frac{\partial^2 w}{\partial y^2}=0 \tag{4}$$

The assumed solution is of the form,

$$w=\sum_{m=1}^{M}\sum_{n=1}^{N}A_{mn}X_m(x)\,Y_n(y) \tag{5}$$

The Ritz method (Whitney, 1987) is applied to obtain the coefficient in the assumed solution. The coefficient is calculated by solving the following $M\times N$ algebraic equations.

$$\begin{aligned}&\sum_{i=1}^{M}\sum_{j=1}^{N}\{D_{11}\int_0^a\frac{d^2X_i}{dx^2}\frac{d^2X_m}{dx^2}dx\int_0^b Y_jY_n dy+D_{12}[\int_0^a X_m\frac{d^2X_i}{dx^2}dx\int_0^b Y_j\frac{d^2Y_n}{dy^2}dy\\&+\int_0^a X_i\frac{d^2X_m}{dx^2}dx\int_0^b Y_n\frac{d^2Y_j}{dy^2}dy]+D_{22}\int_0^a X_iX_m dx\int_0^b\frac{d^2Y_j}{dy^2}\frac{d^2Y_n}{dy^2}dy\\&+4D_{66}\int_0^a\frac{dX_i}{dx}\frac{dX_m}{dx}dx\int_0^b\frac{dY_j}{dy}\frac{dY_n}{dy}dy+2D_{16}[\int_0^a\frac{d^2X_i}{dx^2}\frac{dX_m}{dx}dx\int_0^b Y_j\frac{dY_n}{dy}dy\\&+\int_0^a\frac{dX_i}{dx}\frac{d^2X_m}{dx^2}dx\int_0^b Y_n\frac{dY_j}{dy}dy]+2D_{26}[\int_0^a X_m\frac{dX_i}{dx}dx\int_0^b\frac{dY_j}{dy}\frac{d^2Y_n}{dy^2}dy\\&+\int_0^a X_i\frac{dX_m}{dx}dx\int_0^b\frac{d^2Y_j}{dy^2}\frac{dY_n}{dy}dy]\}A_{ij}=q_0\int_0^a X_m dx\int_0^b Y_n dy\\&(m=1,2,\ldots,M\quad and\quad n=1,2,\ldots,N)\end{aligned} \tag{6}$$

where, for all simply-supported edges,

$$X_m(x)=\sin\frac{m\pi x}{a}\qquad Y_n(y)=\sin\frac{n\pi y}{b}$$
$$X_i(x)=\sin\frac{i\pi x}{a}\qquad Y_j(y)=\sin\frac{j\pi y}{b} \tag{7}$$

From the constitutive equation and equations of motion, the following in-plane stress components,

$$\sigma_x^{(k)}=-z\left(Q_{11}^{(k)}\frac{\partial^2 w}{\partial x^2}+2Q_{16}^{(k)}\frac{\partial^2 w}{\partial x\partial y}+Q_{12}^{k}\frac{\partial^2 w}{\partial y^2}\right) \tag{8}$$

$$\sigma_y^{(k)}=-z\left(Q_{12}^{(k)}\frac{\partial^2 w}{\partial x^2}+2Q_{26}^{(k)}\frac{\partial^2 w}{\partial x\partial y}+Q_{22}^{k}\frac{\partial^2 w}{\partial y^2}\right) \tag{9}$$

$$\sigma_{xy}^{(k)} = -z\left(Q_{16}^{(k)}\frac{\partial^2 w}{\partial x^2} + 2Q_{66}^{(k)}\frac{\partial^2 w}{\partial x \partial y} + Q_{26}^{k}\frac{\partial^2 w}{\partial y^2}\right) \tag{10}$$

and interlaminar shear stress components,

$$\sigma_{xz}^{(k)} = \frac{z^2}{2}\begin{bmatrix} Q_{11}^{(k)}\frac{\partial^3 w}{\partial x^3} + 3Q_{16}^{(k)}\frac{\partial^3 w}{\partial x^2 \partial y} + \\ \left(Q_{12}^{k} + 2Q_{66}^{k}\right)\frac{\partial^3 w}{\partial x \partial y^2} + Q_{26}^{(k)}\frac{\partial^3 w}{\partial y^3} \end{bmatrix} \tag{11}$$

$$\sigma_{yz}^{(k)} = \frac{z^2}{2}\begin{bmatrix} Q_{16}^{(k)}\frac{\partial^3 w}{\partial x^3} + \left(Q_{12}^{k} + 2Q_{66}^{k}\right)\frac{\partial^3 w}{\partial x^2 \partial y} + \\ 3Q_{26}^{(k)}\frac{\partial^3 w}{\partial x \partial y^2} + Q_{22}^{(k)}\frac{\partial^3 w}{\partial y^3} \end{bmatrix} \tag{12}$$

for the ***kth*** layer in the plate can be derived. Finally, the in-plane and interlaminar shear stress components for the ***kth*** layer in the plate can be calculated by substituting the assumed solution, w, into the above equations, Eqn. 12. In the above equations, $Q_{ij}^{(k)}$ $(i, j, = 1,\ 2,\ 6)$ are the transformed stiffness quantities.

3. Failure Criteria

A general representation of failure envelope for laminates can be given by a truncated tensor equation, (Tsai & Wu, 1971; Agarwal & Broutman, 1990).

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1(i, j = 1, 2, 3, 4, 5, 6) \tag{13}$$

The coefficients F_i and F_{ij} are the functions of the unidirectional lamina strengths and defined for the different failure criteria as below.

For maximum stress failure criterion,

$$\begin{aligned} &F_1 = X_t^{-1} - X_c^{-1},\ F_2 = Y_t^{-1} - Y_c^{-1},\ F_{11} = (X_t X_c)^{-1} \\ &F_{12} = -1/2\left(X_t^{-1} - X_c^{-1}\right)\left(Y_t^{-1} - Y_c^{-1}\right),\ F_{22} = \left(Y_t Y_c\right)^{-1} \\ &F_{44} = R^{-2},\ F_{55} = S^{-2},\ F_{66} = T^{-2} \end{aligned} \tag{14}$$

For Tsai-Hill's failure criterion,

$$\begin{aligned} &F_{11} = X_{t,c}^{-2},\ F_{12} = -1/2\left(X_{t,c}^{-2} + Y_{t,c}^{-2}\right),\ F_{22} = Y_{t,c}^{-2} \\ &F_{44} = R^{-2},\ F_{55} = S^{-2},\ F_{66} = T^{-2} \end{aligned} \tag{15}$$

For Hoffman's failure criterion,

$$\begin{aligned} &F_1 = X_t^{-1} - X_c^{-1},\ F_2 = Y_t^{-1} - Y_c^{-1},\ F_{11} = (X_t X_c)^{-1} \\ &F_{12} = -1/2\left(X_t^{-1} - X_c^{-1} + Y_t^{-1} - Y_c^{-1}\right),\ F_{22} = \left(Y_t Y_c\right)^{-1} \\ &F_{44} = R^{-2},\ F_{55} = S^{-2},\ F_{66} = T^{-2} \end{aligned} \tag{16}$$

For maximum strain failure criterion,

$$\begin{aligned} &F_1 = \left(X_t^{-1} - X_c^{-1}\right) - v_{TL}\left(Y_t^{-1} - Y_c^{-1}\right) \\ &F_2 = -v_{TL}E_L E_T^{-1}\left(X_t^{-1} - X_c^{-1}\right) + \left(Y_t^{-1} - Y_c^{-1}\right), \\ &F_{11} = (X_t X_c)^{-1} + v_{TL}\left(X_t^{-1} - X_c^{-1}\right) \\ &\qquad \times \left(Y_t^{-1} - Y_c^{-1}\right) + v_{TL}^2\left(Y_t Y_c\right)^{-1} \\ &F_{12} = -v_{TL}E_L E_T^{-1}(X_t X_c)^{-1} - 1/2\left(1 + v_{TL}^2 E_L E_T^{-1}\right) \\ &\qquad \times \left(X_t^{-1} - X_c^{-1}\right)\left(Y_t^{-1} - Y_c^{-1}\right) \\ &F_{22} = v_{TL}^2 E_L^2 E_T^{-2}(X_t X_c)^{-1} + v_{TL}E_L E_T^{-1} \\ &\qquad \times \left(X_t^{-1} + X_c^{-1}\right)\left(Y_t^{-1} Y_c^{-1}\right) + \left(Y_t Y_c\right)^{-1} \\ &F_{44} = R^{-2},\ F_{55} = S^{-2},\ F_{66} = T^{-2} \end{aligned} \tag{17}$$

For Tsai-Wu's failure criterion,

$$\begin{aligned} &F_1 = X_t^{-1} - X_c^{-1},\ F_2 = Y_t^{-1} - Y_c^{-1},\ F_{11} = (X_t X_c)^{-1} \\ &F_{12} = -1/2\left(\sqrt{X_t X_c Y_t Y_c}\right),\ F_{22} = \left(Y_t Y_c\right)^{-1} \\ &F_{44} = R^{-2},\ F_{55} = S^{-2},\ F_{66} = T^{-2} \end{aligned} \tag{18}$$

where, $X_{t,c}$ and $Y_{t,c}$ are longitudinal and transverse tensile/compressive strengths of a unidirectional lamina, and R, S and T are in-plane and transverse shear strengths of a unidirectional lamina, respectively. It should be mentioned that all of the remaining F_i and F_{ij} tensor coefficient terms are zero.

4. Strength Analysis of a Laminated Plate

In strength analysis of a laminated plate, the mechanical responses of each ply due to an external load such as stress and strain components are fundamental with their corresponding ultimate strength values to predict failure. Thus for a given external load, ply-by-ply strength analysis can be performed by comparing the calculated ply stresses or strains with the corresponding ultimate strengths in the previously described failure criteria.

When independent failure criteria are used, the first-ply-failure will occur if the calculated stresses or strains of a given ply exceed the corresponding ultimate strengths. When polynomial failure criteria are used, the first-ply-failure will occur if Eqn. (13) (it has different coefficient values according to the different failure criteria) reaches a value of unity (Ochoa & Reddy, 1990). In conjunction with the first-ply-failure analysis, Figure 2 shows the general analysis procedure.

5. Numerical Examples and Discussions

The exemplified plate is taken from the middle of the WIG craft close to its centre of gravity. The plate is surrounded by two trans-ring type frames. It is assumed that this plate is simply supported around all its edges and subjected to lateral pressure load.

Four different lamination schemes are considered for the plate and they are as follows,

Lamination-I:
[45°/0°/-45°/0°/45°/(0°)$_2$/(-45°)$_2$/0°]s
Lamination-II:
[45°/0°/-45°/90°/45°/0°/90°/-45°/90°/0°]s
Lamination-III:
[0°/90°/45°/-45°/0°/90°/45°/-45°/0°/45°]s
Lamination-IV:
[90°/0°/45°/-45°/90°/0°/45°/-45°/90°/0°]s

The first-ply-failure strength of the above laminated plates is obtained by applying lateral pressure load from small magnitude until these plates deem to have failures in accordance with the analysis procedure shown in Fig. 2.

As building material, pre-preg carbon/epoxy is used for the plates and its mechanical properties are shown in Table 1.

Results are shown in Table 2. Commonly all the failure criteria detect the plies in outer side of the plates as failed plies. These plies are in tension and it reveals that the plies in tensile side are weaker than the compression or the middle part of the plates with respect to the lateral pressure load.

Among four different lamination schemes, the first-ply-failure is occurred at the 1st ply in case of lamination-I/II and the 3rd ply in case of lamination-III/IV respectively. This implies that the ply having an angle of orientation of 45° is weak ply compared to other angle of orientation plies and this is more evident when 45° ply is placed in the outermost.

Since various failure criteria are used, there are modeling uncertainties as shown by different failure loads and deflections, see Table 2. Regardless of the different lamination

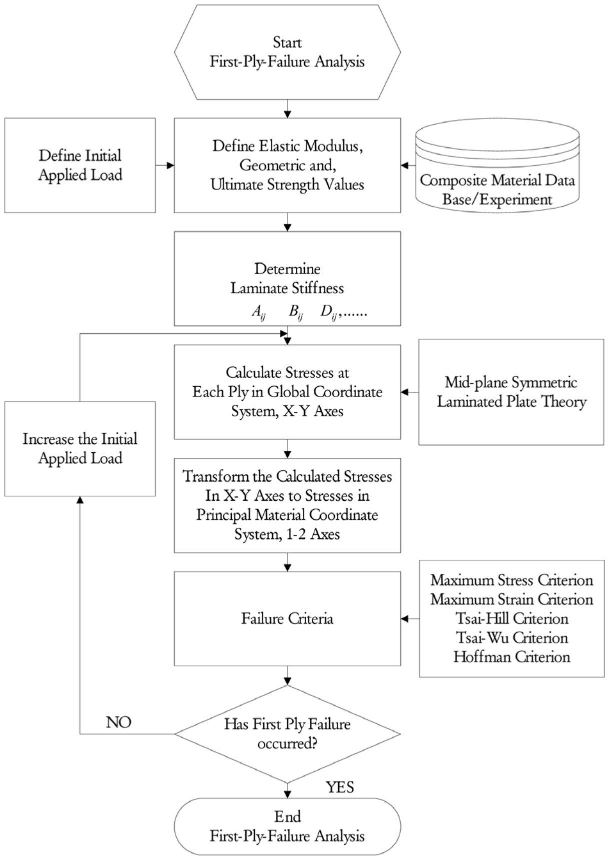

Fig. 2 First-ply-failure analysis procedure

Table 1 Material properties of pre-preg carbon/epoxy used

Materials properties	Values
E_{11}	63.4 GPa
E_{22}	58.0 GPa
G_{12}	56.2 GPa
ν_{12}	0.17
X_t	0.635 GPa
X_C	0.528 GPa
Y_t	0.412 GPa
Y_c	0.304 GPa
R	0.062 GPa
S	0.062 GPa
T	0.115 GPa
$a,\ b$	758mm, 379mm
Thickness	0.2mm (each ply) / 3.8mm (total thickness)

schemes, maximum stress failure criterion yields the most conservative failure loads and Tsai-Hill failure criterion produces the most optimistic failure loads. The same finding can be applied to the first-ply-failure deflections. From this, one can construct the first-ply-failure based design strength and stiffness ranges. For example, lower bounds of 32,690 Pa and upper bounds of 34,057Pa design strength range can be obtained for Lamination-I.

Concerning the effect of stacking sequences to the load resistance of the laminated plates, lamination-III/IV that have 0° and 90° plies in the outermost are more capable of bearing the lateral pressure load than lamination-I/II that have 45° ply in the outermost, see Table 2.

Table 2 Results of analysis

	Failure criteria	Failed ply	Failure load (Pa)	Failure deflection
Lamination I	Max. stress	1st ply	32,690.0	16.84mm
	Max. strain	1st ply	33,536.0	17.28mm
	Hoffman	1st ply	33,136.0	17.07mm
	Tsai-Hill	1st ply	34,057.0	17.55mm
	Tsai-Wu	1st ply	33,459.0	17.24mm
Lamination II	Max. stress	1st ply	32,952.0	16.83mm
	Max. strain	1st ply	33,804.0	17.27mm
	Hoffman	1st ply	33,400.0	17.06mm
	Tsai-Hill	1st ply	34,329.0	17.54mm
	Tsai-Wu	1st ply	33,726.0	17.23mm
Lamination III	Max. stress	3rd ply	42,374.0	21.21mm
	Max. strain	3rd ply	43,487.0	21.76mm
	Hoffman	3rd ply	42,962.0	21.50mm
	Tsai-Hill	3rd ply	44,155.0	22.10mm
	Tsai-Wu	3rd ply	43,388.0	21.72mm
Lamination IV	Max. stress	3rd ply	42,535.0	21.20mm
	Max. strain	3rd ply	43,651.0	21.76mm
	Hoffman	3rd ply	43,125.0	21.49mm
	Tsai-Hill	3rd ply	44,322.0	22.09mm
	Tsai-Wu	3rd ply	43,552.0	21.71mm

Above results are compared with water reaction load information used for the design of the bottom plate of seaplanes, when they land on the water surface, using the rules shown in FAR Part 25. In the regulations, two different wave impact pressure loads are defined: one is for the step landing case and another is for the bow and stern landing cases. Between them, wave impact pressure load for the step landing depicts the lateral pressure load applied to the exemplified plate in this paper.

Eqn. 19 shows the water reaction load factors for seaplanes when they step-land on the water surface according to the FAR Part 25C-527.1 (FAR) (1970).

$$n_w \frac{C_1 V_{so}^2}{\left(Tan^{2/3}\beta\right)\times W^{1/3}} \tag{19}$$

Where n_w is equal to the water reaction divided by seaplane weight, C_1 is empirical seaplane operations factor equal to 0.012, V_{so} is seaplane landing speed in knots, β is angle of dead rise of the bottom plate, and W is seaplane design landing weight in pounds.

Having substituted the design specification of the small WIG craft considered in this paper, V_{so} and β are equal to 43.2 knots and 22°, into Eqn.19 the water reaction load factor of 3.0 can be obtained. From this load factor, with a safety factor of 1.5, wave impact pressure load of 17,955.7 Pa for the bottom plate of seaplane when it does step landing can be obtained. Through a comparison between the first-ply-failure and FAR regulations' results, it is judged that the proposed design of the bottom laminated plates provides enough structural integrity by securing a greater load resistance capability.

6. Concluding Remarks

In this paper the first-ply-failure analysis is applied to the bottom laminated plate of the WIG craft where direct contact with the water surface occurs during step landing.

Use of different failure criteria produces the design strength and stiffness ranges of the laminated plates in terms of lower and upper bounds. Additionally, the stacking sequences of the laminated plates are varied to investigate the effect of different laminations to the strength and stiffness of the laminated plates. These numerical findings are compared with wave impact pressure load calculated according to the FAR(Federal Aviation Regulations) Part 25 for seaplanes. The comparison reveals that the proposed structural design of the laminated plates is satisfying by acquiring a greater load resistance capability.

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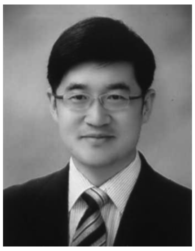

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