Cooperative Multi-relay Scheme for Secondary Spectrum Access

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Abstract

In this paper, we propose a cooperative multi-relay scheme for a secondary system to achieve spectrum access along with a primary system. In the primary network, a primary transmitter (PT) transmits the primary signal to a primary receiver (PR). In the secondary network, N secondary transmitter-receiver pairs (ST-SR) selected by a centralized control unit (CCU) are ready to assist the primary network. In particular, in the first time slot, PT broadcasts the primary signal to PR, which is also received by STs and SRs. At STs, the primary signal is regenerated and linearly combined with the secondary signal by assigning fractions of the available power to the primary and secondary signals respectively. The combined signal is then broadcasted by STs in a predetermined order. In order to achieve diversity gain, STs, SRs and PT will combine received replicas of the primary signal**,** using selection combining technique (SC). We derive the exact outage probability for the primary network as well as the secondary network. The simulation results are presented to verify the theoretical analyses.

Keywords: Cognitive radios, spectrum sharing, cooperative communication, decode-and-forward relaying, selection combining, outage probability

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1. Introduction

Nowadays, due to rapid increase in the number of wireless devices, spectrum scarcity becomes a critical issue. To overcome this problem, Mitola [1] first presented a concept, called cognitive radio (CR), to improve spectrum utilization. In CR networks [2][3][4][5][6], the primary users (PUs) have the right to access the licensed bands at any time, while secondary users (SUs) can only access these bands without causing interference to the primary users. Due to the lower spectrum access priority, SUs must opportunistically sense the presence and absence of the primary users. Once there is no transmission/reception activity of primary users, SUs can use the licensed spectrum. An alternative model, called Property-rights or spectrum leasing, has been proposed in [4][5][6]. In this model, PUs lease a part of the spectrum resources to SUs in exchange for appropriate remuneration.

For the remuneration, the primary link attempts to obtain a better quality of service in terms of achivable rate (or outage probability). It is well-known that cooperative communication can improve the channel capacity and achieve higher diversity gain under Rayleigh fading environment [7][8]. In conventional cooperative protocols [7][8][9][10][11], relay(s) is used to enhance the reliability of data transmission for source-destination link. Thus, in CR networks, the secondary users can play the role as relays and exploit a fraction of leased spectrum to improve the quality of service for primary link as well as to transmit their own data.

Various cooperative relaying schemes for secondary spectrum access have been proposed in [12][13]. In [12], the primary link leases the owned bandwidth for a fraction of time to achieve the benefit from enhanced quality of service. A subset of secondary transmitters cooperates to transmit the primary signal to primary receiver via distributed space-time coding. However, in this proposal, the primary system fully controls the spectrum sharing mechanism. The implementation of this model, using the game theory to optimize the achivable rate and requiring instantaneous channel state information (CSI) of both primary and secondary systems, is a difficult work. In [13], Yang Han *et al* proposed a cooperative decode-and-forward relaying for secondary system. In this model, the secondary transmitter, after decoding successfully the primary signal received from primary transmitter, will combine linearly the primary signal and secondary signal by assigning fractions of the available power. The authors of [13] also derived the approximate expressions to determine the outage probability of the primary and secondary systems.

In this paper, we extend the single-relay model proposed in [13] to multi-relay model. Also, in this proposal, each secondary transmitter will combine linearly the primary signal with its own signal and then broadcasts the combined signal in a predetermined order. Each ST, SR and PR can thus receive many replicas of the primary signal and then attempts to decode it using selection combining technique (SC). The benefit of SC technique, compared with maximal combining technique (MRC), is reduced hardware complexity at each receiver. It just chooses the best signal among all received replicas for further processing and neglects the remaining ones. In this paper, we derive the exact expressions to evaluate the performance of primary network as well as secondary network in terms of outage probability. The simulation results are then presented to validate the theoretical analyses.

The rest of the paper is organized as follows. The system model is described in Section 2 and performance analysis is discussed in Section 3. In Section 4, we will show the simulation results and Section 5 concludes the paper.

2. System Model

In this paper, we assume that the secondary network has a centralized control unit (CCU) to control the secondary network operations. It is also assumed that the CCU is aware of the positions of SUs and it can select several STs from the secondary network to help the data transmission of primary network. In **Fig. 1**, we consider a single pair of primary transmitter (PT) and receiver (PR) and a group of N secondary transmitter-receiver pairs selected by the CCU. Assume that the channels over links $\mathrm{PT} \rightarrow \mathrm{PR}$, $\mathrm{PT} \rightarrow \mathrm{ST}$, $\mathrm{PT} \rightarrow \mathrm{SR}$ and $\mathrm{ST} \rightarrow \mathrm{SR}$ are Rayleigh flat fading. Each node has a single half duplex radio and a single antenna. Due to the half duplex constraint, each transmitter must transmit on separate channels. Hence, for medium access, a time-division channel allocation including $N+1$ time slots is occupied in order to realize orthogonal channels. The CCU will assign these time slots for the selected secondary transmitters as follows.

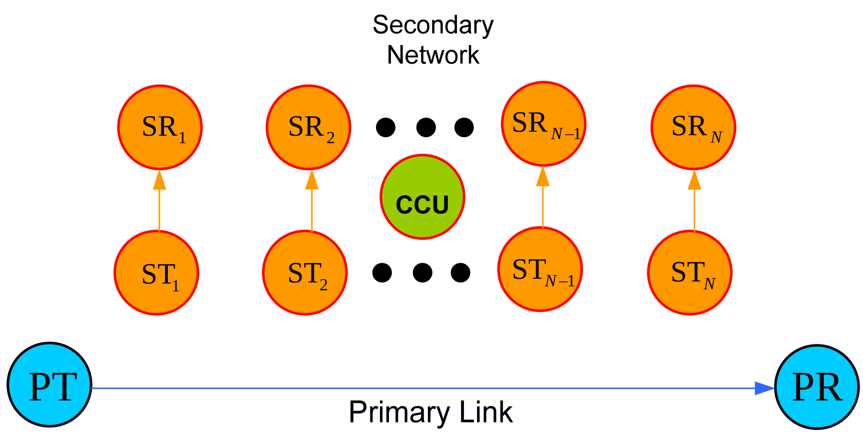

Fig. 1. Cooperative multi-relay scheme for secodary network.

Without loss of generality, we assume that ST_1 is nearest to the PT and ST_N is furthest to PT. In the first time slot, PT transmits the primary signal x_p to PR, which is also received by N STs and N SRs. If ST_1 decodes x_p unsuccessfully, it will keep silent in the second time slot. Otherwise, ST_1 will combine linearly the primary signal x_p and its signal $x_{s,1}$ by assigning fractions α_1 and $1-\alpha_1$ of the available power to x_p and $x_{s,1}$, respectively. Then the combined signal will be broadcasted at the second time slot. Generally, for ST_j $(1 \le j \le N)$, it will combine all received replicas from PT and the previous STs by using selection combining technique (SC). If it decodes correctly, it will combine the primary signal x_p and its signal $x_{s,j}$ with fractions α_j and $1-\alpha_j$ of the available power to x_p and $x_{s,j}$, respectively. ST_j will then broadcast the combined signal in the $(j+1)^{\text{th}}$ time slot. At PR, SC technique is also used to decode the primary signal. Similar to [13], at each SR, interference cancellation is first applied to cancel the primary component and then its secondary signal is retrieved.

In this paper, we assume that all the nodes (primary and secondary) are synchronized. Such synchronization can be achieved through MAC layer control signals. However, the detail of the medium access control policy is beyond the scope of the paper.

3. Performance Analysis

The signal received at node *j* due to the transmission of node *i* is given by

$$r_{i,j} = \sqrt{P} h_{i,j} x_i + \eta_j \tag{1}$$

where AWGN noise η_j at the receiver *j* has variance N_0, $h_{i,j}$ is fading coefficient between node *i* and node *j*, x_i is the signal transmitted by node *i* and *P* is transmit power. In this paper, we assume that the transmit power is same for PT and STs.

From (1), the instantaneous signal to noise ratio (SNR) is determined as follows:

$$\gamma_{i,j} = \frac{P|h_{i,j}|^2}{N_0} = \bar{\gamma}|h_{i,j}|^2 \tag{2}$$

where $\bar{\gamma} = P/N_0$ is average SNR.

In (2), $|h_{i,j}|^2$ has exponential distribution with parameter $\lambda_{i,j}$. To take path loss into account, we can model the variance of channel coefficient between node *i* and node *j* as a function of distance between two nodes [14]. Therefore, the parameter $\lambda_{i,j}$ can be expressed as

$$\lambda_{i,j} = d_{i,j}{}^{\beta} \tag{3}$$

where β is path loss exponent that varies from 2 to 6 and $d_{i,j}$ is the distance between node *i* and node *j*.

3.1 Outage Analysis

We consider the secondary transmitter-receiver pair ST$_j$- SR$_j$ $(1 < j \le N)$. Let us denote D_j as set of secondary transmitters ST$_i$ $(1 \le i < j)$ decoding successfully the primary signal. Note that D_j is a random set and the number of nodes in set D_j is a random variable *n*, i.e., $0 \le n \le j-1$. Assume that $D_j = \{\mathrm{ST}_{k_1}, \mathrm{ST}_{k_2}, ..., \mathrm{ST}_{k_n}\}$, where $1 \le k_1 < k_2 < ... < k_n \le j-1$; and set of secondary transmitters decoding incorrectly the primary signal is $F_j = \{\mathrm{ST}_{l_1}, \mathrm{ST}_{l_2}, ..., \mathrm{ST}_{l_{j-1-n}}\}$ with $1 \le l_1 < l_2 < ... < l_{j-1-n} \le j-1$. For each value of *n*, there are $\binom{j-1}{n}$ possible sets of size *n* and hence, we have total 2^{j-1} possible sets of D_j. Now, we will calculate the probability for each set D_j.

Consider node ST_{k_m} $(1 \le m \le n)$ belonging to set D_j; it is obvious that ST_{k_m} receives *m* replicas of the primary signal: one from PT and (*m*-1) from $\mathrm{ST}_{k_1}, \mathrm{ST}_{k_2}, ..., \mathrm{ST}_{k_{m-1}}$. Because ST_{k_g} $(1 \le g \le m-1)$ decodes successfully the primary signal, it combines linearly the primary signal x_p and its own signal x_{s,k_g}. Therefore, the signal transmitted by ST_{k_g} is given as

$$z_{\mathrm{ST}_{k_g}} = \sqrt{\alpha_{k_g} P} x_p + \sqrt{\left(1-\alpha_{k_g}\right) P} x_{s,k_g} \tag{4}$$

The signal received at ST_{k_m} due to the transmission of ST_{k_g} is

$$r_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} = h_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} z_{\mathrm{ST}_{k_g}} + \eta_{\mathrm{ST}_{k_m}} = \sqrt{\alpha_{k_g} P} h_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} x_p + \sqrt{\left(1-\alpha_{k_g}\right)P} h_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} x_{s,k_g} + \eta_{\mathrm{ST}_{k_m}} \quad (5)$$

Assume that the channel $h_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}$ can be estimated at the primary transmitter ST_{k_m} by using standard preamble-aided channel estimation technique. From (5), the instantaneous SNR can be calculated as

$$\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} = \frac{\alpha_{k_g} P \,|\, h_{ST_{k_g},ST_{k_m}} \,|^2}{\left(1-\alpha_{k_g}\right) P \,|\, h_{ST_{k_g},ST_{k_m}} \,|^2 + N_0} = \frac{\bar{\gamma}\alpha_{k_g} \,|\, h_{ST_{k_g},ST_{k_m}} \,|^2}{\bar{\gamma}\left(1-\alpha_{k_g}\right) |\, h_{ST_{k_g},ST_{k_m}} \,|^2 + 1} \quad (6)$$

In (6), it is noted that ST_{k_m} must have explicit knowledge of the factor α_{k_g}. Now, the instantaneous SNR at the output of the selection combiner in ST_{k_m} is given by

$$\gamma_{\mathrm{ST}_{k_m}} = \max_{g=1,2,\dots,m-1}\left(\gamma_{\mathrm{PT},\mathrm{ST}_{k_m}}, \gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}\right) \quad (7)$$

Therefore, the achievable rate between PT and ST_{k_m} is determined as follows:

$$R_{\mathrm{ST}_{k_m}} = \frac{1}{N+1}\log_2\left(1+\gamma_{\mathrm{ST}_{k_m}}\right) \quad (8)$$

where the factor of $N+1$ accounts for the fact that the overall transmission is split into $N+1$ time slots.

Because decoding at ST_{k_m} is successful, $R_{\mathrm{ST}_{k_m}}$ is larger than target rate R of the system. Consequently, the probability of this case is calculated by

$$\begin{aligned}\Pr\left(R_{\mathrm{ST}_{k_m}} \ge R\right) &= \Pr\left(\gamma_{\mathrm{ST}_{k_m}} \ge \tau\right) = 1-\Pr\left(\max_{g=1,2,\dots,m-1}\left(\gamma_{\mathrm{PT},\mathrm{ST}_{k_m}}, \gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}\right) < \tau\right) \\ &= 1-\Pr\left(\gamma_{\mathrm{PT},\mathrm{ST}_{k_m}} < \tau\right)\prod_{g=1}^{m-1}\Pr\left(\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} < \tau\right)\end{aligned} \quad (9)$$

where $\tau = 2^{(N+1)R} - 1$.

Relying on (2) and (3), the probability $\Pr\left(\gamma_{\mathrm{PT},\mathrm{ST}_{k_m}} < \tau\right)$ in (9) can be given as

$$\Pr\left(\gamma_{\mathrm{PT},\mathrm{ST}_{k_m}} < \tau\right) = \Pr\left(|\, h_{\mathrm{PT},\mathrm{ST}_{k_m}} \,|^2 < \rho\right) = 1-\exp\left(-\lambda_{\mathrm{PT},\mathrm{ST}_{k_m}}\rho\right) \quad (10)$$

where $\rho = \tau / \bar{\gamma}$.

Now, in order to calculate $\Pr\left(\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}} < \tau\right)$ in (9), we must find the cumulative density function (CDF) of the random variable $\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}$. Indeed, using the definition of CDF, we have

$$F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}}(\chi) = \Pr\left(\frac{\overline{\gamma}\,\alpha_{k_g}\,|h_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}|^2}{\overline{\gamma}\left(1-\alpha_{k_g}\right)|h_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}|^2+1} < \chi\right)$$

$$= \begin{cases} 0; & \text{if } \chi < 0 \\ 1-\exp\left(-\dfrac{\lambda_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}\chi}{\overline{\gamma}\left(\alpha_{k_g}-\left(1-\alpha_{k_g}\right)\chi\right)}\right); & \text{if } 0 \le \chi < \dfrac{\alpha_{k_g}}{\left(1-\alpha_{k_g}\right)} \\ 1; & \text{if } \chi \ge \dfrac{\alpha_{k_g}}{\left(1-\alpha_{k_g}\right)} \end{cases} \tag{11}$$

From (10) and (11), (9) is written as

$$\Pr\left(R_{\mathrm{ST}_{k_m}} \ge R\right) = 1-\left(1-\exp\left(-\lambda_{\mathrm{PT},\mathrm{ST}_{k_m}}\rho\right)\right)\prod_{g=1}^{m-1} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}}(\tau) \tag{12}$$

We should note that for the case $m=1$, (12) reduces to $\Pr\left(R_{\mathrm{ST}_{k_1}} \ge R\right) = \exp\left(-\lambda_{\mathrm{PT},\mathrm{ST}_{k_1}}\rho\right)$. Therefore, the probability for each set D_j can be calculated as follows:

$$P\left(D_j\right) = \prod_{m=1}^{n}\Pr\left(R_{\mathrm{ST}_{k_m}} \ge R\right)$$
$$= \exp\left(-\lambda_{\mathrm{ST}_{k_m},\mathrm{PT}}\rho\right)\prod_{m=2}^{n}\left[1-\left(1-\exp\left(-\lambda_{\mathrm{PT},\mathrm{ST}_{k_m}}\rho\right)\right)\prod_{g=1}^{m-1} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{k_m}}}(\tau)\right] \tag{13}$$

As mentioned above, corresponding to a set D_j, we have a set F_j. In the following, the probability of each set F_j will be derived.

Consider node ST_{l_b} $\left(1 \le b \le j-1-n\right)$ belonging to the set F_j; it is assumed that $k_{m-1} < l_b < k_m \left(1 < m < n\right)$. In this case, ST_{l_b} receives *m* replicas of the primary signal: one from PT and (*m*-1) from $\mathrm{PT}_{k_1}, \mathrm{PT}_{k_2}, ..., \mathrm{PT}_{k_{m-1}}$. Using a similar method as above, the probability for the unsuccessful decoding at ST_{l_b} is given by

$$\Pr\left(R_{\mathrm{ST}_{l_b}} < R\right) = \Pr\left(\max_{g=1,2,...,m-1}\left(\gamma_{\mathrm{PT},\mathrm{ST}_{l_b}}, \gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{l_b}}\right) < \tau\right) = \left(1-\exp\left(-\lambda_{\mathrm{PT},\mathrm{ST}_{l_b}}\rho\right)\right)\prod_{g=1}^{m-1} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{l_b}}}(\tau) \tag{14}$$

In case that $l_b < k_1$, ST_{l_b} only receives the primary signal from PT, (14) reduces to

$$\Pr\left(R_{\mathrm{ST}_{l_b}} < R\right) = 1-\exp\left(-\lambda_{\mathrm{PT},\mathrm{ST}_{l_b}}\rho\right) \tag{15}$$

In addition, in case that $l_b > k_n$, node ST_{l_b} receives replicas of primary signal from all nodes belonging to the set D_j, hence (14) can be rewritten as

$$\Pr\left(R_{\mathrm{ST}_{l_b}} < R\right) = \left(1 - \exp\left(-\lambda_{\mathrm{PT,ST}_{l_b}}\rho\right)\right)\prod_{g=1}^{n} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{l_b}}}(\tau) \tag{16}$$

From (14)-(16), the probability for each set F_j can be expressed generally as follows:

$$P\left(F_j\right) = \prod_{b=1}^{j-n-1}\Pr\left(R_{\mathrm{ST}_{l_b}} < R\right) = \prod_{b=1}^{j-n-1}\left[\left(1 - \exp\left(-\lambda_{\mathrm{PT,ST}_{l_b}}\rho\right)\right)\prod_{g=1}^{m-1} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_{l_b}}}(\tau)\right] \tag{17}$$

Similar to the case of $l_b > k_n$, it is obvious that ST_j and SR_j also receive signals from all nodes belonging to the set D_j, hence the probability that the decoding at nodes ST_j and SR_j is unsuccessful is calculated respectively as

$$P_{\mathrm{ST}_j}^{\mathrm{out},x_p} = \left(1 - \exp\left(-\lambda_{\mathrm{PT,ST}_j}\rho\right)\right)\prod_{g=1}^{n} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{ST}_j}}(\tau) \tag{18}$$

$$P_{\mathrm{SR}_j}^{\mathrm{out},x_p} = \left(1 - \exp\left(-\lambda_{\mathrm{PT,SR}_j}\rho\right)\right)\prod_{g=1}^{n} F_{\gamma_{\mathrm{ST}_{k_g},\mathrm{SR}_j}}(\tau) \tag{19}$$

Note that with $j = 1$, (18) and (19) reduce to $P_{\mathrm{ST}_1}^{\mathrm{out},x_p} = 1 - \exp\left(-\lambda_{\mathrm{PT,ST}_1}\rho\right)$ and $P_{\mathrm{SR}_1}^{\mathrm{out},x_p} = 1 - \exp\left(-\lambda_{\mathrm{PT,SR}_1}\rho\right)$, respectively. Once ST_j decodes correctly the primary signal, the combined signal of the primary signal and its own signal is transmitted and given by

$$z_{\mathrm{ST}_j} = \sqrt{\alpha_j P}x_p + \sqrt{\left(1-\alpha_j\right)P}x_{s,j} \tag{20}$$

The received signal at node SR_j due to the transmission of ST_j is

$$r_{\mathrm{ST}_j,\mathrm{SR}_j} = h_{\mathrm{ST}_j,\mathrm{SR}_j} z_{\mathrm{ST}_j} + n_{\mathrm{SR}_j} = \sqrt{\alpha_j P}h_{\mathrm{ST}_j,\mathrm{SR}_j}x_p + \sqrt{\left(1-\alpha_j\right)P}h_{\mathrm{ST}_j,\mathrm{SR}_j}x_{s,j} + n_{\mathrm{SR}_j} \tag{21}$$

As discussed in [13], if SR_j also decodes successfully the primary signal, the component $\sqrt{\alpha_j P}h_{\mathrm{ST}_j,\mathrm{SR}_j}x_p$ can be canceled out from (21) and we have

$$r'_{\mathrm{ST}_j,\mathrm{SR}_j} = \sqrt{\left(1-\alpha_j\right)P}h_{\mathrm{ST}_j,\mathrm{SR}_j}x_{s,j} + n_{\mathrm{SR}_j} \tag{22}$$

Thus, the achievable rate between ST_j-SR_j link is written as follows:

$$R_{\mathrm{ST}_j,\mathrm{SR}_j} = \frac{1}{N+1}\log_2\left(1 + \bar{\gamma}\left(1-\alpha_j\right)|h_{\mathrm{ST}_j,\mathrm{SR}_j}|^2\right) \tag{23}$$

From (23), the probability that node SR_j retrieves $x_{s,j}$ incorrectly is calculated by

$$P_{\mathrm{SR}_j}^{out,x_{s,j}} = \Pr\left(R_{\mathrm{ST}_j,\mathrm{SR}_j} < R\right) = 1 - \exp\left(-\frac{\lambda_{\mathrm{ST}_j,\mathrm{SR}_j}\rho}{\left(1-\alpha_j\right)}\right) \tag{24}$$

Note that SR_j can not receive $x_{s,j}$ successfully if SR_j or ST_j is not able to decode x_p correctly or ST_j-SR_j link is in outage. Therefore, from (13), (17), (18), (19) and (24), the total outage probability at SR_j is given by

$$P_{\mathrm{SR}_j}^{out} = \sum_{D_j} P\left(D_j\right)\times P\left(F_j\right)\times\left[1 - \left(1 - P_{\mathrm{ST}_j}^{\mathrm{out},x_p}\right)\left(1 - P_{\mathrm{SR}_j}^{\mathrm{out},x_p}\right)\left(1 - P_{\mathrm{SR}_j}^{out,x_{s,j}}\right)\right] \tag{25}$$

Now, defining the outage probability of secondary system as the probability that all secondary pairs are in outage, we have

$$P_{\text{Secondary}}^{out} = \prod_{j=1}^{N} P_{\text{SR}_j}^{out} \tag{26}$$

Consider the primary link; we denote set of STs having the successful decoding and unsuccessful decoding of the primary signal by D and F, respectively. Assume that $D = \left\{ \text{ST}_{k_1}, \text{ST}_{k_2}, ..., \text{ST}_{k_n} \right\}$ and $F = \left\{ \text{ST}_{l_1}, \text{ST}_{l_2}, ..., \text{ST}_{l_{N-n}} \right\}$, where $0 \le n \le N$, $1 \le k_1 < ... < k_n \le N$ and $1 \le l_1 < ... < l_{N-n} \le N$. Similarly to (13), (17) and (18), we obtain

$$P(D) = \prod_{m=1}^{n} \left[1 - \left(1 - \exp\left(-\lambda_{\text{ST}_{k_m},\text{PR}} \rho \right) \right) \prod_{g=1}^{m-1} F_{\gamma_{\text{ST}_{k_g},\text{PR}}}(\tau) \right] \tag{27}$$

$$P(F) = \prod_{b=1}^{N-n} \left[\left(1 - \exp\left(-\lambda_{\text{PT},\text{ST}_{l_b}} \rho \right) \right) \prod_{g=1}^{m-1} F_{\gamma_{\text{ST}_{k_g},\text{ST}_{l_b}}}(\tau) \right] \tag{28}$$

$$P_{\text{PR}}^{\text{out}} = \left(1 - \exp\left(-\lambda_{\text{PT,PR}} \rho \right) \right) \prod_{g=1}^{n} F_{\gamma_{\text{ST}_{k_g},\text{PR}}}(\tau) \tag{29}$$

Using the theorem on total probability, the average outage probability of primary system is given by

$$P_{\text{Primary}}^{out} = \sum_{D} P(D) \times P(F) \times P_{\text{PR}}^{\text{out}} \tag{30}$$

3.2 Diversity Order

Proposition 1: Node ST_j $\left(1 \le j \le N\right)$ assigns fractions α_j and $1-\alpha_j$ of the transmit power P to primary signal and its own signal. If α_j satisfies the condition $\frac{\alpha_j}{1-\alpha_j} \le \tau$, ST_j is called a non-diversity relay. Among N selected STs, if there are $q\left(0 \le q \le N\right)$ non-diversity relays, the achievable diversity gain of the primary network is $N+1-q$.

Proof: We assume that $D = \left\{ \text{ST}_{k_1}, \text{ST}_{k_2}, ..., \text{ST}_{k_n} \right\}$, $F = \left\{ \text{ST}_{l_1}, \text{ST}_{l_2}, ..., \text{ST}_{l_{N-n}} \right\}$, and there are $w\left(0 \le w \le n\right)$ non-diversity relays in set D and $q-w$ non-diversity relays in set F. Without loss of generality, assume that w non-diversity relays belonging to set D and $q-w$ non-diversity relays belonging to set F are $\left\{ \text{ST}_{k_1}, \text{ST}_{k_2}, ..., \text{ST}_{k_w} \right\}$ and $\left\{ \text{ST}_{l_1}, \text{ST}_{l_2}, ..., \text{ST}_{l_{q-w}} \right\}$, respectively. From (11) and (27), at high SNR regime $\overline{\gamma}$, $P(D)$, $P(F)$ and $P_{\text{PR}}^{\text{out}}$ in (27), (28) and (29) can be approximated as

$$P(D) \approx 1 \tag{31}$$

$$P(F) \approx \begin{cases} c_{1,D} \left(\overline{\gamma} \right)^{-(N-n)}; & \text{if } m \le w+1 \\ c_{2,D} \left(\overline{\gamma} \right)^{-(N-n)(m-w)}; & \text{if } m > w+1 \end{cases} \tag{32}$$

$$P_{\text{PR}}^{\text{out}} \approx c_{3,D} \left(\overline{\gamma} \right)^{-(n-w+1)} \tag{33}$$

where $c_{1,D}=\prod_{b=1}^{N-n}\lambda_{\mathrm{PT,ST}_{l_b}}\tau$, $c_{2,D}=\prod_{b=1}^{N-n}\lambda_{\mathrm{PT,ST}_{l_b}}\tau\prod_{g=w+1}^{m-1}\frac{\lambda_{\mathrm{ST}_{k_g},\mathrm{ST}_{l_b}}\tau}{\alpha_{k_g}-\left(1-\alpha_{k_g}\right)\tau}$ and

$c_{3,D}=\lambda_{\mathrm{PT,PR}}\tau\prod_{g=w+1}^{n}\frac{\lambda_{\mathrm{ST}_{k_g},\mathrm{PR}}\tau}{\alpha_{k_g}-\left(1-\alpha_{k_g}\right)\tau}$.

Relying on (31)-(33), we can approximate the outage probability of primary link for each possible set of D as

$$P(D)\times P(F)\times P_{\mathrm{PR}}^{\mathrm{out}}\approx\begin{cases}c_{1,D}c_{3,D}\left(\overline{\gamma}\right)^{-(N-w+1)}; & \text{if } m\le w+1\\ c_{1,D}c_{3,D}\left(\overline{\gamma}\right)^{-(N-n)(m-w)-(n-w+1)}; & \text{if } m>w+1\end{cases}\tag{34}$$

Furthermore, in case that all secondary transmitters decode sucessfully or $D=\{\mathrm{ST}_1,\mathrm{ST}_2,\ldots,\mathrm{ST}_N\}$ and $F=\{\phi\}$, the outage probability of this case is given by

$$P(D)\times P(F)\times P_{\mathrm{PR}}^{\mathrm{out}}=\left(1-\exp\left(-\lambda_{\mathrm{PT,PR}}\rho\right)\right)\prod_{j=1}^{N}\left(1-\exp\left(-\lambda_{\mathrm{ST}_j,\mathrm{PR}}\rho\right)\right)\tag{35}$$

At high SNR $\overline{\gamma}$, (35) can be approximated by

$$P(D)\times P(F)\times P_{\mathrm{PR}}^{\mathrm{out}}\approx c_{4,D}\left(\overline{\gamma}\right)^{-(N-q+1)}\tag{36}$$

where $c_{4,D}=\lambda_{\mathrm{PT,PR}}\tau\prod_{j=q+1}^{N}\left(\lambda_{\mathrm{ST}_j,\mathrm{PR}}\tau\right)$.

Futhermore, it is easy to see that $(N-n)(m-w)+n-w+1\ge N-w+1\ge N-q+1$, hence, from (30), (34) and (36), we can approximate $P_{\mathrm{Primary}}^{out}$ at high SNR $\overline{\gamma}$ as

$$P_{\mathrm{Primary}}^{out}=\sum_{D}P(D)\times P(F)\times P_{\mathrm{PR}}^{\mathrm{out}}\approx c\left(\overline{\gamma}\right)^{-(N-q+1)}\tag{37}$$

where c is a constant.
Finally, the diversity order is determined by [11]

$$\text{Diversity order}=\lim_{\overline{\gamma}\to\infty}-\frac{\log\left(P_{\mathrm{Primary}}^{out}\right)}{\log\left(\overline{\gamma}\right)}=\lim_{\overline{\gamma}\to\infty}-\frac{\log\left(c\left(\overline{\gamma}\right)^{-(N-q+1)}\right)}{\log\left(\overline{\gamma}\right)}=N-q+1\tag{38}$$

Proposition 2: For each ST_j-SR_j link, the achivable diversity order is 1 and hence the diversity gain of the secondary network is N.

Proof: As in Section 3.1, we define $D_j=\{\mathrm{ST}_{k_1},\mathrm{ST}_{k_2},\ldots,\mathrm{ST}_{k_n}\}$ and $F_j=\{\mathrm{ST}_{l_1},\mathrm{ST}_{l_2},\ldots,\mathrm{ST}_{l_{j-1-n}}\}$. It is assumed that there are $w\,(0\le w\le n)$ non-diversity relays in set D_j and without loss of generality, we can assume that they are $\{\mathrm{ST}_{k_1},\mathrm{ST}_{k_2},\ldots,\mathrm{ST}_{k_w}\}$. At first, we approximate (13), (17) at high $\overline{\gamma}$ as

$$P\left(D_j\right) \approx 1 \tag{39}$$

$$P\left(F_j\right) \approx \begin{cases} c_{1,D_j}\left(\overline{\gamma}\right)^{-(j-n-1)}; & \text{if } m \le w+1 \\ c_{2,D_j}\left(\overline{\gamma}\right)^{-(j-n-1)(m-w)}; & \text{if } m > w+1 \end{cases} \tag{40}$$

where $c_{1,D_j} = \prod_{b=1}^{j-n-1} \lambda_{\mathrm{PT,ST}_{l_b}} \tau$ and $c_{2,D_j} = \prod_{b=1}^{j-n-1} \lambda_{\mathrm{PT,ST}_{l_b}} \tau \prod_{g=w+1}^{n} \frac{\lambda_{\mathrm{ST}_{k_g},\mathrm{ST}_{l_b}} \tau}{\alpha_{k_g} - \left(1-\alpha_{k_g}\right)\tau}$.

Next, from (18), (19) and (24), the expression $1-\left(1-P_{\mathrm{ST}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{out,x_{s,j}}\right)$ in (25) can be calculated approximately by

$$1-\left(1-P_{\mathrm{ST}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{out,x_{s,j}}\right) \approx \begin{cases} c_{3,D_j}\left(\overline{\gamma}\right)^{-1}; & \text{if } m \le w+1 \\ c_{4,D_j}\left(\overline{\gamma}\right)^{-1}; & \text{if } m > w+1 \end{cases} \tag{41}$$

where $c_{3,D_j} = \left(\lambda_{\mathrm{PT,ST}_j} + \lambda_{\mathrm{PT,SR}_j} + \frac{\lambda_{\mathrm{ST}_j,\mathrm{SR}_j}}{1-\alpha_j}\right)\tau$ and $c_{4,D_j} = \frac{\lambda_{\mathrm{ST}_j,\mathrm{SR}_j}\tau}{1-\alpha_j}$.

From (39)-(41), the outage probability for each possible set of D_j is approximated as

$$P\left(D_j\right)P\left(F_j\right)\left[1-\left(1-P_{\mathrm{ST}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{out,x_{s,j}}\right)\right] \approx \begin{cases} c_{1,D_j}c_{3,D_j}\left(\overline{\gamma}\right)^{-(j-n)}; & \text{if } m \le w+1 \\ c_{2,D_j}c_{4,D_j}\left(\overline{\gamma}\right)^{-(j-n-1)(m-w)-1}; & \text{if } m > w+1 \end{cases} \tag{42}$$

Note that when all STs, i.e., $\mathrm{ST}_1, \mathrm{ST}_2, ..., \mathrm{ST}_{j-1}$ decode the primary signal successfully, (42) is calculated as follows:

$$P\left(D_j\right)P\left(F_j\right)\left[1-\left(1-P_{\mathrm{ST}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{\mathrm{out},x_p}\right)\left(1-P_{\mathrm{SR}_j}^{out,x_{s,j}}\right)\right] \approx \begin{cases} c_{1,D_j}c_{3,D_j}\left(\overline{\gamma}\right)^{-1}; \text{if } m \le w+1 \\ c_{2,D_j}c_{4,D_j}\left(\overline{\gamma}\right)^{-1}; \text{if } m > w+1 \end{cases} \tag{43}$$

For all remaining cases of set D_j, we have $(j-n-1)(m-w)+1 \ge j-n \ge 1$, hence the outage probability for ST_j-SR_j link $P_{\mathrm{SR}_j}^{out}$ in (25) can be approximated at high $\overline{\gamma}$ by

$$P_{\mathrm{SR}_j}^{out} \approx c_j\left(\overline{\gamma}\right)^{-1} \tag{44}$$

where c_j is a constant.

Therefore, the outage probability of secondary system in (26) can be calculated approximately as

$$P_{\mathrm{Secondary}}^{out} = \prod_{j=1}^{N} P_{\mathrm{SR}_j}^{out} \approx \left(\overline{\gamma}\right)^{-N} \prod_{j=1}^{N} c_j \tag{45}$$

Applying the definition of diversity order [11], the diversity gains of ST_j-SR_j link and secondary system are easily determined by 1 and N, respectively.

4. Simulation Results

In this section, we use the Monte-Carlo simulation to verify theoretical results. We assume that the distance between PT and PR, and the distances between ST_j and SR_j are normalized to 1. In addition, STs are placed on the line between PT and PR such that the distance between ST_j and PT equals $\frac{j}{N+1}$. We set the path loss exponent β to 3 and the target rate R to 1 in all simulations.

Fig. 2 shows the outage probability of the primary network as a function of average SNR $\bar{\gamma}$ in dB. In this simulation, the number of selected STs N varies from 0 to 3 and the fractions of transmit power to primary signal are set to 0.95 for all STs. In case of N=0, PT transmits the primary signal to PR directly, without the help of STs. It can be seen from the figure that the simulation and theoretical results match very well with each other. In addition, for all cases, the performance of primary link employing cooperative transmission from STs is better than that of direct transmission (N=0) at high SNR regime.

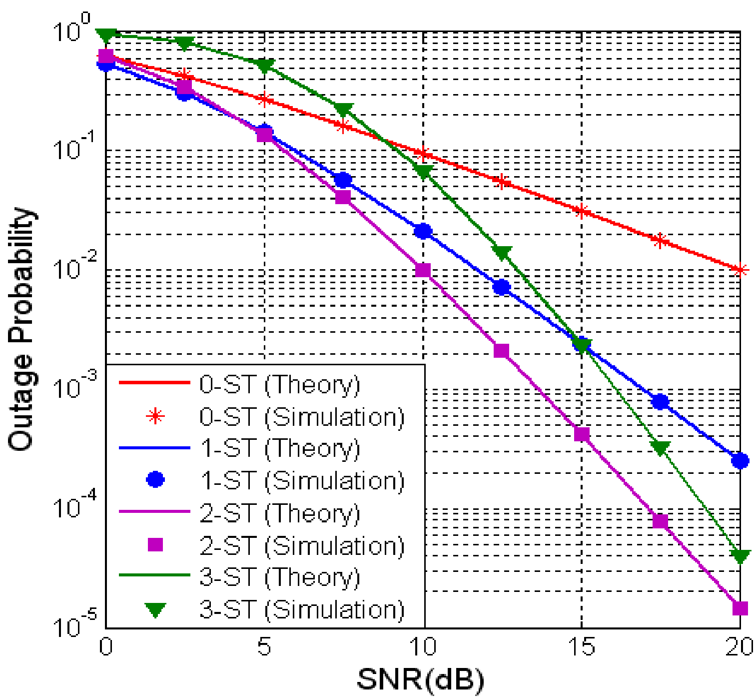

Fig. 2. The outage probability for primary network.

In **Fig. 3**, we investigate the effect of the fraction of transmit power to primary signal assigned by STs on the performance of primary network. In this figure, the results are presented by theoretical calculations. It can be observed that in case that $[\alpha_1 \ \alpha_2] = [0.8 \ 0.8]$, the performance is worst. It is due to the fact that in this case $\tau > \frac{\alpha_1}{1-\alpha_1}$ and $\tau > \frac{\alpha_2}{1-\alpha_2}$, hence the achievable diversity order is equal to 1, follows the statement of proposition 1. For the case that $[\alpha_1\ \alpha_2]=[0.9\ 0.8]$ (or $[\alpha_1\ \alpha_2]=[0.8\ 0.9]$), due to $\frac{\alpha_1}{1-\alpha_1}>\tau\left(\text{ or } \frac{\alpha_2}{1-\alpha_2}>\tau\right)$, the diversity gain increases 1 from the help of ST_1 (ST_2). In the last case $[\alpha_1\ \alpha_2]=[0.9\ 0.9]$, the performance is best because the diversity gain of 3 can be achievable.

In **Fig. 4** and **5**, the outage performances of the secondary network are evaluated and compared. In **Fig. 4**, we assume the CCU selects 3 STs for cooperation. It is also assumed that each selected ST_j, j=1,2,3, uses the same fraction α of transmit power to primary signal. As we can see, when we decrease the value of α, the performance of secondary network increases. It is because the fraction $1-\alpha$ of transmit power assigned to secondary signals increases with decreasing of α. In addition, the simulation and theoretical results again match very well.

The theoretical results are presented in **Fig. 5** to determine the diversity order of the secondary network. In this figure, the number of selected STs is set to 4, while fraction α changes. It can be observed that for all values of α, the diversity order does not change and equals to 4. This is in accordance to the proposition 2.

In **Fig. 6**, **7**, and **8**, we present the outage probability of each ST-SR pair. In particular, we assume that 3 STs are selected to help PT to transmit the primary signal to PR. As expected, the results from simulation and theory are in excellent agreement. It can be seen from these figures that the performance of ST_j-SR_j pair increases with decreasing $\alpha_j\ (1\le j\le 3)$. Furthermore, the performance of ST_1 –SR_1 pair does not depend on α_2 and α_3 while that of ST_2–SR_2 pair just depends on α_1 and that of ST_3-SR_3 pair depends on α_1 and α_2.

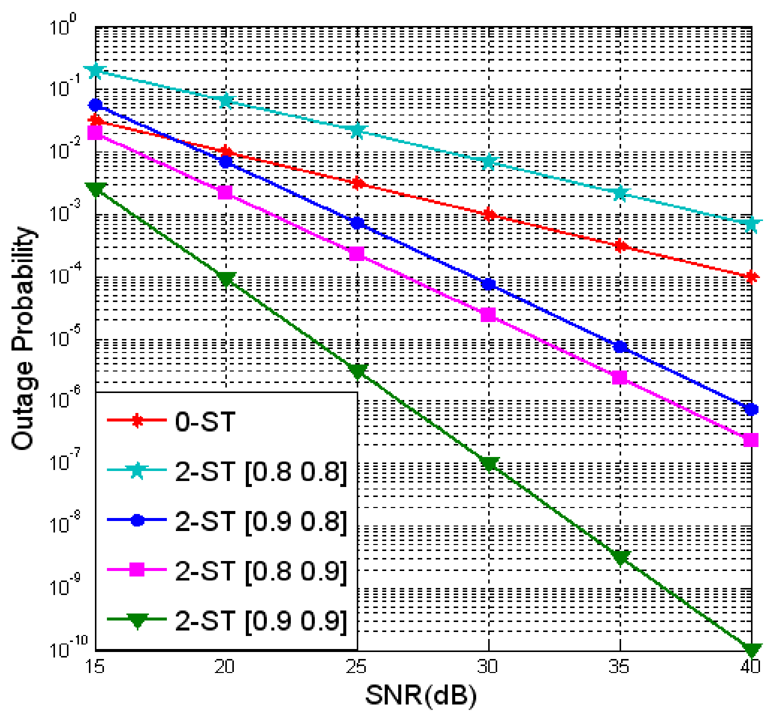

Fig. 3. The outage probability for primary network.

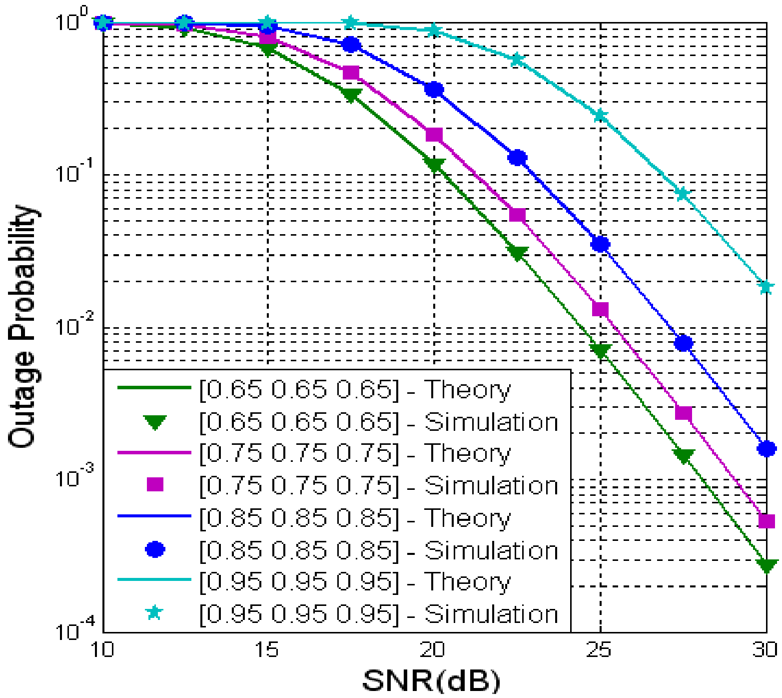

Fig. 4. The outage probability for secondary network.

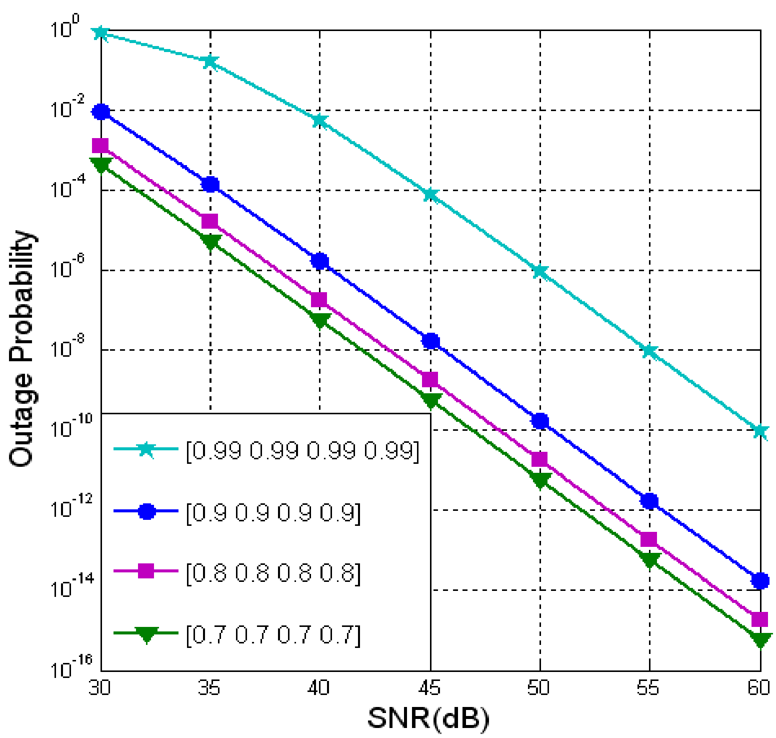

Fig. 5. The outage probability for secondary network.

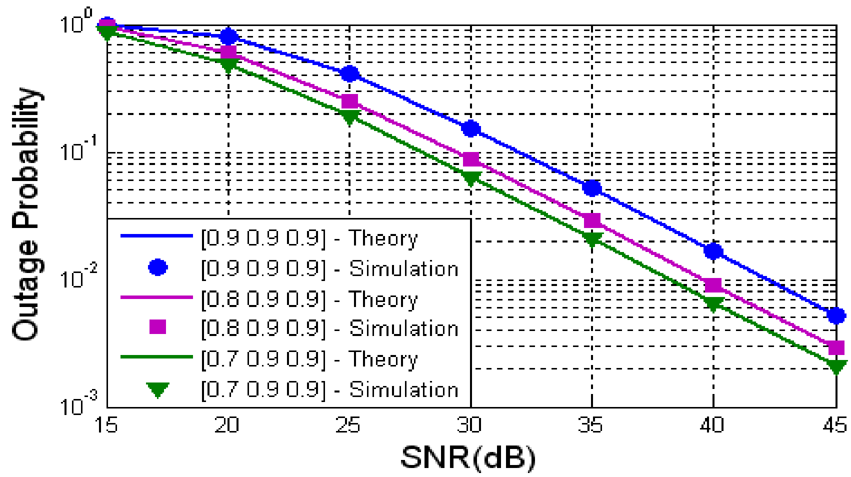

Fig. 6. The outage probability of the first ST-SR pair.

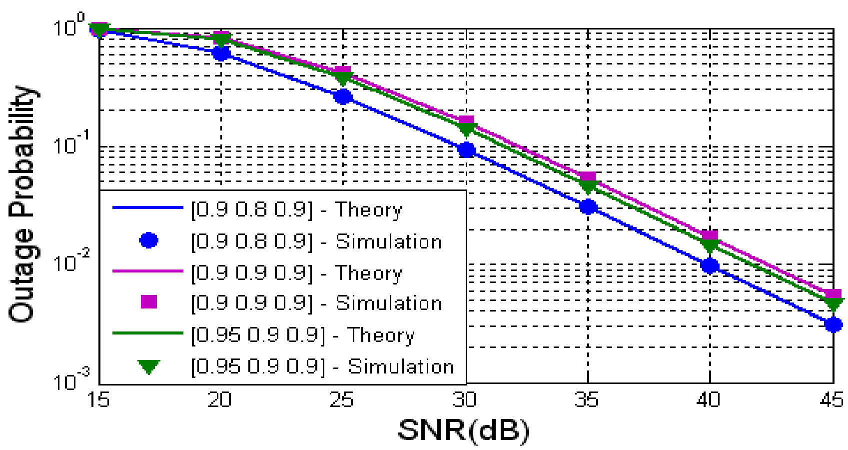

Fig. 7. The outage probability of the second ST-SR pair.

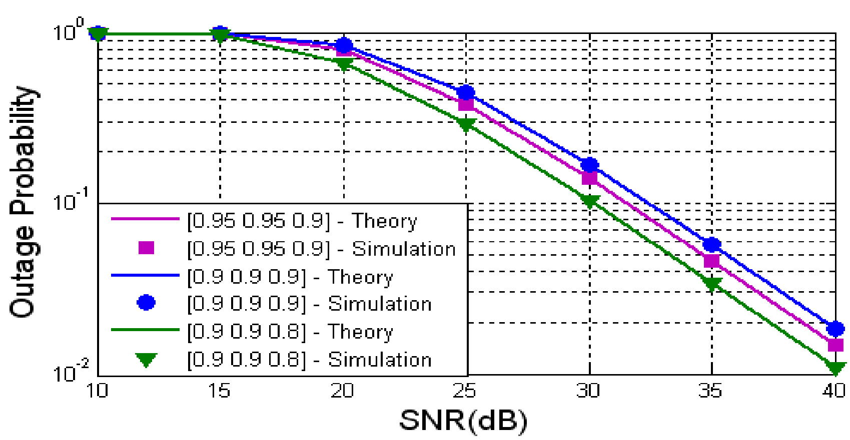

Fig. 8. The outage probability of the third ST-SR pair.

5. Conclusions

In this paper, a cooperative multi-relay scheme for the secondary system was proposed. We also presented the exact outage probability expressions for the primary network as well as secondary network. Then, the validity was verified by a variety of Monte-Carlo simulations. The simulation results showed that the performance of the primary network employing the proposed protocol was enhanced significantly when compared with the system that does not cooperate with secondary nodes. Furthermore, cooperation of secondary users not only improves the outage performance but also achieves higher diversity order for secondary system.

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