Sum Rate Approximation of Zero-Forcing Beamforming with Semi-Orthogonal User Selection

Janghoon Yang, Seunghun Jang, and Dong Ku Kim

Abstract: **In this paper, we present a closed-form approximation of the average sum rate of zero-forcing (ZF) beamforming (BF) with semi-orthogonal user selection (SUS). We first derive the survival probability associated with the SUS that absolute square of the channel correlation between two users is less than the orthogonalization level threshold (OLT). With this result, each distribution for the number of surviving users at each iteration of the SUS and the number of streams for transmission is calculated. Secondly, the received signal power of ZF-BF is represented as a function of the elements of the upper triangular matrix from QR decomposition of the channel matrix. Thirdly, we approximate the received signal power of ZF-BF with the SUS as the maximum of scaled chi-square random variables where the scaling factor is approximated as a function of both OLT and the number of users in the system. Putting all the above derivations and order statistics together, the approximated ergodic sum rate of ZF-BF with the SUS is shown in a closed form. The simulation results verify that the approximation tightly matches with the sample average for any OLT and even for a small number of users.**

Index Terms: **Broadcasting channel (BC), multiple-input multiple-output (MIMO), sum rate, zero-forcing (ZF).**

I. INTRODUCTION

Transmission with multi-antenna in broadcasting channel (BC) can significantly improve the capacity by selecting users and multiplexing data properly. The capacity region of the multiple-input multiple-output (MIMO) BC channel is known to be achieved by dirty paper coding (DPC) [1]. However, feasible channel coding for DPC is still under investigation. Since the basic principle of DPC is interference cancellation, DPC-like precoding schemes such as zero-forcing DPC (ZF-DPC) [2] and sphere encoding with regularized channel inversion [3] tend to be nonlinear and too complicated to be applicable to the commercial system. In addition to precoding, user selection is also complex. In DPC, every transmit covariance matrix will be optimally decided and users with non-zero trace of the transmit covariance matrix will be selected for transmission. Given a precoding scheme, the optimal user selection requires precoding for all possible combination of users, which results in a complexity which is too huge to be realized in a practical system.

As an alternative, ZF-beamforming (BF) with some simplified user selections has been proposed. Use scheduling over subset of users with ZF-BF was shown to achieve near optimal performance in MIMO broadcasting channel [4]. A greedy user selection with ZF-BF was proposed such that its sum rate was shown to increase with the same rate for increasing SNR as greedy ZF-DPC [5]. Later, a generalized greedy user selection with ZF-BF was shown to achieve the asymptotic optimal sum rate [6]. The ZF-BF with the semi-orthogonal user selection (SUS) was shown to be asymptotically optimal for a large number of users [7]. Comparison of the average received transmit power of ZF-BF with different user selection methods showed that the performance of norm based user selection with much less complexity is comparable to that of the SUS when the order of multiplexing is small [8]. The SUS over the subset of users having large channel magnitude [9] was shown to perform as well as the conventional SUS [7] while significantly reducing the complexity especially for a large number of users. Several joint schedulings with block diagonalization (BD), a generalization of ZF-BF to the receiver with multiple receive antennas were also proposed [10], [11].

However, to the best of the author's knowledge, thorough analysis of the sum rate of ZF-BF with user selection has not been made yet. [7] presented a lower bound of the sum rate based on the lower bound of the received signal power of ZF-BF. However, the bound is meaningful only when the orthogonalization level threshold (OLT) is small. Among existing user selection algorithms for ZF-BF, the SUS is found to be simple and efficient. However, due to the candidate selection process and associated beamforming, it is very difficult to derive the exact average sum rate. In this paper, we focus on the derivation of the approximated average sum rate of ZF-BF with the SUS in a numerically manageable closed form. We first derive the received signal power of ZF-BF, and distributions of the number of surviving users at each iteration of the SUS, and the number of streams for transmission. On the basis of these results, we approximate the average sum rate of ZF-BF with the SUS such that it can include the effect of the candidate user selection and the number of users in the system. This approximation will be particularly useful for choosing a proper OLT for a given system condition. The numerical results show that the proposed approximation tightly matches with various system conditions and that the sum rate performance based on the OLT chosen from approximation provides nearly the same performance as one with optimal OLT.

This paper is composed as follows. In Section II, the system model is presented. ZF-BF and the SUS are overviewed in Section III. The approximation for the average sum rate of ZF-BF with the SUS is made in Section IV while the detailed approximation is presented in the appendix. The proposed approximation is numerically compared with the sample average

Manuscript received September 17, 2008; approved for publication by Alain Sibille, Division II Editor, March 21, 2010.

This work was supported by Mid-career Researcher Program through NRF grant funded by the MEST (No. 2010-0000172).

J. Yang is with the Department of Newmedia, Korean German Institute of Technology, Seoul, Korea, email: jhyang@kgit.ac.kr.

S. Jang and D. K. Kim are with School of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea, email: {shjang, dkkim}@yonsei.ac.kr.

1229-2370/10/$10.00 © 2010 KICS

in Section V. Some concluding remarks are made in Section VI.

II. SYSTEM MODEL

We consider a single cell multi-user MIMO downlink channel with a base station (BS) of M transmit antennas and K mobile stations (MSs) of a single receive antenna. It is assumed that perfect channel state information (CSI) is available to the receiver and the transmitter. For simplicity, it is also assumed that each user experiences the statistically homogenous channel which is modeled as a circularly symmetric complex Gaussian vector with independently and identically distributed (i.i.d.) elements having zero mean and unit variance. The received signal at user k can be expressed as

$$r_k = \mathbf{h}_k^H \mathbf{x} + n_k \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the transmit signal with total transmit power constraint $tr(E\{\mathbf{x}\mathbf{x}^H\}) = P$, a channel from the BS to the user k, $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ is a channel vector from the BS to user k, and n_k is an additive white Gaussian noise with zero mean and unit variance.

When beamforming is applied as a transmission scheme, the received signal at user $\pi(m)$ where $\pi(m)$ is the user index occupying the mth beam can be expressed as

$$\begin{aligned} r_{\pi(m)} =& \sqrt{p_m}\mathbf{h}_{\pi(m)}^H \mathbf{u}_{\pi(m)} s_{\pi(m)} \\ &+ \sum_{j\neq m}^{M_0} \sqrt{p_j}\mathbf{h}_{\pi(m)}^H \mathbf{u}_{\pi(j)} s_{\pi(j)} + n_{\pi(m)} \end{aligned} \tag{2}$$

where p_m is the transmit power for the mth beam, $\mathbf{u}_{\pi(m)}$ is the mth beamforming vector with unit norm, $s_{\pi(j)}$ is the information symbol for user $\pi(j)$ with unit power, and M_0 is total number of beams. With ZF-BF, SINR at the user $\pi(m)$ will be

$$\gamma_{\pi(m)} = p_m \left|\mathbf{h}_{\pi(m)}^H \mathbf{u}_{\pi(m)}\right|^2. \tag{3}$$

From (3), it is noted that the SINR depends on the beamforming vector and power allocation, which are determined from the selected users for transmission. Throughout this paper, equal power allocation is assumed for simplicity, i.e., $p_m = P/M_0$ for $m = 1, \cdots, M_0$. In the next section, we will briefly review how users can be selected with the SUS.

III. ZERO-FORCING BEAMFORMING WITH SEMI-ORTHOGONAL USER SELECTION

In this section, we reproduce the algorithm explained in [7] to clarify the operation of the SUS which will be analyzed in the subsequent sections. ZF-BF with the SUS is summarized in Fig. 1. Roughly speaking, the SUS consists of selecting the user with the best metric among the semi-orthogonal users in the candidate set, and updating it for the next iteration. More specifically, among the semi-orthogonal users in the candidate set $S_{C,m}$ at the mth iteration, the SUS selects the user with the largest norm of the effective channel $\bar{\mathbf{h}}_{k,m}$ which is a projection of $\mathbf{h}_k$ to the null space of the subspace spanned by the channel vectors of the selected users in the preceding iterations. Then, it

Step 1: $S_{S,0} = \{\}, \bar{\mathbf{h}}_{k,1} = \mathbf{h}_k,\ k \in S_{C,1} = \{1, 2, \cdots, K\},\ m = 1$
Step 2: User selection for scheduling

$$\pi(m) = \arg\max_{k\in S_{C,m}} \left\|\bar{\mathbf{h}}_{k,m}\right\|^2$$

Step 3: Semi-orthogonal users selection

$S_{S,m} = S_{S,m-1} \cup \{\pi(m)\}$

$\mathbf{v}_m = \bar{\mathbf{h}}_{\pi(m),m} / \left\|\bar{\mathbf{h}}_{k,m}\right\|^2$

$S_{S,m+1} = S_{C,m} - \left\{k \middle| \left|\mathbf{v}_m^H \cdot \bar{\mathbf{h}}_{k,m}\right|^2 / \left\|\bar{\mathbf{h}}_{k,m}\right\|^2 > \delta, k \in S_{C,m}\right\}$

$\bar{\mathbf{h}}_{k,m+1} = \bar{\mathbf{h}}_{k,m} - (\mathbf{v}_m^H \cdot \mathbf{h}_{k,m})\mathbf{v}_m$ for $k \in S_{C,m+1}$

Step 4: Terminating condition verification

If $|S_{C,m+1}| == 0$ or $|S_{S,m}| == M$

$M_0 = m$ and end

else

$m = m + 1$, and go to Step 2

Fig. 1. Semi-orthogonal user selection algorithm for ZF-BF in multi-antenna system.

updates the candidate set $S_{C,m+1}$ for the next iteration such that users of which the absolute square of the channel correlation with the channel of the selected user for scheduling at the mth iteration is larger than OLT are removed from the candidate set $S_{C,m}$. The algorithm is finished when either the size of the candidate set for the next iteration is empty or the size of the set of selected users $S_{S,m}$ is M. In this paper the correlation is taken between $\bar{\mathbf{h}}_{\pi(m),m}$ and $\bar{\mathbf{h}}_{k,m}$ for mathematical rigorousness in the following analysis while it was done between $\bar{\mathbf{h}}_{\pi(m),m}$ and $\mathbf{h}_{k,m}$ in [7]. Since this modification has an effect only on formulating the set of semi-orthogonal users, the basic principle of the SUS originally proposed in [7] is likely to be kept. This issue will be revisited through simulations as described in Section V.

For the set of selected users S_S, the set of ZF-BF vectors $\mathbf{U} = [\mathbf{u}_{\pi(1)}, \cdots, \mathbf{u}_{\pi(M_0)}]$ will be determined in the following way

$$\mathbf{U} = \mathbf{H}(S_S)(\mathbf{H}(S_S)^H\mathbf{H}(S_S))^{-1}\mathbf{\Lambda}(S_S) \tag{4}$$

where $M \geq M_0$, $\mathbf{H}(S_S) = [\mathbf{h}_{\pi(1)}, \cdots, \mathbf{h}_{\pi(M_0)}]$, and $\mathbf{\Lambda}(S_S)$ is a diagonal matrix with the mth element $[\mathbf{\Lambda}(S_S)]_{m,m} = 1/\sqrt{[(\mathbf{H}(S_S)^H\mathbf{H}(S_S))^{-1}]_{m,m}}$. It was observed that there is a tradeoff between the beamforming gain and multi-user diversity gain depending on the selection of the OLT [5]. However, the effect of the OLT on the sum rate has not been analyzed properly. In the next section, the approximated sum rate analysis of the ZF-BF with SUS will be developed.

IV. AVERAGE SUM RATE APPROXIMATION OF ZF-BF WITH THE SUS

In this section, we provide an approximate sum rate analysis of ZF-BF with the SUS in the following way. First, the probability that each user survives in orthogonality condition will be addressed. With this probability, the average number of streams for given threshold and the average received signal power of the scheduled user will be approximately analyzed from the approximation of the received signal power of ZF-BF with the SUS. Finally the approximated sum rate will be expressed in a closed form by exploiting all preceding results.

A. Survival Probability

In this subsection, we solve the probability that absolute square of the correlation between two users is less than OLT, which we call "survival probability." This probability can be easily approximated by exploiting Theorem 1 in [12].

Theorem 1: Let $\mathbf{h} \in \mathbb{C}^M$ and $\mathbf{h}' \in \mathbb{C}^M$ be i.i.d. zero-mean circularly symmetric complex Gaussian vector with identity covariance matrix. The random variable $C = \frac{|\mathbf{h}^H \mathbf{h}'|^2}{\|\mathbf{h}\|^2 \|\mathbf{h}'\|^2}$ where $\|\cdot\|$ is the l^2-norm of the vector inside, has the beta distribution with parameters 1 and $M-1$. The corresponding probability density function $f_C(x)$ of C is given by

$$f_C(x) = (M-1)(1-x)^{M-2},\ 0 \le x \le 1. \tag{5}$$

Proof: Let $\mathbf{v} = \mathbf{h}/\|\mathbf{h}\|$. For a fixed $\mathbf{v}$, $C' = \frac{|\mathbf{v}^H \mathbf{h}'|^2}{\|\mathbf{h}'\|^2}$ was shown to have beta distribution with parameters 1 and $M-1$ [12]. From the rule of total probability, $P(\rho \le x) = \int P(C' \le x|\mathbf{v}) f(\mathbf{v}) d\mathbf{v}$ where $\mathbf{v}$ was proved to be uniformly distributed over the unit-norm sphere [12]. Thus, $P(C \le x) = P(C' \le x|\mathbf{v})$ which proves the theorem. □

Since effective channel $\bar{\mathbf{h}}_{k,m}$ at the mth iteration is in the orthogonal subspace of the subspace spanned by the channels of users which have been selected in the previous iterations, $\|\bar{\mathbf{h}}_{k,m}\|^2$ has chi-square distribution with $2(M-m+1)$ degrees of freedom. From Theorem 1, random variables for comparing with OLT at the mth iteration have the beta distribution with parameters 1 and $M-m$. Consequently, the survival probability at the mth iteration for candidate users in $S_{C,m}$ can be expressed in the following way

$$p_{s,m}(\delta) = 1 - (1-\delta)^{M-m}. \tag{6}$$

B. Approximation of the Received Signal Power of ZF-BF with the SUS

In this subsection, we provide the exact received signal power of ZF-BF in terms of the elements of the upper triangular matrix from the QR decomposition of $\mathbf{H}(S_S)$. On the basis of this result, we approximate the received signal power of ZF-BF with the SUS. Let QR decomposition of $\mathbf{H}(S_S)$ be

$$\mathbf{H}(S_S) = \mathbf{Q}(S_S)\mathbf{R}(S_S) \tag{7}$$

where $\mathbf{Q}(S_S)$ is a unitary matrix and $\mathbf{R}(S_S)$ is a upper triangular matrix. The received signal power of the user $\pi(n)$ with unit power transmission can be expressed as given in [6]

$$p_{r,n} = \frac{1}{\|\mathbf{g}_n\|^2} \tag{8}$$

where $\mathbf{g}_n$ is the nth row vector of the inverse matrix of $\bar{\mathbf{R}}(S_S)$ with dimension $M_0 \times M_0$ which is the submatrix of $\mathbf{R}(S_S)$ such that $\mathbf{R}(S_S) = \begin{bmatrix} \bar{\mathbf{R}}(S_S)^T & \mathbf{0}^T_{(M-M_0)\times M_0} \end{bmatrix}^T$. We can easily calculate (8) from the following lemma.

Lemma 1: The received signal power of ZF-BF for the nth user whose channel is the nth column vector of $\mathbf{H}(S_S)$ can be calculated as

$$p_{r,n} = \frac{|r_{n,n}|^2}{1 + \sum_{d=1}^{M_0-n} \left| \sum_{k=0}^{d-1} b^{(n)}_{n+k} \bar{r}_{n+k,r+d} \right|^2} \tag{9}$$

$$b^{(n)}_{n+d} = \begin{cases} 1, & \text{for } d = 0, \\ \sum_{k=0}^{d-1} b^{(n)}_{n+k} \bar{r}_{n+k,n+d}, & \text{for } d > 0 \end{cases} \tag{10}$$

where $r_{i,j}$ is the element of the $\bar{\mathbf{R}}(S_S)$ in the ith row and the jth column, and $\bar{r}_{n+k,n+d} = r_{n+k,n+d}(r_{n+d,n+d})^{-1}$

Proof: See Appendix A. □

It can be explicitly observed from (9) that the received signal power experiences ZF-BF loss as the correlation between channel increases. Even though (9) is an exact expression of the received signal power with ZF-BF, it is very hard to derive the statistical property exactly when it is combined with the SUS. This is because the received signal powers of the user selected in the preceding iterations are affected by the selection of users in the subsequent iterations. Thus, we provide another form of the received signal power and make approximation for the received signal power of ZF-BF with the SUS.

The received signal power of ZF-BF in (9) and (10) can be easily modified as follows

$$p_{r,n} = \frac{|r_{1,1}|^2}{|r^{-1}_{n,n} r_{1,1}|^2 + \sum_{d=1}^{M_0-n} \left| \sum_{k=0}^{d-1} c^{(n)}_{n+k} \bar{r}_{n+k,r+d} \right|^2} \tag{11}$$

$$c^{(n)}_{n+d} = \begin{cases} r^{-1}_{n,n} r_{1,1}, & \text{for } d = 0, \\ \sum_{k=0}^{d-1} c^{(n)}_{n+k} \bar{r}_{n+k,n+d}, & \text{for } d > 0. \end{cases} \tag{12}$$

Let $p_{r,n}(k)$ denote the received signal power of the user k with ZF-BF when the user k is scheduled at the nth iteration of the SUS and the total number of streams with the SUS is M_0. Mimicking the user selection step of the SUS, we approximate the received signal power of ZF-BF in the nth iteration of the SUS as

$$p_{r,n,sus} \approx \max_{k \in S_{C,n}} p_{r,n}(k) \tag{13}$$

$$\overset{(d)}{\approx} \max_{k \in S_{C,n}} \frac{\chi^2_{2M}(k)}{\alpha_{n,M_0}(K,\delta)} \tag{14}$$

where $\overset{(d)}{\approx}$ denotes the approximation in distribution, $\chi^2_{2M}(k)$ is i.i.d. chi-square random variable with $2M$ degrees of freedom, and $\alpha_{n,M_0}(K,\delta)$ is a constant representing the ZF-BF loss associated with the SUS. This approximation will be naturally valid only when the OLT is very small, which means that channels of the scheduled users are nearly orthogonal. Even though we can not mathematically rigorously argue further the validity of this approximation for other cases, it will be shown that the average sum rate approximation is very accurate in the numerical verification.

When the SUS procedure is considered, it is desired that $\alpha_{n,M_0}(K,\delta)$ decreases with K and increases with δ. Keeping

$$\begin{aligned} R_{ave} &\approx \sum_{m'=1}^{M} P_{M_0}(m') \sum_{m=1}^{m'} E\left\{\log_2\left(1+\frac{P}{m'\sigma_n^2}\max_{k\in S_{C,m}} \frac{\chi^2_{2M}(k)}{\alpha_{m,m'}(K,\delta)}\right)\right\} \\ &\le \sum_{m'=1}^{M} P_{M_0}(m') \sum_{m=1}^{m'} \log_2\left(1+\frac{P}{m'\sigma_n^2}E\left\{\max_{k\in S_{C,m}} \frac{\chi^2_{2M}(k)}{\alpha_{m,m'}(K,\delta)}\right\}\right) \\ &\approx \sum_{m'=1}^{M} P_{M_0}(m') \sum_{m=1}^{m'} \log_2\left(1+\frac{P}{m'\sigma_n^2}\frac{1}{\alpha_{m,m'}(K,\delta)}\sum_{n=1}^{K-m+1} p_{ave,m,n}\frac{P_{N_{m-1}}(n,\delta,K)}{\lambda_{m-1}(n,\delta,K)}\right) \end{aligned} \quad (22)$$

this in mind, we derive $\alpha_{n,M_0}(K,\delta)$ in Appendix B as

$$\alpha_{n,M_0}(K,\delta) \approx 1+\frac{n-1}{M-n+1}\frac{1}{1+\beta K}+\sum_{d=1}^{M_0-n} m^{(n)}_{n+d,c^2} \quad (15)$$

$$m^{(n)}_{n+d,c^2} \approx \sum_{k=0}^{d-1} m^{(n)}_{n+k,c^2}\frac{\rho(1,M-n-d+1+I_{n+d}(0),\delta)}{M-n-d+1+I_{n+d}(0)} \quad (16)$$

where $m^{(n)}_{n,c^2} = 1+\frac{n-1}{M-n+1}\frac{1}{1+\beta K}$, $I_a(b)$ is the indicator function which has a value of 1 only when $a=b$, and 0 otherwise, and $\rho(1,a,\delta)$ is the conditional mean of the F-distributed random variable with parameters $(1,a)$ with the condition that the corresponding random variable is less than or equal to δ, which is defined in Appendix B. Since $\rho(1,M-n-d+1,\delta)$ is an increasing function of δ and $\frac{1}{1+\beta K}$ is a decreasing function of K, it satisfies the desired property. It is also noted that since $\rho(1,M-n-d+1,\delta)$ and $m^{(n)}_{n,c^2}$ are the increasing functions of n, so will $\alpha_{n,M_0}(K,\delta)$ be. β controls the effect of K on the ZF-BF loss with the SUS, which will be defined heuristically through numerical evaluation in Section V.

C. The Average Number of Streams

ZF-BF with the SUS does not always guarantee maximum usage of the available degrees of freedom. This mainly results due to the insufficient number of users when the OLT is so small that most of users are rejected from the candidate set. The average number of streams $M_{0,ave}$ with SUS can be expressed as

$$M_{0,ave} = \sum_{m=1}^{M} mP_{M_0}(m) \quad (17)$$

where $P_{M_0}(m)$ is the probability that the number of streams M_0 of ZF-BF with the SUS is m. To calculate this probability, we first derive the probability distribution of the number of surviving users at each iteration.

Since each user has the same survival probability at each iteration, the probability distribution of the number of surviving users at the end of the first iteration follows binominal distribution as follows

$$P_{N_1}(k,\delta,K) = \binom{K-1}{k}(p_{s,1}(\delta))^k(1-p_{s,1}(\delta))^{K-1-k}. \quad (18)$$

However, it is not the binominal distribution at the end of mth iteration for $m>1$, since the number of users at the $(m-1)$th iteration is not fixed. Thus, the probability that there are k users at the end of the mth iteration can be calculated with total probability as follows

$$P_{N_m}(k,\delta,K) = \sum_{k'=k+1}^{K-m+1} P_{N_{m-1}}(k',\delta,K)\binom{k'-1}{k} \times (p_{s,m}(\delta))^k(1-p_{s,m}(\delta))^{k'-1-k}. \quad (19)$$

Since the scheduled user at the mth iteration is removed from the candidate set, $(k'-1)$ users are considered when k' users had survived at the $(m-1)$th iteration. From this distribution, $P_{M_0}(m)$ can be easily calculated. It is noted that the number of streams is m for $m<M$ is equivalent to the event that the number of users at the end of the mth iteration is zero. Thus, this can be expressed as

$$P_{M_0}(m) = \begin{cases} P_{N_m}(0,\delta,K), & \text{for } m<M \\ 1-\sum_{m=1}^{M-1} P_{M_0}(m), & \text{for } m=M. \end{cases} \quad (20)$$

By inserting (20) in (17), the average number of streams can be easily calculated as

$$M_{0,ave} = M - \sum_{m=1}^{M-1}(M-m)P_{N_m}(0,\delta,K). \quad (21)$$

D. Approximation of the Average Sum Rate of ZF-BF with the SUS

In this section, we derive a closed form approximation of the average sum rate R_{ave} for ZF-BF with the SUS. It can be expressed using (14) as (22) on the top of this page, where $P_{N_0}(k,\delta,K)$ is 1 for $k=K$ and 0 otherwise, and

$$\begin{aligned} p_{ave,m,n} &= E\left\{\max_{k\in S_{C,m}} \chi^2_{2(M-m+1)}(k)\,\big|\,|S_{C,m}|=n\right\} \\ &\approx \left(\left(\varsigma_n+\frac{\sum_{i=0}^{4}A_i\varsigma_n^i}{\sum_{i=0}^{4}B_i\varsigma_n^i}\right)\sqrt{\frac{1}{9M}}+1-\frac{1}{9M}\right)^3 \end{aligned} \quad (23)$$

$$\lambda_m(k,\delta,K) = \sum_{k=1}^{K-m+1} P_{N_m}(k,\delta,K). \quad (24)$$

The inequality in (22) follows from Jensen's inequality, and the approximation in (23) is derived in Appendix C.

One may have a simpler form of approximation by replacing the random variables with their average values as follows

$$R'_{ave} \approx \sum_{m=1}^{\bar{M}_{0,ave}} \log_2\left(1+\frac{P}{\bar{M}_{0,ave}\sigma_n^2}\frac{p_{ave,m,\bar{N}_{ave,m}}}{\alpha_{m,m'}(K,\delta)}\right) \quad (25)$$

where $\bar{M}_{0,ave}$ and $\bar{N}_{ave,m}$ are the nearest integers of $M_{0,ave}$ and $N_{ave,m}$ which is the average number of surviving users at the end of the $(m-1)$th iteration of the SUS. It can be calculated as

$$N_{ave,m} = \sum_{k=1}^{K-m+1} k\frac{P_{N_{m-1}}(k,\delta,K)}{\lambda_{m-1}(k,\delta,K)}. \quad (26)$$

The accuracy of the proposed approximations will be evaluated numerically in the next section.

E. Further Approximations for Asymptotic Cases

Since the approximations presented in (22) and (25) are rather complex, it is not straightforward to observe the physical meaning with the associated parameters. However, the complicated approximation in the analysis of the MIMO system may be simply expressed for some special or asymptotic cases [17]. To get some insight from this approximation, we examine some asymptotic cases based on (25), which is simpler than (22). For asymptotic high SNR, (25) can be further approximated from $\log(1+x)\approx\log(x)$ for $x \gg 1$

$$R'_{ave} \overset{P\to\infty}{\approx} \bar{M}_{0,ave}\log_2(P) + \log_2\left(\prod_{m=1}^{\bar{M}_{0,ave}} \theta_{m,\bar{M}_{0,ave}}\right) \quad (27)$$

where $\theta_{m,\bar{M}_{0,ave}} = \frac{1}{\bar{M}_{0,ave}\sigma_n^2}\frac{p_{ave,m,\bar{N}_{ave,m}}}{\alpha_{m,\bar{M}_{0,ave}}(K,\delta)}$. It is noted that the sum rate increases logarithmically with increasing transmit power for the given number of transmit antennas at high SNRs.

For asymptotic low SNR, it can be rearranged from $\log(1+x)\approx x$ for $x \ll 1$

$$R'_{ave} \overset{P\to 0}{\approx} P\left[\sum_{m=1}^{\bar{M}_{0,ave}} \frac{\theta_{m,\bar{M}_{0,ave}}}{\log(2)}\right]. \quad (28)$$

Unlike the asymptotic high SNR case, the ergodic sum rate increases linearly with increasing transmit power for the given number of transmit antennas at low SNRs.

$\theta_{m,\bar{M}_{0,ave}}$ can be further approximated using the extreme value theory in [16] for large K and small δ such that $\bar{M}_{0,ave} = M$ and $\alpha_{m,\bar{M}_{0,ave}}(K,\delta) \ll 1$

$$\theta_{m,\bar{M}_{0,ave}} \approx \frac{1}{M\sigma_n^2}\log(\bar{N}_{ave,m}) + O(\log\log(\bar{N}_{ave,m})). \quad (29)$$

By inserting (29) into (25), the ergodic sum rates with asymptotic number of users and small OLT, can be expressed as

$$R'_{ave} \overset{\substack{K\to\infty \\ \delta\to 0}}{\approx} \sum_{m=1}^{M}\log_2\left(1+\frac{P}{M\sigma_n^2}\log(\bar{N}_{ave,m})\right). \quad (30)$$

This result is in line with the result in [7] which shows the multiuser diversity gain of $\log K$ for the large number of users.

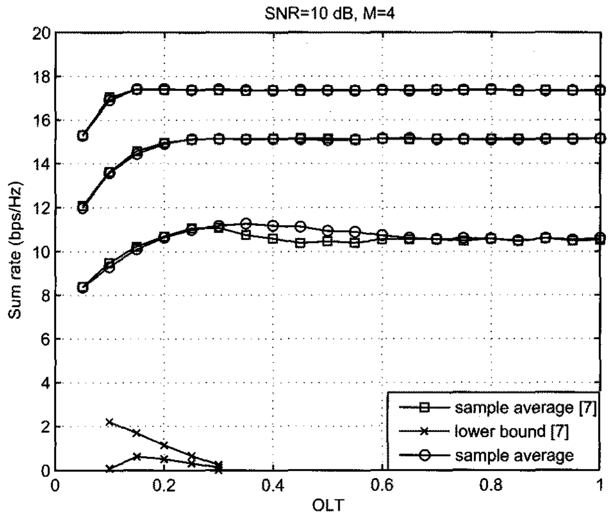


Fig. 2. Comparion of ZF-BF with the proposed SUS and one in [7].

V. NUMERICAL RESULTS

In this section, the proposed approximation of the average sum rate performance of ZF-BF with the SUS is verified by comparing it with the sample average. The sample average for each simulation was obtained by averaging out 1000 independent channel realizations. In the figures, "approximation-1" and "approximation-2" refer to the numerical evaluations of (22) and (25), respectively.

In Fig. 2, the average sum rate of ZF-BF with the SUS in Section II was compared with the one in [7]. They showed almost the performance at any OLT and any number of users. Slight difference can be observed for some OLTs for $K = 8$ when the OLT is between 0.3 to 0.6. This is because the performance of ZF-BF with the SUS is sensitive to the user selection when the number of users is small. We also plotted the lower bound with a positive real value in [7], which is

$$R_{ave,[7]} = \sum_{i=1}^{M}\log_2\left(1+\frac{P}{M}\frac{\log\frac{KI_\delta(i-1,M-i+1)}{(M-i)!}}{1+\frac{(M-1)^4\delta}{1-(M-1)\delta}}\right) \quad (31)$$

where $I_z(a,b)$ is the regularized incomplete beta function. It can be observed that this bound is too loose to be a good approximation for the actual average sum rate and it has real value only for small OLTs.

To validate the proposed approximation, β in (15) needs to be specified. For a system setup which consists of every possible combination of $K = 16, 64, 256$, $M = 2, 4, 8$, SNR (dB) $= 0, 10, 20$, and $\delta = 0.1, 0.5, 0.9$, the normalized mean squared error (MSE) was calculated for βs ranging from 0.005 to 0.1 by 0.005 step, averaged out and plotted in Fig.-3. It can be observed that the average normalized MSE is pretty small for both approximations. From this result, we set β to be 0.035 for the remaining evaluation steps of the approximations in which minimum average normalized MSE occurred.

In Fig. 4, the sum-rate approximation of ZF-BF with the SUS is compared with the sample average as the OLT increases on the condition that the number of transmit antennas is 4, and SNR is 10 dB. "approximation-3" refers to (22) with $\beta = 0$,

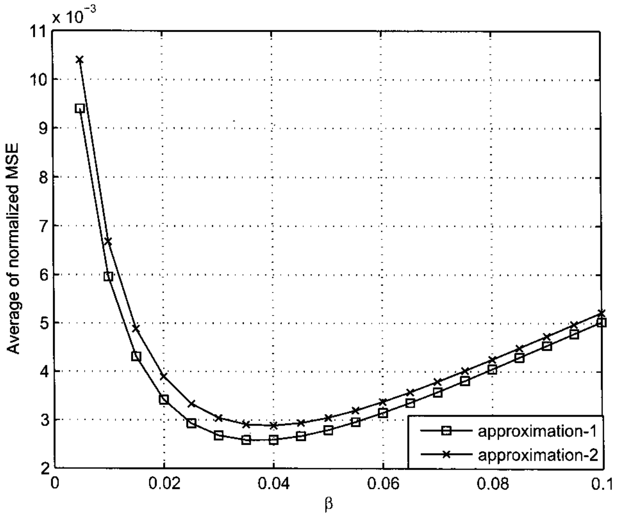


Fig. 3. Average sum rate approximation of ZF-BF with the SUS for increasing OLTs.

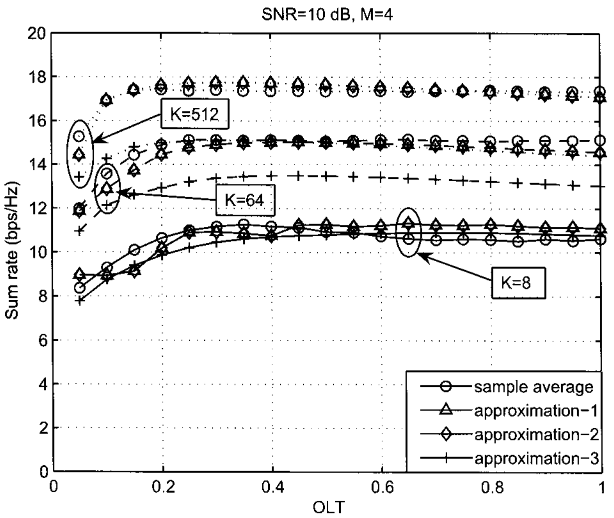


Fig. 4. Average sum rate approximation of ZF-BF with the SUS for increasing OLTs.

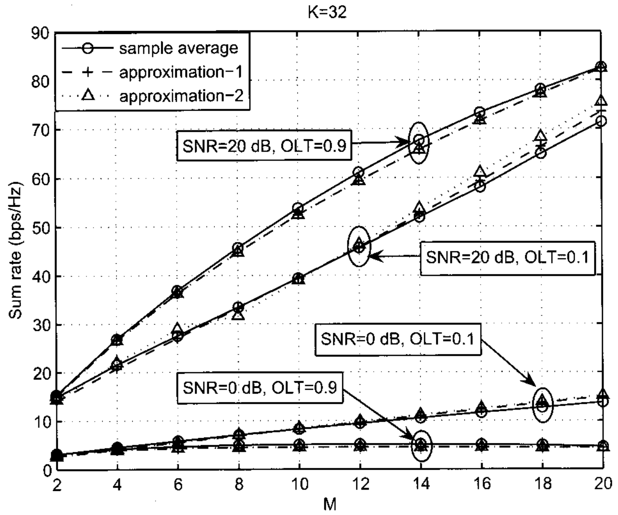


Fig. 5. Average sum rate approximation of ZF-BF with the SUS for increasing numbers of transmit antennas.

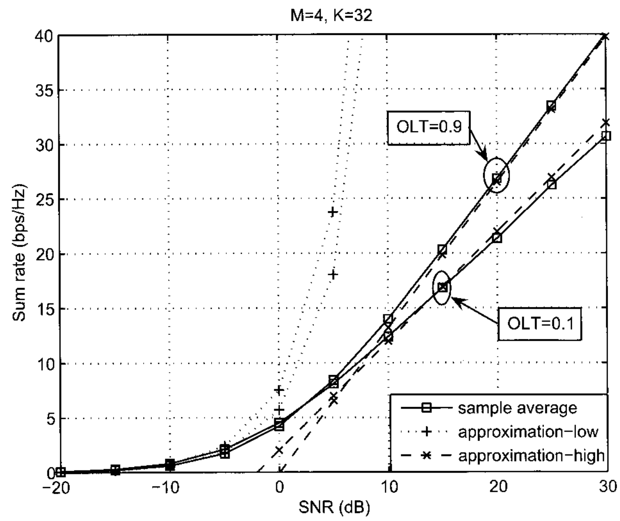


Fig. 6. Average sum rate approximation of ZF-BF with the SUS for asymptotic low and high SNRs.

which shows that without properly decreasing the ZF-BF loss, it tends to underestimate the average sum rate. This is because the proposed approximation treats the ZF-BF loss as a constant term even though there can be interbeam interference diversity additionally. With $\beta = 0.035$, It can be seen that the proposed approximation provides a tight match with sample average for various system conditions and OLTs. It is also noted that the simpler approximation (25) is almost identical to (22) for large OLTs. When the OLT is large, the number of streams is likely to be the same as the number of transmit antennas most of the time, and the large number of users will survive. That is, for a large OLT, $\bar{M}_{0,ave} \approx M$, and $P_{M_0}(M) \approx 1$ will make (22) and (25) almost the same expression, which results in almost identical approximation for large OLTs.

In Fig. 5, the accuracy of the approximation is shown over increasing numbers of transmit antennas. The number of users was set to be 32 for a more practical system condition. The approximation is very tight for all Ms and OLTs, which corroborates the proposed approximations especially for the large systems. In Fig. 6, the asymptotic approximations were compared with the sample averages. It can be observed that the asymptotic low SNR approximation is tightly matched to the sample average for up to -5 dB while the high SNR approximation starts to match well from the SNR of 10 dB or so, which shows the linear scaling with SNR in dB scale.

In Fig. 7, the sample average of sum rate of ZF-BF with the SUS was compared for differently selected OLTs when the number of transmit antenna is 4. The sample averages were evaluated for OLT from 0.05 to 1 by 0.05 step and the best performance was selected for a given system condition. We also numerically evaluated (22) and (25) for the same OLTs and selected the OLTs such that the approximated sum rate for the selected OLTs is larger than that for other OLTs. For these OLTs, the sample average of the sum rate of ZF-BF with the SUS was compared. It is observed that the selection of OLT from the approximation provides nearly the same performance as one based on the sam-

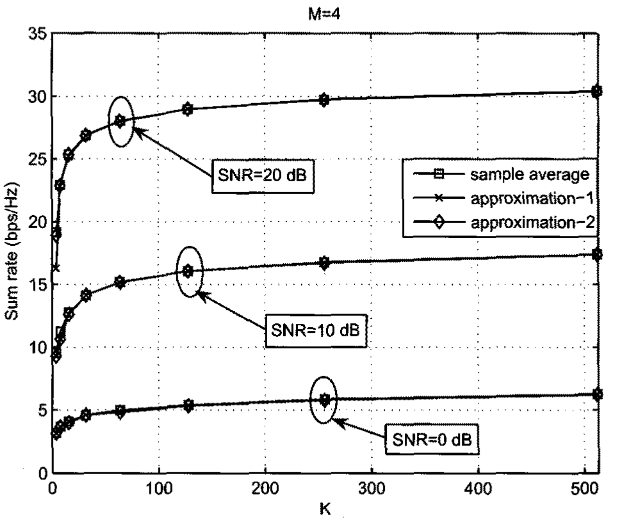


Fig. 7. Comparison of average sum rate of ZF-BF with the SUS for optimally chosen OLTs from the sample average, and ones from the approximation.

ple averages. Even though the proposed approximation of the average sum rate is not concave with OLT, numerical evaluation shows that it shows the largest value when the OLT is close to 1, which is often the case with the sample average. Thus, one can benefit from choosing the proper OLT from the numerical evaluation of the approximation rather than resorting extensive simulation to get the sample average for various OLTs.

VI. CONCLUSIONS

In this paper, we analyzed the performance of ZF-BF with the SUS for in the nonasymptotic regime. The survival probability, distributions of the number of streams, and the number of users at each iteration of the SUS were derived. From these analysis, approximation for the sum rate of ZF-BF with the SUS was proposed such that it can capture the effect of OLTs for any system setup. The numerical results show that the proposed approximation is accurate for all considered system environments.

Even though ZF-BF with the SUS is a simple and efficient precoding and user selection scheme for MIMO BC channels, there are many issues remaining associated with this scheme. Especially, the sum rate analysis associated with fair and QoS scheduling need to be studied more analytically in future research in order to characterize this scheme in a more practical communication system.

APPENDICES

A. Derivation of the Received Signal Power of ZF-BF

Let $\bar{\mathbf{R}}_n(S_S) \in \mathbb{C}^{n\times n}$ be the first subblock matrix of $\bar{\mathbf{R}}(S_S)$ such that

$$\bar{\mathbf{R}}(S_S) = \begin{bmatrix} \bar{\mathbf{R}}_n(S_S) & \mathbf{F}_n(S_S) \\ \mathbf{0} & \mathbf{D}_n(S_S) \end{bmatrix}. \tag{32}$$

By using the upper diagonal structure of the matrix, $\bar{\mathbf{R}}_{n+1}(S_S)^{-1}$ can be expressed as

$$\bar{\mathbf{R}}_{n+1}(S_S)^{-1} = \begin{bmatrix} \bar{\mathbf{R}}_n(S_S)^{-1} & -\bar{\mathbf{R}}_n(S_S)^{-1}\mathbf{r}_{n+1}r^{-1}_{n+1,n+1} \\ \mathbf{0}_{1\times n} & r^{-1}_{n+1,n+1} \end{bmatrix} \tag{33}$$

where $\mathbf{r}_{n+1} = \mathbf{Q}_{n+1}(S_S)^H \mathbf{h}_{\pi(n+1)}$, and $\mathbf{Q}_{1:(n+1)}(S_S) \in \mathbb{C}^{M\times(n+1)}$ is a subblock matrix of $\mathbf{Q}(S_S)$ such that $\mathbf{Q}(S_S) = [\mathbf{Q}_{1:n+1}(S_S)\ \mathbf{Q}_{(n+2):M_0}(S_S)]$. The calculation of $p_{r,n}$ is associated with the last $M_0 - n + 1$ elements of the vector $\mathbf{g}_n$ which can be represented using (33) as

$$[\mathbf{g}_n]_{n+d} = \begin{cases} r^{-1}_{n,n}, & \text{for } d = 0, \\ -\frac{[\bar{\mathbf{R}}_{n+d-1}(S_S)^{-1}\mathbf{r}_{n+d}]_n}{r_{n+d,n+d}}, & \text{for } d > 0 \end{cases} \tag{34}$$

where $[\cdot]_i$ is the ith element of the vector in the bracket. For $d > 0$, $|[\mathbf{g}_n]_{n+d}|^2$ can be expressed as

$$\begin{aligned} |[\mathbf{g}_n]_{n+d}|^2 &= \left| -\frac{\sum_{k=0}^{d-1} [\bar{\mathbf{R}}_{n+d-1}(S_S)^{-1}]_{n,n+k} r_{n+k,r+d}}{r_{n+d,n+d}} \right|^2 \\ &= \left| \frac{\sum_{k=0}^{d-1} [\bar{\mathbf{R}}_{n+k-1}(S_S)^{-1}\mathbf{r}_{n+k} r^{-1}_{n+k,n+k}]_n r_{n+k,n+d}}{r_{n+d,n+d}} \right|^2 \end{aligned} \tag{35}$$

where $[\mathbf{A}]_{i,j}$ is the element of the matrix $\mathbf{A}$ in the ith row and the jth column, and the second equality follows from (33). Let $\frac{r_{n,n}[\bar{\mathbf{R}}_{n+k-1}(S_S)^{-1}\mathbf{r}_{n+k}]_{n,n}}{r_{n+k,n+k}}$ be denoted by $b^{(n)}_{n+k}$. Then, (35) can be rewritten as

$$\begin{aligned} |[\mathbf{g}_n]_{n+d}|^2 &= \left|r^{-1}_{n,n}\right|^2 \left| \sum_{k=0}^{d-1} b^{(n)}_{n+k}\bar{r}_{n+k,n+d} \right|^2 \\ &= \left|r^{-1}_{n,n}\right|^2 \left|b^{(n)}_{n+d}\right|^2 \end{aligned} \tag{36}$$

where the second equation follows from the definition of $b^{(n)}_{n+d}$ and (34). From this equation, recursion for solving $b^{(n)}_{n+d}$ can be arranged as

$$b^{(n)}_{n+d} = \begin{cases} 1, & \text{for } d = 0 \\ \sum_{k=0}^{d-1} b^{(n)}_{n+k}\bar{r}_{n+k,n+d}, & \text{for } d > 0. \end{cases} \tag{37}$$

By putting together (34) and (36) into (8), it can be expressed as

$$p_{r,n} = \frac{1}{\sum_{d=0}^{M_0-n} |[\mathbf{g}_n]_{n+d}|^2} = \frac{|r_{n,n}|^2}{1 + \sum_{d=1}^{M_0-n} \left| \sum_{k=0}^{d-1} b^{(n)}_{n+k}\bar{r}_{n+k,r+d} \right|^2} \tag{38}$$

$$\alpha_{n,M_0}(K,\delta) = \sum_{d=1}^{M_0-n}\sum_{k=0}^{d-1} E\left\{\left|c_{n+k}^{(n)}\right|^2 |C_s\right\} E\left\{|\bar{r}_{n+k,r+d}|^2 |C_s\right\} + E\left\{\frac{r_{1,1}^2}{r_{n,n}^2}|C_s\right\}. \quad (40)$$

$$\alpha_{n,M_0}(K,\delta) \approx 1 + \frac{n-1}{M-n+1}f(K) + \sum_{d=1}^{M_0-n}\sum_{k=0}^{d-1} m_{n+k,b^2}^{(n)} \frac{\rho(1, M-n-d+1+I_{n+d}(0),\delta)}{M-n-d+1+I_{n+d}(0)} \quad (47)$$

B. Derivation of $\alpha_{n,M_0}(K,\delta)$

One natural way of choosing $\alpha_{n,M_0}(K,\delta)$ is using the average of the ZF-BF loss in the consideration of the SUS, which can be expressed as

$$\alpha_{n,M_0}(K,\delta) = E\left\{\sum_{d=1}^{M_0-n}\left|\sum_{k=0}^{d-1} c_{n+k}^{(n)}\bar{r}_{n+k,r+d}\right|^2 |C_s\right\} + E\left\{\frac{r_{1,1}^2}{r_{n,n}^2}|C_s\right\} \quad (39)$$

where $C_s = \{|\mathbf{q}_i^H\bar{\mathbf{h}}_{\pi(n),i}|^2 < \|\bar{\mathbf{h}}_{\pi(n),i}\|^2\delta$, for $n = 1,\cdots,M_0$, and $i = 1,\cdots,n-1\}$. C_s is added in condition of the expectation to reflect the effect of the SUS. Since $c_{n+k}^{(n)}$ and $\bar{r}_{n+k,r+d}$ are uncorrelated and cross terms in the absolute square of (39) are uncorrelated, $\alpha_{n,M_0}(K,\delta)$ can be rearranged as (40) on the top of this page.

Let us approximate $E\left\{|\bar{r}_{n+k,r+d}|^2 |C_s\right\}$ with its lower bound first. It can be rewritten as

$$\begin{aligned} &E\left\{|\bar{r}_{n+k,r+d}|^2 |C_s\right\} \\ &= E\left\{|\bar{r}_{n+k,r+d}|^2 \,|\, \frac{|\mathbf{q}_{n+k}^H\bar{\mathbf{h}}_{\pi(n+d),n+k}|^2}{|\bar{\mathbf{h}}_{\pi(n+d),n+k}|^2} < \delta\right\} \\ &\geq E\left\{|\bar{r}_{n+k,r+d}|^2 \,|\, \frac{|\mathbf{q}_{n+k}^H\bar{\mathbf{h}}_{\pi(n+d),n+k}|^2}{|\bar{\mathbf{h}}_{\pi(n+d),n+d}|^2} < \delta\right\} \end{aligned} \quad (41)$$

where inequality follows from $|\bar{\mathbf{h}}_{\pi(n+d),n+d}|^2 < |\bar{\mathbf{h}}_{\pi(n+d),n+k}|^2$. Since $|\bar{r}_{n+k,r+d}|^2 = \frac{|\mathbf{q}_{n+k}^H\bar{\mathbf{h}}_{\pi(n+d),n+k}|^2}{|\bar{\mathbf{h}}_{\pi(n+d),n+d}|^2}$ where the nominator is chi-square distributed with 2 degrees of freedom, and the denominator is chi-square distributed with $2(M-n-d+1)$ degrees of freedom, $(M-n-d+1)|\bar{r}_{n+k,n+d}|^2$ has the F-distribution with parameters $(1, M-n-d+1)$ for $M-n-d > 0$. Let $\kappa_{2m,2n}$ and $F_\kappa(t)$ be F-distributed random variable with parameters (m,n) and corresponding CDF. The conditional mean of this random variable can be easily calculated as

$$\begin{aligned} &E\{\kappa_{2m,2n}|\kappa_{2m,2n} < t\} \\ &\triangleq \rho(m,n,t) \\ &= \frac{m}{m+1}\frac{x_2^{m+1}F_1(m+1, m+n; m+2; -mx/n)}{x_2^{m}F_1(m, m+n; m+1; -mx/n)} \end{aligned} \quad (42)$$

where ${}_2F_1(a,b,c,x)$ is Gauss hypergeometric function. From the above equation, the lower bound of $E\left\{|\bar{r}_{n+k,r+d}|^2 |C_s\right\}$ can be calculated as

$$E\left\{|\bar{r}_{n+k,r+d}|^2 |C_s\right\} \geq \frac{\rho(1, M-n-d+1,\delta)}{M-n-d+1}. \quad (43)$$

However, when $M-n-d = 0$, $E\left\{\frac{|\bar{\mathbf{h}}_{\pi(n+d),n+k}|^2}{|\bar{\mathbf{h}}_{\pi(n+d),n+d}|^2}\right\} = \infty$ which is improper in the approximation. Thus, for this specific case, we make another approximation as

$$E\left\{|\bar{r}_{n+k,r+d}|^2 |C_s\right\} \geq \frac{\rho(1,2,\delta)}{2}. \quad (44)$$

From (12), $E\left\{\left|c_{n+d}^{(n)}\right|^2 |C_s\right\} \triangleq m_{n+d,c^2}^{(n)}$ can be calculated by the exploitation of approximation in (43) and (44) as

$$\begin{aligned} m_{n+d,c^2}^{(n)} &= \sum_{k=0}^{d-1} m_{n+k,c^2}^{(n)} E\left\{|\bar{r}_{n+k,r+d}|^2 |C_{sus}\right\} \\ &= \sum_{k=0}^{d-1} m_{n+k,c^2}^{(n)} \frac{\rho(1, M-n-d+1+I_{n+d}(0),\delta)}{M-n-d+1+I_{n+d}(0)}. \end{aligned} \quad (45)$$

It is noted that $m_{n+d,c^2}^{(n)}$ does not depend on K. In order to reflect the dependency of the ZF-BF loss on the number of users in the system associated with the SUS, $m_{n,c^2}^{(n)}$ can be approximated in the following way

$$m_{n,c^2}^{(n)} = E\left\{\frac{r_{1,1}^2}{r_{n,n}^2}|C_s\right\} \approx \frac{E\{r_{1,1}^2\}}{E\{r_{n,n}^2\}} \approx 1 + \frac{M}{M-n+1}f(K) \quad (46)$$

where the first approximation follows from Jensen's inequality with assumption of independence of $r_{1,1}^2$ and $r_{n,n}^2$ under the condition of C_s, and $f(K)$ is added in the second approximation so that it can decrease with increasing K to represent the improved beamforming efficiency with increasing K. It can be defined in various forms. For simplicity, we propose $f(K) = \frac{1}{1+\beta K}$. Proper choice of β will determine accuracy of the approximation. By putting together (45) and (46) into (39), $\alpha_{n,M_0}(K,\delta)$ can be approximated as (47) on the top of this page.

C. Approximation of the First Moment of the Maximum among n i.i.d. Chi-Square Random Variables

When Y_k is an i.i.d. random variable, the first moment of the order statistics, can be approximated with quantile approximation [13]

$$E\left\{\max_{1\leq k\leq n} Y_k\right\} \approx F_Y^{-1}\left(\frac{K-c_1}{K+c_2}\right) \quad (48)$$

where $F_Y^{-1}(\cdot)$ is the inverse CDF of the random variable Y_k, and c_1 and c_2 are refining parameters. Since the inverse CDF of the chi-square distribution is not known, we introduce Wilson-Hilferty approximate deviate [15] to calculate it through the inverse of the normal distribution

$$F_G(g) = F_X(x) \Rightarrow g \approx \frac{(x/q)^{1/3} - 1 + \frac{2}{9q}}{\left(\frac{2}{9q}\right)^{1/2}} \quad (49)$$

where $F_G(g)$ is the CDF of the Gaussian random variable. By applying the relationship in (48) to (49), it can be further approximated as

$$E\left\{\max_{1\le k\le n} \chi_q^2\right\} \approx \left(F_G^{-1}\left(\frac{n-3/8}{n+1/4}\right)\sqrt{\frac{2}{9q}} + 1 - \frac{2}{9q}\right)^3 \quad (50)$$

where $F_G^{-1}(\cdot)$ is the inverse function of $F_G(g)$, and $c_1 = 3/8$ and $c_2 = 1/4$ are known to provide a good approximation [13]. Finally, it can be approximately evaluated in a closed form from [14] as

$$F_G^{-1}\left(\frac{n-3/8}{n+1/4}\right) \approx \left(\varsigma_n + \frac{\sum_{i=0}^{4} A_i \varsigma_n^i}{\sum_{i=0}^{4} B_i \varsigma_n^i}\right) \quad (51)$$

where polynomial coefficients A_i and B_i are defined in [14], and $\varsigma_n = \sqrt{-2\log(1 - \frac{n-3/8}{n+1/4})}$. From (50) and (51), the average of $\max_{k\in S_{C,m}} \chi_{2M}^2(k)$ with $|S_{C,m}| = n$ can be approximately calculated as

$$E\left\{\max_{k\in S_{C,m}} \chi_{2M}^2(k)) \,|\, |S_{C,m}| = n\right\} \approx \left(\left(\varsigma_n + \frac{\sum_{i=0}^{4} A_i \varsigma_n^i}{\sum_{i=0}^{4} B_i \varsigma_n^i}\right)\sqrt{\frac{1}{9M}} + 1 - \frac{1}{9M}\right)^3. \quad (52)$$

REFERENCES

[1] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," in *Proc. IEEE ISIT*, Chicago, USA, July 2004.

[2] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Guassian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691–1706, July 2003.

[3] B. M. Hochwald, C. B. Peel, and A. L. Windlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—Part II: Perturbation," *IEEE Trans. Commun.*, vol. 53, no. 3, pp. 537–544, Mar. 2005.

[4] C. Swannack, E Uysal-Biyikoglu, and G. W. Wornell, "Low complexity multiuser scheduling for maximizing throughput in the MIMO broadcasting channel," in *Proc. Allerton Conf.*, USA, Oct. 2004.

[5] G. Dimic and N. D. Sidiropoulos, "On downlink beamforming with greedy user selection: Performance analysis and a simple new algorithm," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3857–3868, Oct. 2005.

[6] J. Wang, D. J. Love, and M. D. Zoltowski, "User selection with zero-forcing beamforming achieves the asymptotically optimal sum rate," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3713–3726, Aug. 2008.

[7] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Commun.*, vol. 24, no.3 . pp. 528–541, Mar. 2006.

[8] X. Zhang, E. A. Jorswieck, B. Ottersen, and A. Paulraj, "User selection schemes in multiple antenna broadcast channels with guaranteed performance," in *Proc. SPAWC*, Helsinki, Finland, June 2007.

[9] A. Kahn, R. Vesilo, and I. B. Collins, "Efficient user selection algorithms for wireless broadcast channels," in *Proc. AusWireless*, Sydney, Australia, Aug. 2007.

[10] J. Zhang, Y. Li, S. Zhou, and J. Wang, "Low complexity scheduling algorithms for downlink multiuser MIMO system," *IEICE Trans. Commun.*, vol. E90-B, no. 2, pp. 358–363, Feb. 2007.

[11] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, Jr., and B. L. Evans, "Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization," *IEEE Trans. Signal Process.*, vol. 54, no. 9, pp. 3658–3663, Sept. 2006.

[12] J. C. Roh and B. D. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: A VQ-based approach," *IEEE Trans. Inf. Theory.*, vol. 52, pp. 1101–1112, Mar. 2006.

[13] H. A. David and H. N. Nagaraja, *Order Statistics*. Hoboken, NJ: Wiley, 2003.

[14] R. E. Odeh and J. O. Evans, "The percentage points of the normal distribution," *Applied Statistics*, vol. 23, no. 1., pp. 96–97, 1974.

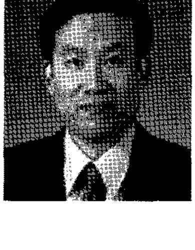
[15] D. L. Wallace, "Bounds on normal approximations to Student's and the chi-square distributions," *Ann. Mathematical Statistics*, vol. 30, no. 4, pp. 1121–1130, Dec. 1959.

[16] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast cahnnels with partial side information," *IEEE Trans. Inf. Theory,* vol. 51, pp. 506–522, Feb. 2005.

[17] C. Yuen and B. M. Hochwald, "Achieving near-capacity at low SNR on a multiple-antenna multiple-user channel," *IEEE Trans. Commun.,* vol. 57, no.1, pp. 69–74, Jan. 2009.

Janghoon Yang received the Ph.D. in Electrical Engineering from the University of Southern California, Los Angeles, USA, in 2001. He is currently an Assistant Professor at Department of Newmedia, Korean German Institute of Technology, Seoul, Korea. From 2001 to 2006, He was with communication R&D center, Samsung Electronics. From 2006 to 2009, he was a Research Assistant Professor at the Department of Electrical and Electronic Engineering, Yonsei University. He has published numerous papers in the area of multi-antenna transmission and signal processing. His research interests include wireless system and network, artificial intelligence, neuroscience, and brain computer interface.

Seunghun Jang received M.S. in Electrical and Electronics Engineering from Yonsei University, Seoul, Korea. He is currently working towards the Ph.D. degree in the Department of Electrical and Electronic Engineering, Yonsei University, Seoul, Korea. His research interests include relay, transceiver optimization, and information theory.



Dong Ku Kim received the Ph.D. in Electrical Engineering from the University of Southern California, Los Angeles, USA, in 1992. He is currently a Professor at Electrical and Electronic Engineering, Yonsei University, Seoul, Korea. From 1992 to 1994, he was with Cellular Infrastructure Group of Motorola. Since then, he has been with Yonsei Unviersity. He has published numerous papers in the area of CDMA and multi-antenna transmission. His research interests include wireless system, relay system, and information theory.