Estimating BP Decoding Performance of Moderate-Length Irregular LDPC Codes with Sphere Bounds

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ABSTRACT

This paper estimates belief-propagation (BP) decoding performance of moderate-length irregular low-density parity-check (LDPC) codes with sphere bounds. We note that for moderate-length $(10^3 \le N \le 4\times 10^3)$ irregular LDPC codes, BP decoding performance, which is much worse than maximum likelihood (ML) decoding performance, is well matched with one of loose upper bounds, i.e., sphere bounds.

We introduce the sphere bounding technique for particular codes, not average bounds. The sphere bounding estimation technique is validated by simulation results. It is also shown that sphere bounds and BP decoding performance of irregular LDPC codes are very close at bit-error-rates (BERs) P_b of practical importance $(10^{-5} \le P_b \le 10^{-4})$.

Key Words : Sphere Bounds, Irregular LDPC Codes, BP Decoding, ML Decoding, Upper Bounds

Ⅰ. Introduction

Iterative decoding[1-5] represents a great advancement in communications theory because of their excellent performance. Low-density parity-check (LDPC) codes[1,2] are preferred because of efficient parallel hardware implementation as well as their excellent performance. Numerous simulations and bounds have demonstrated their remarkable performance.

Especially the belief-propagation (BP)[6] decoding performance of irregular LDPC codes of long block length $(N \ge 10^5)$ is near Shannon limit [7]. It can be conjectured that BP decoding performance of irregular LDPC codes of long block length $(N \ge 10^5)$ is close to maximum-likelihood (ML) decoding performance. For moderate-lengths $(10^3 \le N \le 4\times 10^3)$, however, it is generally believed that BP decoding performance of irregular LDPC codes is far from ML decoding performance. This motivates us to find one of loose upper bounds for estimating BP decoding performance. we introduce the sphere bounding technique for particular codes, not average bounds. It is also shown that sphere bounds[8] are well matched to the BP decoding performance for this purpose.

The remainder of this paper is organized as follows. In Section II, we describe sphere bounds used in this paper. In Section III, we present sphere bounds for irregular LDPC codes and BP decoding simulation results. In Section IV, we conclude the paper.

Ⅱ. Sphere Bounds

Consider a linear binary (N, K) block code C, where N is the codeword length and K the information frame length.

For a given code C, d is the Hamming weight of a code word, d_{min} is the minimum distance of the code and $Q(\cdot)$ is the complementary unit variance Gaussian distribution function. The sphere bound[8] on bit-error-rate (BER) with ML code word decoding is given by

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$$P_b \leq \sum_{d=d_{\min}}^{N-K+1} \min\{e^{-E(c,d,\beta)}, e^{Ng(\delta)}Q(\sqrt{2cd})\} \quad (1)$$

In (1), the exponent $E(c,d,\beta)$ is defined as

$$E(c,d,\beta) \equiv -g(\delta) + \frac{1}{2}ln[\beta+(1-\beta)e^{2g(\delta)}] + \delta\beta c \quad (2)$$

where the optimum β is

$$\beta \equiv \frac{2\delta c-(1-e^{-2g(\delta)})}{2\delta c(1-e^{-2g(\delta)})}. \quad (3)$$

Parameters are defined as $\delta \equiv d/N$, $c \equiv r(E_b/N_0)$ with E_b being the energy per information bit and N_0 being the one-sided noise spectral density, and

$$g(\delta) \equiv \frac{1}{N}\ln\left\{\sum_w \frac{w}{K} A_{w,d}\right\}. \quad (4)$$

In (4), the input-output weight distribution (IOWD) $A_{w,d}$ denotes the number of code words for an input sequence weight w and output code word weight d. In order to calculate this sphere bound to a particular code, $A_{w,d}$ should be obtained for that particular code, which is usually very complicated. Therefore, an upper bound is obtained using an ML estimated IOWD[9], which is given by

$$\widehat{A_{w,d}} = \binom{K}{w}\frac{k}{N_s} \quad (5)$$

where k is the number of codewords with the Hamming weight d for input sequences of the Hamming weight w among N_s generated sample codewords. In order to calculate an ML estimated input-output weight distribution, a specific encoder is required. Using Gaussian elimination, we can easily obtain LDPC encoders. Sample codewords are randomly generated.

Ⅲ. Simulation Results

We consider irregular LDPC codes with the degree distribution pair which has the maximum variable degree d_v of 9[7]. The block lengths N are 1000, 2000, 3000, and 4000. The rate for these LDPC codes is 1/2. In each case, the maximum allowable number of iterations is 100. A specific encoder is constructed so that BER is given for systematic bits. The parity check matrices were not chosen entirely randomly. The degree-two nodes were made loop-free and all of them correspond to nonsystematic bits. Short cycles were avoided. For an ML estimated IOWD, the number of randomly generated sample codewords N_s is 10000 for each $w=1,2,\cdots,K$.

Figure 1. to 4. show that for moderate-lengths ($10^3 \leq N \leq 4\times10^3$), BP decoding performance, which is much worse than ML decoding performance, is well matched with one of loose upper bounds, i.e., sphere bounds. Generally upper bounds have water-fall shape over low SNR region. Thus a gap between an upper bound and simulation performance is observed. Since an upper bound is bounding on ML decoding performance and BP decoding is sub-optimal, over high SNR region, BP decoding performance is naturally worse than the upper bound. We also observed that sphere bounds estimate BP decoding performance very closely at BERs of practical importance ($10^{-5} \leq P_b \leq 10^{-4}$).

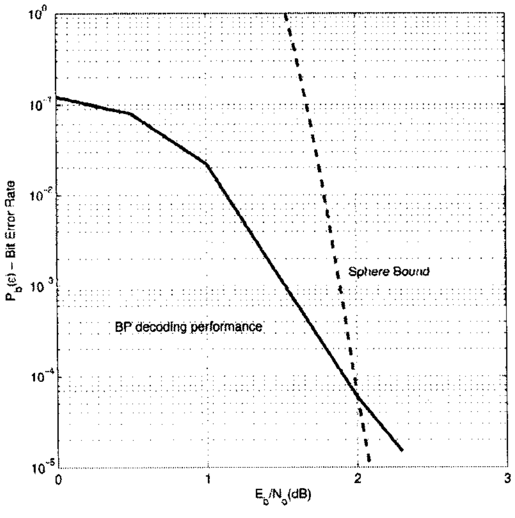

Fig. 1. Estimating BP decoding performance with sphere bound ($N=10^3$)

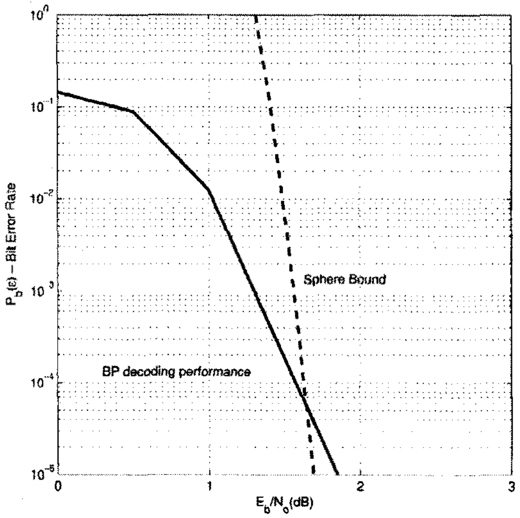

Fig. 2. Estimating BP decoding performance with sphere bound ($N=2\times10^3$)

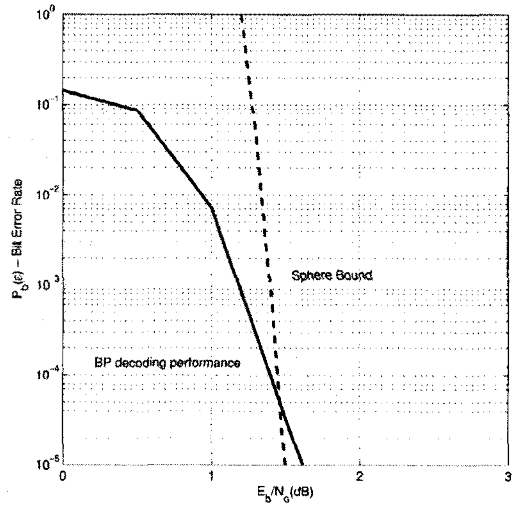

Fig. 3. Estimating BP decoding performance with sphere bound ($N=3\times10^3$)

For $N=10^3$, the sphere bound is about 0.3 dB better that the BP decoding simulation. For $N=2\times10^3$, the sphere bound is about 0.25 dB better that the BP decoding simulation. For $N=3\times10^3$, the sphere bound is about 0.1 dB better that the BP decoding simulation. For $N=4\times10^3$, the sphere bound is about 0.1 dB better that the BP decoding simulation. Thus, the estimation error is within 0.3 dB for all block lengths considered at such BERs.

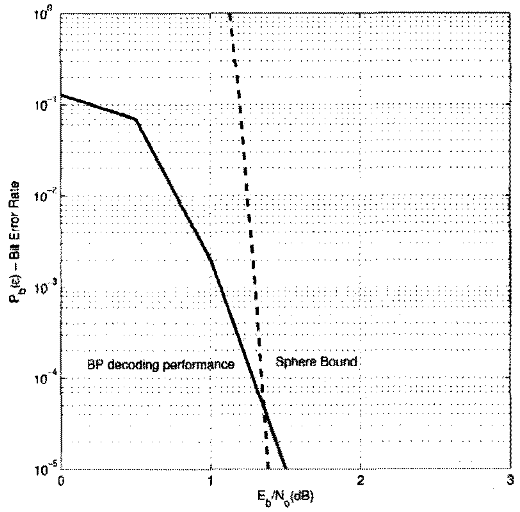

Fig. 4. Estimating BP decoding performance with sphere bound ($N=4\times10^3$)

Ⅳ. Conclusion

This paper has estimated BP decoding performance of moderate-length irregular LDPC codes with sphere bounds. We noted that for moderate-length ($10^3\le N\le 4\times10^3$) irregular LDPC codes, BP decoding performance, which is much worse than ML decoding performance, is well matched with one of loose upper bounds, i.e., sphere bounds. Simulation results were in good agreement with the analytic sphere bounds. It was also shown that sphere bounds and BP decoding performance were very close at BERs of practical importance ($10^{-5}\le P_b\le10^{-4}$). Thus sphere bounds could be used to estimate BP decoding performance of irregular LDPC codes without BP decoding simulation so that we might evaluate code performance fast and easily.

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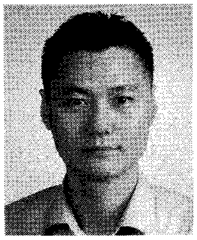

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