Quasi-Orthogonal Space-Time Block Codes Designs Based on Jacket Transform

Wei Song, Moon Ho Lee, Mustafa M. Matalgah, and Ying Guo

Abstract: **Jacket matrices[1], motivated by the complex Hadamard matrix, have played important roles in signal processing, communications, image compression, cryptography, etc. In this paper, we suggest a novel approach to design a simple class of space-time block codes (STBCs) to reduce its peak-to-average power ratio. The proposed code provides coding gain due to the characteristics of the complex Hadamard matrix, which is a special case of Jacket matrices. Also, it can achieve full rate and full diversity with the simple decoding. Simulations show the good performance of the proposed codes in terms of symbol error rate. For generality, a kind of quasi-orthogonal STBC may be similarly designed with the improved performance.**

Index Terms: **Discrete Fourier transform (DFT) matrix, full rate, Hadamard transform, Jacket matrix, multiple-input multiple-output (MIMO) system, quasi-orthogonal space-time block code (STBC).**

I. INTRODUCTION

The multiple-input and multiple-output (MIMO) communication systems provide more potential capacity gains than that of the single-antenna wireless communication system [1]. To approach the capacity of the MIMO system, many types of space-time block codes (STBCs) have been intensively studied [1]–[5]. Since the orthogonal STBCs have suggested a linear decoding complexity, several elegant designs of STBCs have been reported by using group and representation theory of groups [6], [7]. To absorb STBCs from orthogonal designs as a special case, Hassibi and Hochwald [8] introduced the linear codes in space and time called *linear dispersion codes*. To achieve good performance with full rate and full diversity, the STBC designed using unitary matrices have been investigated in [9]–[12]. However, in all previous STBC cases the patterns of the codes design are fixed.

It is known that orthogonal space-time codes have linear decoding complexity with full diversity. However, Tarokh *et al.* have already proved that there is not a full-rate complex orthogonal design for four antennas [4]. To solve this problem, Jafarkhani suggested a quasi-orthogonal design for STBC [5], but pairs of the transmitted symbols need be decoded separately. Subsequently, Jafarkhani and Khan presented the full rate diagonal block code with diversity two [1], which is a full-rate and full diversity diagonal block code based on coordinate interleaved orthogonal design (CIOD) in [13] and [14]. Compared with other STBCs, the diagonal block codes have the high peak-to-average power ratio (PAPR). Unfortunately, it is undesirable for practical application because of the usage of zero-element in transmission matrix.

Currently, Jacket matrix, which includes Hadamard matrix, discrete Fourier transform (DFT) matrix, etc., are being intensely investigated [15]–[17]. Since a Jacket matrix can be decomposed into multiplication of a Hadamard matrix and a sparse matrix, the Fourier matrix $\mathcal{F}_N$, which is yielded from the DFT with the form $X(n) = \sum_{m=0}^{N-1} x(m)W^{nm}$, can be expressed as

$$\mathcal{F}_N = (W^{nm}) = \mathcal{C}_N \mathcal{S}_N \mathcal{P}_N \tag{1}$$

where $W = e^{-i\frac{2\pi}{N}}$ for $0 \leq n, m \leq N-1$, $\mathcal{C}_N$ is a Hadamard matrix, $\mathcal{S}_N$ is a sparse diagonal block matrix, and $\mathcal{P}_N$ is a permutation matrix.

Using the matrix $\mathcal{P}_N$, one may design various patterns for the quasi-orthogonal STBC, which can be generally described as

$$\tilde{\mathcal{Q}} = \mathcal{Q}\mathcal{P}_N. \tag{2}$$

Since $|\det \mathcal{P}_N| = 1$, the revision with multiplication of $\mathcal{P}_N$ does not change the performance of the yielded codes.

To enrich the patterns of the families of STBCs, we suggest an elegant model for the designation of the STBC with transmission matrices being pairwise-row-orthogonal and modulating block diagonal STBC. In fact, the proposed STBCs are not strict orthogonal STBC, so we call them quasi-orthogonal STBC. This kind of quasi-orthogonal STBCs can be linearly decoded and reduce PAPR perfectly. It also has an advantage of being designed fast via Jacket transforms.

This paper is outlined as follows. In Section II, we describe the system model, which is the foundation of the present approaches for constructing STBC. In Section III, we state the encoding approaches in detail. In Section IV, we suggest the decoding analysis for the designed codes. In Section V, simulations results are presented by comparing itself with the previous codes. In Section VI, the extension of constructions are given can be generated with efficiency. Finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL

Consider a multiple antennas communication system with M transmitting and N receiving antennas. Let $\mathbf{H} = [h_1, h_2, \cdots, h_M]^T$ be the channel vector over the M channel uses. The employed channel in this paper is assumed to be a quasi-static

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[1] http://en.wikipedia.org/wiki/Jacket_matrix.

Rayleigh flat fading channel. Let the signal constellation be $\mathcal{V} = \{V_1, V_2, \cdots, V_L\}$, and $\mathcal{Q}(V)$ be the transmitted code matrix, where $V \in \mathcal{V}$. Then, the received signal vector $\mathbf{r}$ is written as

$$\mathbf{r} = \sqrt{\mathbf{E}_s}\mathcal{Q}\mathbf{H} + \mathbf{n} \qquad (3)$$

where $\mathbf{H}$ is the $M \times N$ channel matrix of Rayleigh-fading coefficients, $\mathbf{E}_s$ is the average energy at each receive antenna, and $\mathbf{n}$ is the noise modelled as independent samples of a zero-mean complex Gaussian random variable with the variance $N_0/2$ per dimension.

For the communication system presented in (3), the pairwise block error probability under the maximum-likelihood (ML) decoder should satisfy the following constraints [11],

$$P_e \leq \left[\prod_{m=1}^{M}(1 + \frac{E_s}{4N_0}\sigma_m^2)\right]^{-N} \qquad (4)$$

where σ_m denotes the mth singular values of the matrix $(\mathcal{Q} - \mathcal{Q}')$. At high SNR, the above inequality becomes

$$P_e \leq \left(\frac{E_s}{4N_0}\right)^{-MN} \frac{1}{|\det(\mathcal{Q} - \mathcal{Q}')|^{-N}}. \qquad (5)$$

Define a diversity product λ [12] (coding gain [13]), i.e.,

$$\lambda = \min_{\mathcal{Q} \neq \mathcal{Q}'} \left|\det\left[(\mathcal{Q} - \mathcal{Q}')^H(\mathcal{Q} - \mathcal{Q}')\right]\right|^{1/2} \qquad (6)$$

where $(\cdot)^H$ denotes the Hermitian, and $\mathcal{Q}$ and $\mathcal{Q}'$ are the code and error-code words matrix, respectively. To design a 'good' STBC, one needs that λ in (6) reaches the maximum.

III. ENCODING APPROACHES

Following the encoding criteria, we propose a novel approach for the designation of the STBC for MIMO communication system in this section.

Firstly, we divide randomly the signal constellation $\mathcal{V} = \{V_1, V_2, \cdots, V_L\}$ into K subset $\{A_1, A_2, \cdots, A_K\}$. For example, we may construct a generator similar to Alamounti's generator. After that, we encode the STBC code

$$\mathcal{Q} = \mathcal{C}\mathcal{S} = \mathcal{C}\Lambda(A_1, A_2, \cdots, A_K) \qquad (7)$$

where the symbol Λ denotes a diagonal block matrix with block elements A_i for $1 \leq i \leq K$, and $\mathcal{C}$ is a Hadamard matrix such that $\mathcal{C}^H\mathcal{C} = nI_n$ for an identity matrix I_n. It is clear that one difference between Λ and the other STBCs is its high PAPR because of the use of zero in Λ. The proposed approach can reduce its PAPR and provide a coding gain. Simple calculations give

$$\mathcal{Q}^H\mathcal{Q} = \Lambda(A_1^H A_1, A_2^H A_2, \cdots, A_K^H A_K). \qquad (8)$$

Since the Hadamard matrix is an orthogonal matrix, one may obverse that there are no interferences between any different signal constellation subsets A_i and A_j for $i \neq j$. This property is available for the decoding procedure. Evidently, if $\mathcal{S}^H\mathcal{S} = \sigma I_n$, the code $\mathcal{Q}$ is an orthogonal code; otherwise it is a quasi-orthogonal STBC with interference only in the various signal constellation subsets. According to (6), the diversity product of $\mathcal{Q}$ can hence be calculated as

$$\begin{aligned}\lambda &= \min_{\mathcal{Q} \neq \mathcal{Q}'} |\det(\mathcal{Q} - \mathcal{Q}')^H(\mathcal{Q} - \mathcal{Q}')|^{\frac{1}{2}} \\ &= \min_{\mathcal{S} \neq \mathcal{S}'} \left[|\det(\mathcal{C})| \prod_{i=1}^{K} |\det(\mathcal{S}_i - \mathcal{S}'_i)|\right].\end{aligned} \qquad (9)$$

It shows that the total diversity product depends only on the multiplication of diversity product of various subsets. Since controlling determinants of various subsets is easier than that of a high order matrix, the proposed approach is more useful for the orthogonal or quasi-orthogonal STBC designs. In addition, the performance of the proposed STBC is controllable.

As an example, we focus on the four transmit antennas case. The transmit code matrix may be written as

$$\mathcal{Q}_{p_4} = \mathcal{C}_4\mathcal{S}_4 \qquad (10)$$

where $\mathcal{S}_4 = diag\{Q_{12}, Q_{34}\}$. It is obvious that

$$\mathcal{S}_4^H\mathcal{S}_4 = diag(Q_{12}^H Q_{12}, Q_{34}^H Q_{34}). \qquad (11)$$

To design a quasi-orthogonal STBC, we begin with a simple combination with Alamounti's generator [2] as the block-elements of code matrix $\mathcal{S}_4$, i.e.,

$$Q_{ij} = \begin{pmatrix} x_i & x_j \\ -x_j^* & x_i^* \end{pmatrix} \qquad (12)$$

where $j = i + 1$ for any positive integer i. According to (11) with respect to orthogonal designs of STBC [4], It is obvious that the $\mathcal{S}_4$ is a full-rate code with diversity two [1]. So, the proposed code also has similar properties. If we replace $\mathcal{S}_4$ by diagonal CIOD or asymmetric CIOD (ACIOD) with full rate and full diversity [13], [14], where $x_i = \mathrm{Re}\{x_i\} + j\mathrm{Im}\{x_{(i+2)_4}\}$ and $(a)_k$ denotes a (mod k), the $\mathcal{Q}_{p_4}$ with low PAPR can achieve the full rate and full diversity, and the coding gain is improved. However, the proposed codes always have linear decoding complexity, one difference is that the proposed code based on CIOD diagonal code need to change signals before transmitting and after decoding. Without a loss of generality, we select the conventional diagonal block matrix in (10) to construct the proposed code and analyze decoding algorithms.

In the proposed approach, the code matrix $\mathcal{Q}_p$ in (10) may be rewritten as

$$\begin{aligned}\mathcal{Q}_{p_4} &= \mathcal{C}_4\mathcal{S}_4 = \mathcal{C}_4 \cdot diag\{Q_{12}, Q_{34}\} \\ &= \begin{pmatrix} x_1 - x_2^* & x_2 + x_1^* & x_3 - x_4^* & x_4 + x_3^* \\ x_1 + x_2^* & x_2 - x_1^* & x_3 + x_4^* & x_4 - x_3^* \\ x_1 - x_2^* & x_2 + x_1^* & -x_3 + x_4^* & -x_4 - x_3^* \\ x_1 + x_2^* & x_2 - x_1^* & -x_3 - x_4^* & -x_4 + x_3^* \end{pmatrix}\end{aligned} \qquad (13)$$

where $\mathcal{C}_4 = \mathcal{C}_2 \otimes \mathcal{C}_2$ is a 4-order Sylvester Hadamard matrix with the 2-order Hadamard matrix

$$\mathcal{C}_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

where '⊗' denotes Kronecker production. Assuming x_i and $\widetilde{x}_i$ denote the transmitted signal and the receiving signal, respectively, we get

$$\mathcal{Q}_{p_4}^H \mathcal{Q}_{p_4} = diag\{a_0, a_0, a_1, a_1\} \tag{14}$$

where $a_0 = 4\sum_{i=1}^{2} |\widehat{x}_i|^2$, $a_1 = 4\sum_{i=3}^{4} |\widehat{x}_i|^2$, and $\widehat{x}_i = x_i - \widetilde{x}_i$. Therefore, the diversity product is obtained,

$$\lambda_p = \min\{(a_0 \cdot a_1)\}. \tag{15}$$

IV. DECODING ANALYSIS

According to the orthogonality of pairwise rows of the quasi-orthogonal code matrix in (13), the decoding algorithms can be derived from an STBC with complex signal constellations.

Let $\in_t$ denote the permutations of the symbols from the first row to the tth row of the transmission matrix. The column position of x_i in the tth row is represented by $\in_t(i)$, and the sign of x_i in the tth row is denoted by $sgn_t(i)$. We assume that the channel coefficients $h_{i,j}(t)$ are constant, i.e., $h_{i,j}(t) = h_{i,j}$. Based on the orthogonality of the yielded quasi-orthogonal code matrix, the decision statistics $\widetilde{x}_i$ of the transmitted signal x_i can be constructed as [4]

$$\widetilde{x}_i = \sum_{t\in\eta(i)} \sum_{j=1}^{N} sgn_t(i)\tilde{r}_t^j \tilde{h}_{j,\in_t(i)}^* \tag{16}$$

where $\eta(i)$ is the set of rows of the transmission matrix including x_i,

$$\tilde{r}_t^j(i) = \begin{cases} r_t^j, & x_i \text{ belongs to the } t\text{th row of } \mathcal{Q}_P \\ (r_t^j)^*, & x_i^* \text{ belongs to the } t\text{th row of } \mathcal{Q}_P, \end{cases}$$

$$\tilde{h}_{j,\in_t(i)}^* = \begin{cases} h_{j,\in_t(i)}^*, & x_i \text{ belongs to the } t\text{th row of } \mathcal{Q}_P \\ h_{j,\in_t(i)}, & x_i^* \text{ belongs to the } t\text{th row of } \mathcal{Q}_P. \end{cases}$$

Since the value of $\widetilde{x}_i$ only depends on the code symbol x_i, given the received signals, the path coefficients and the structure of the transmission matrix, minimizing the maximum likelihood (ML) matric

$$\sum_{t=1}^{M}\sum_{j=1}^{N} | r_t^j - \sum_{i=1}^{M} h_{j,i}x_t^i |^2$$

is equivalent to minimizing each individual decision metric

$$|\widetilde{x}_i - x_i|^2 + \left(\sum_{t=1}^{M}\sum_{j=1}^{N} |h_{j,t}|^2 - 1\right)|x_i|^2. \tag{17}$$

Now applying (17) to the 4×4 STBC $\mathcal{Q}_\mathcal{P}$, we obtain the following decision statistics:

$$\widetilde{x}_1 = 4x_1\rho_1 + \sum_{j=1}^{N}[h_{j,1}^*\eta(0,0,0,0) + h_{j,2}\eta(0,2,0,2)],$$

$$\widetilde{x}_2 = 4x_2\rho_1 + \sum_{j=1}^{N}[h_{j,2}^*\eta(0,0,0,0) + h_{j,1}\eta(2,0,2,0)],$$

$$\widetilde{x}_3 = 4x_3\rho_3 + \sum_{j=1}^{N}[h_{j,3}^*\eta(0,0,2,2) + h_{j,4}\eta(0,2,2,0)],$$

$$\widetilde{x}_4 = 4x_4\rho_3 + \sum_{j=1}^{N}[h_{j,4}^*\eta(0,0,2,2) + h_{j,3}\eta(2,0,0,2)] \tag{18}$$

where $\rho_\mu = \sum_{i=\mu}^{\mu+1}\sum_{j=1}^{N} |h_{j,i}|^2$ for $\mu \in \{1,3\}$ and $\eta(a_1, a_2, a_3, a_4) = \sum_{i=1}^{4}(\tau)^{a_i}n_i^j$ for $a_i \in \{0,1,2,3\}$ and $\tau^2 = -1$, where n_i^j denotes the i.i.d. complex addition white Gaussian noise (AWGN) of the ith time slot at the jth receive antenna. Essentially, the decision statistics $\widetilde{x}_i$ is only a function of x_i for $1 \le i \le 4$. Consequently, the maximum likelihood decoding rule can be separated into four independent decoding rules each for x_i, which amounts for having a simple decoding algorithm at the receiver based only on linear processing of the transmitted signal, and hence providing remarkably reduced complexity.

Generally, we substitute a 4×4 DFT matrix $\mathcal{C}_{D_4}$, in which entries are composed of the elements in $\{\pm 1, \pm\tau\}$, for Hadamard $\mathcal{C}_4$ in (13). It is known that matrix $\mathcal{C}_{D_4}$, which is a complex Hadamard matrix, is a kind of Jacket matrices with orthogonal rows [15]–[17], and can hence be written as

$$\mathcal{C}_{D_4} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -\tau & -1 & \tau \\ 1 & -1 & 1 & -1 \\ 1 & \tau & -1 & -\tau \end{pmatrix}. \tag{19}$$

Since the two matrices $\mathcal{C}_{D_4}$ and $\mathcal{C}_4$ have the property $\mathcal{C}_{D_4}^H\mathcal{C}_{D_4} = \mathcal{C}_4^H\mathcal{C}_4$, we get the following decision statistics,

$$\widetilde{x}_1 = 4x_1\rho_1 + \sum_{j=1}^{N}[h_{j,1}^*\eta(0,0,0,0) + h_{j,2}\eta(0,3,2,1)],$$

$$\widetilde{x}_2 = 4x_2\rho_1 + \sum_{j=1}^{N}[h_{j,2}^*\eta(0,0,0,0) + h_{j,1}\eta(2,1,0,3)],$$

$$\widetilde{x}_3 = 4x_3\rho_3 + \sum_{j=1}^{N}[h_{j,3}^*\eta(0,2,0,2) + h_{j,4}\eta(0,1,2,3)],$$

$$\widetilde{x}_4 = 4x_4\rho_3 + \sum_{j=1}^{N}[h_{j,4}^*\eta(0,2,0,2) + h_{j,3}\eta(2,3,0,1)]. \tag{20}$$

It implies that the performance of the two transmission matrices $\mathcal{C}_{D_4}$ and $\mathcal{C}_4$ are the same as the simulation plotted in Fig. 1.

V. SIMULATION RESULTS

In this section, we show the simulated performance of the proposed quasi-orthogonal STBCs for the radiation-power-limited communication system with the assumption of Rayleigh flat fading channels.

We consider four transmitted antennas and one receiving antenna based on the ML decoding algorithm. The noises of the channel are independent samples of a zero-mean complex Gaussian random variable with a variance of $1/(2\text{SNR})$ per complex dimension. Fig. 1 shows the simulations of the performance of the proposed codes by comparing them to Jafarkhani's code [5], Tarokh's orthogonal $1/2$-rate (or $3/4$-rate) code [4], conventional diagonal code $\mathcal{S}_4$ in (10) and CIOD diagonal code [13] for the transmission of 2 bits/s/Hz (or 3 bits/s/Hz).

Simulation results show that the suggested quasi-orthogonal codes outperform Jafarkhani's code and the $1/2$-rate (or $3/4$-rate) orthogonal codes for a range of SNR. It is obvious that the performances of our codes have been improved at low SNR. However, the performance of the proposed codes start to deteriorate when the SNR value exceeds 22 dB (or 20 dB) for the 2 bits/s/Hz (or 3 bits/s/Hz) transmission rates. When we construct the proposed code by using the CIOD diagonal code, it outperforms the Jafarkhani's code at whole SNR.

In fact, compared with the utilized diagonal codes, the proposed codes can increase the coding gain without changing diversity gain. The proposed codes have better performances with low PAPR than its diagonal codes due to coding gain. At high SNR, the half-rate code outperform the others codes due to an increase in diversity gain.

Compared with Jafarkhani's code, from the diversity product λ_p in (15), the proposed codes with full rank increase the coding gain. They have better performances at low SNR. At high SNR while selecting the conventional diagonal code as block matrix, the proposed codes have full rate and full rank with diversity two and diversity gain two due to lost transmit diversity. The transmit diversity of the full rate Jafarkhani's code, which makes use of pairs of symbols decoding without sacrificing the performance, is four with the minimum rank two. From simulation results, the performance of Jafarkhani's code is very close to full diversity due to the pairs of symbols decoding. It implies that the performance of the proposed code based on the conventional diagonal code performs a little worse than that of Jafarkhani's code. While selecting the diagonal CIOD code as block matrix, the diversity gain of the proposed codes is always equal to four. Furthermore, the proposed code based on the diagonal CIOD code outperforms Jafarkhani's code at whole SNR due to coding gain. The analysis fits well in with the simulation results.

VI. CONSTRUCTION-EXPANDING OF STBC

In above sections, we have constructed a STBC through designing a quasi-orthogonal transmission matrix based on a 4×4 Sylvester Hadamard transform. This method can dispel the zero elements of the diagonal block transmitted matrix, and hence reduce its peak-to-average power ratio. In this section, we suggest two approaches to generalize the proposed STBC. Also, in order to improve the transmit diversity, in the following proposals we should select ACIOD [14] codes as block diagonal matrix because of the full rate CIOD of size exists if and only if size equals to 2, 3, or 4.

For one hand, a Sylvester Hadamard is a real Hadamard matrix with each entry being either '$+1$' or '-1' in mathematics [15]–[17]. This kind of matrix has the order being a multiple of 2 or 4. Following the four transmission antennas approach, we first construct the general $4m$-order transmission antennas orthogonal STBC for any positive integer $m \geq 2$. As an example, taking $m = 2$, we may have

$$\mathcal{Q}_{p_8} = \mathcal{C}_8 \mathcal{S}_8 \tag{21}$$

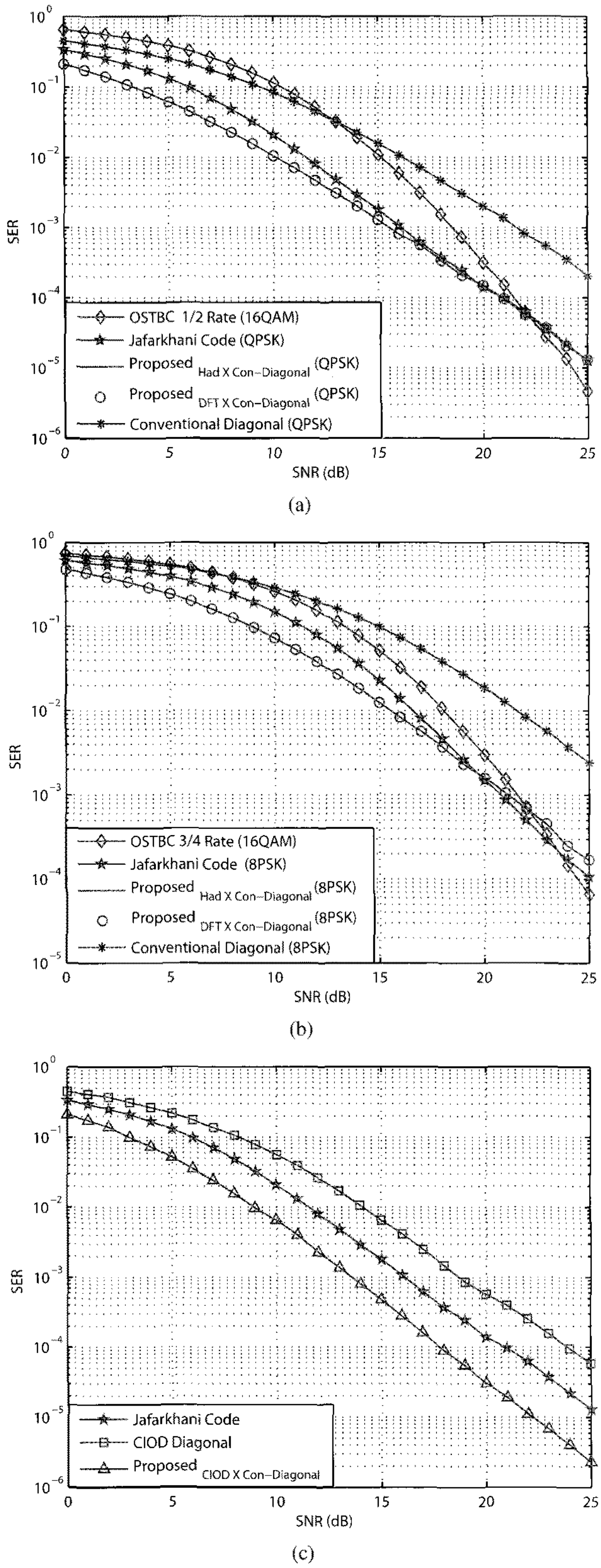

Fig. 1. Performances of different transmission matrices using four transmitting antennas and one receiving antenna over Rayleigh flat fading channel: (a) Transmission rate: 2 bits/s/Hz, (b) transmission rate: 3 bits/s/Hz, and (c) transmission rate: 2 bits/s/Hz for 4-QAM.

where $\mathcal{C}_8 = \mathcal{C}_4 \otimes \mathcal{C}_2$ and $\mathcal{S}_8$ is an 8-order ACIOD diagonal code.

According to the fast construction (or decomposing) algorithm for the block Jacket matrix construction, for $n = n_1^s n_2^m$

the high-order matrix $\mathcal{C}_n$ can be constructed by using the recursive relationship of identity matrices and lower-order block Jacket matrices $\mathcal{C}_{n_1}$ and $\mathcal{C}_{n_2}$, i.e.,

$$\mathcal{C}_{n_1^s n_2^m} = \{\mathcal{I}_{n_1^s} \otimes (\prod_{i=1}^{m} \mathcal{I}_{n_2^{m-i}} \otimes \mathcal{C}_{n_2} \otimes \mathcal{I}_{n_2^{i-1}})\} \times \{(\prod_{i=1}^{s} \mathcal{I}_{n_1^{s-i}} \otimes \mathcal{C}_{n_1} \otimes \mathcal{I}_{n_1^{i-1}}) \otimes \mathcal{I}_{n_2^m}\} \quad (22)$$

where $\mathcal{I}$ denotes identity matrix. The n-order transmission antennas quasi-orthogonal STBC $\mathcal{Q}_{p_n}$ can be constructed fast from

$$\mathcal{Q}_{p_n} = \mathcal{C}_n \mathcal{S}_n \quad (23)$$

where $\mathcal{S}_n$ is n-order ACIOD diagonal code.

For another, we can construct the generalized n-order transmission antennas quasi-orthogonal STBC $\mathcal{Q}_{p_n}$ using the lower-order complex Hadamard matrix with respect to (22). For example, taking $\mathcal{C}_{D_4}$ of (19) and $\mathcal{C}_{D_6}$ described by

$$\mathcal{C}_{D_6} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & \tau & -\tau & -\tau & \tau \\ 1 & \tau & -1 & \tau & -\tau & -\tau \\ 1 & -\tau & \tau & -1 & \tau & -\tau \\ 1 & -1 & -\tau & \tau & -1 & \tau \\ 1 & \tau & -\tau & -\tau & \tau & -1 \end{pmatrix}, \quad (24)$$

we construct the 24-order complex Hadamard matrix $\mathcal{C}_{D_{24}}$, i.e., $\mathcal{C}_{D_{24}} = \mathcal{C}_{D_4} \otimes \mathcal{C}_{D_6}$. Thus, the 24-order transmission antennas orthogonal STBC $Q_{p_{24}}$ can be constructed from $Q_{p_{24}} = \mathcal{C}_{D_{24}} \mathcal{S}_{24}$, where $\mathcal{S}_{24}$ is selected from ACIOD for $n = 24$.

For $n = n_1^s n_2^m$, the n-order quasi-orthogonal STBC can be fast constructed by using well-known the 2-order Alamouti scheme. This quasi-orthogonal STBC is given by

$$\mathcal{Q}_n = \mathcal{C}_{D_n} \mathcal{S}_n \quad (25)$$

where $\mathcal{C}_{D_n}$ is a complex Hadamard matrix constructed from (22) and $\mathcal{S}_n$ is the same diagonal matrix as in (23).

We note that all of the constructed n-order quasi-orthogonal transmission matrices have the same property. Since any two rows of transmission matrix are orthogonal, the interferences can be removed from different antennas, which simplifies the decoding algorithm of the yielded STBC. These kinds of codes have an advantage of being designed fast by employing Kronecker production and recursive relationship of identity matrices and successively lower-order block matrices.

VII. CONCLUSIONS

In conclusion, we have investigated a novel design of STBC based on Jacket transforms. The present STBCs enjoy the advantage that it can be constructed efficiently with good performance. Using this approach, we derive several proposal codes to enrich their family. Simulation shows that the performances of the proposed STBCs outperform Jafarkhani's code and other orthogonal codes with rate less than one over a wide range of SNR.

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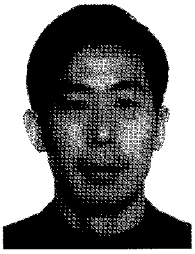

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