# A Novel Subspace Tracking Algorithm and Its Application to Blind Multiuser Detection in Cellular CDMA Systems

Imran Ali, Doug Nyun Kim, Yun-Jeong Song, and Naeem Zafar Azeemi

*Abstract:* **In this paper, we propose and develop a new algorithm for the principle subspace tracking by orthonormalizing the eigenvectors using an approximation of Gram-Schmidt procedure. We carry out a novel mathematical derivation to show that when this approximated version of Gram-Schmidt procedure is added to a modified form of projection approximation subspace tracking deflation (PASTd) algorithm, the eigenvectors can be orthonormalized within a linear computational complexity. While the PASTd algorithm tries to extracts orthonormalized eigenvectors, the new scheme orthonormalizes the eigenvectors after their extraction, yielding much more tacking efficiency. We apply the new tracking scheme for blind adaptive multiuser detection for non-stationary cellular CDMA environment and use extensive simulation results to demonstrate the performance improvement of the proposed scheme.**

*Index Terms:* **Code division multiple access (CDMA), Gram-Schmidt procedure, multiuser detection, subspace tracking.**

## I. INTRODUCTION

There has been a great interest in the subspace methods as they have been demonstrated to be applied in a variety of signal processing areas such as channel estimation and multiuser detection for code division multiple access (CDMA) systems, pattern recognition, source localization, adaptive filtering etc. Subspace methods primarily rely on separating the signal and noise subspaces by estimation of a few eigenvectors corresponding to signal (or noise) subspace and their corresponding eigenvalues and then compute the parameter of interest [1]–[5]. Although the traditional ways of obtaining the eigen components such as eigen-value decomposition (EVD) and singular value decomposition (SVD) are still the most efficient methods of eigen analysis, however, they have turned out to be non-practical for many of the signal processing applications due to their inherent computational complexity. To address the complexity problem, different approaches have been taken to estimate the principle subspace [6]–[12]. Out of these, projection approximation subspace tracking (PAST) [6] has been acknowledged as one of the most robust and efficient method to estimate the signal subspace under the mild conditions, with linear complexity of $O(NK)$, where $N$ is the dimension of noisy data and $K$ is the number of desired eigen components. However, in a rapidly changing dynamic environment, such as cellular CDMA system, this algorithm turns out to be relatively slow in converging to signal subspace and in many situations, it keeps oscillating rather than converging at all [13]. This is because PAST approximates the projection of current input vector on the columns of the weight matrix in current iteration with projection of current input vector on weight matrix in previous iteration. Hence the iterative minimization of this approximated cost function using method of least squares estimates cannot guarantee the orthonormality of eigenvectors and the algorithm cannot always lead to the convergence of weight matrix into the signal subspace. Within [6], the author proposes that any standard orthonormalization procedure can be recursively applied to the updated correlation matrix of input vector, with complexity of $O(NK^2)$ but such a scheme leads to poor numerical properties due to bad conditioning of correlation matrix, especially at low SNR. More recently, orthogonal PAST (OPAST) [13] and exponential window fast approximated power iteratioin (FAPI) [8] algorithms have been reported in the literature. The OPAST, as the name suggests, is an orthonormalized version of PAST algorithm and it can track eigenvectors very efficiently with same linear complexity as PAST algorithm. The FAPI algorithm is based upon power iteration method [14] and in our simulation results, we found its tracking capability to be as much efficient as that of OPAST and it also has same linear complexity, i.g., $O(NK)$. Although both of these two algorithms can track the dominant eigenvectors, they can not track corresponding eigenvalues, thus they can be treated as examples of plain learning rather than coupled learning. In many subspace applications such as blind multiuser detection [15], eigenvalues are also needed to be computed. Estimation of eigenvalues from eigenvectors increases the computational complexity up to $O(N^2K)$.

In this paper, we propose a new scheme for the tracking of eigen components by adding an orthonormalization scheme to a modified form of PAST deflation (PASTd) algorithm. Starting from an approximation of Gram-Schmidt procedure developed by Oja and Karhunen [16], [17], we derive a new framework to show that eigenvectors extracted by PASTd algorithm can be orthonormalized, so that both eigenvectors and eigenvalues can be extracted efficiently, within linear computational complexity. We implement the subspace multiuser detection using the subspace components obtained by the proposed scheme and demonstrate performance improvement.

## II. SYSTEM MODEL

Since this paper is focused on application of subspace tracking in CDMA communication system, so we develop a signal

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model for a cellular CDMA system; however, the algorithm can equally well be applied to any subspace application. We consider a synchronous CDMA system, with $K$ active users in the cell and processing gain of $N$. The data bit of $k$th user at time $t$, denoted as $b_k(t)$, is transmitted after getting spread by the spreading code of $k$th user given as $\mathbf{c}_k = [c_1^k, c_2^k, \cdots, c_N^k]^T$. The received signal will be the superposition of $K$ user's signals each of which passing through $L$ path channels given by [18],

$$r(t) = \sum_{k=1}^{K} A_k b_k(t) \sum_{l=1}^{L} g_{k,l} c_{k,l}(t - \tau_{k,l}) + \delta n(t) \quad (1)$$

where $A_k$ is the received amplitude of the $k$th user, $g_{k,l}$, $s_{k,l}$, and $\tau_{k,l}$ are the channel gain, received signature, and propagation delay of $k$th user from $l$th path, respectively, and $n(t)$ additive white Gaussian noise process. Here, we assume that $T_m \gg \tau_{k,s}$, where $T_m$ is symbol time and $\tau_{k,s}$ is the $k$th user's delay spread and in such situations, intersymbol interference (ISI) can be neglected [18], [19]. The received signal is then sampled at chip rate, and collected as a series of $N \times 1$ sampled vectors corresponding to one bit duration, so that sampled vector at any general sampling instant is given as

$$\mathbf{r} = \mathbf{CGAb} + \mathbf{n} \quad (2)$$

where $N \times L$ vector $\mathbf{C}_k$ is given as $\mathbf{C}_k = [\mathbf{c}_{k,1}, \mathbf{c}_{k,2}, \cdots, \mathbf{c}_{k,L}]$ so that $\mathbf{c}_{k,l}$ is spreading vector of $k$th user from $l$th path and we define $N \times KL$ matrix, $\mathbf{C} := [\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_K]$. Similarly the channel gain matrix, $\mathbf{G}$, is defined as $\mathbf{G} := [\mathbf{G}_1, \mathbf{G}_2, \cdots, \mathbf{G}_K]$, so that $\mathbf{G}_k = [g_{k,1}, g_{k,2}, \cdots, g_{k,L}]^H$ and its last $KL - L$ rows have all zeros entries to make its order $KL \times K$ for multiplication compatibility. We define $\mathbf{A} := diag(A_1, A_2, \cdots, A_K)$ and $\mathbf{b} = [b_1, b_2, \cdots, b_K]^T$. The product of matrices $\mathbf{C}$ and $\mathbf{G}$ is $N \times K$ matrix $\mathbf{D}$, i.g., $\mathbf{D} := \mathbf{CG}$, so that each of its column, $\mathbf{d}_k$, is effective signature of the $k$th user. Thus, composite received vector is given as

$$\mathbf{r} = \sum_{k=1}^{K} A_k \mathbf{b}_k \mathbf{d}_k + \mathbf{n}. \quad (3)$$

The autocorrelation matrix for the received input vector is given as

$$\mathbf{R} = E\{\mathbf{r}\mathbf{r}^H\} = \mathbf{DAD}^H + \delta^2 I_N. \quad (4)$$

The EVD of the $N \times N$ autocorrelation matrix, $\mathbf{R}$, can be written as

$$\mathbf{R} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \mathbf{U}_s \Lambda_s \mathbf{U}_s^H \quad (5)$$

where $\Lambda_s = diag(\lambda_1, \lambda_2, \cdots, \lambda_K)$ contains the $K$ largest eigenvalues; their corresponding eigenvectors are the columns of $\mathbf{U}_s$ and remaining $N - K$ eigenvalues, all equal to $\delta^2$, are diagonal entries of $\Lambda_s$ and their corresponding eigenvectors are columns of $\mathbf{U}_n$. Several parameters of crucial importance have been shown to be estimated from signal subspace $\{u_k, \lambda_k\}, k = 1, 2, \cdots, K$, such as channel state information, blind multiuser detection, direction of arrival in MIMO applications [1], [4], [15], etc.

The signal in (3) contains interference from $K - 1$ CDMA users. To detect $k$th user's bit corresponding to $t$th sampling instant, $b_k(t)$, from received sampled vector $\mathbf{r}(t)$, the multiuser detector, $\mathbf{m}(t)$, is computed. There are numerous ways to compute multiuser detector; the one considered in this paper is based upon signal subspace obtained at the $t$th sampling interval [15]

$$\mathbf{m}_1(t) = \frac{\mathbf{U}_s(t)\Lambda_s^{-1}(t)\mathbf{U}_s^H(t)\mathbf{d}_1(t)}{\mathbf{d}_1^H \mathbf{U}_s(t)\Lambda_s^{-1}(t)\mathbf{U}_s^H(t)\mathbf{d}_1(t)} \quad (6)$$

so that, $b_k(t) = \text{sign}(\mathbf{m}_k^H(t)\mathbf{r}(t))$. It can be noted that the multiuser detector in (7) needs only the timing and effective signature of desired user (besides the signal subspace components), so it is completely blind.

Table 1. The PASTd algorithm.

| | |
|---|---|
| $\mathbf{x}_1(t) = \mathbf{r}(t)$ | |
| FOR $k = 1 : K$ | |
| $y_k(t) = \mathbf{u}_k^H(t-1)\mathbf{x}_k(t)$ | (I) |
| $\lambda_k(t) = \beta\lambda_k(t-1) + \|y_k(t)\|^2$ | (II) |
| $\tau_k(t) = (\frac{y_k^H(t)}{\lambda_k(t)})$ | (III) |
| $\mathbf{m}_k(t) = [\mathbf{x}_k(t) - \mathbf{u}_k(t-1)y_k(t)]\tau_k(t)$ | (IV) |
| $\mathbf{u}_k(t) = \mathbf{u}_k(t-1) + \mathbf{m}_k(t)$ | (V) |
| $\mathbf{x}_{k+1}(t) = \mathbf{x}_k(t) - \mathbf{u}_k(t)y_k(t)$ | (VI) |
| END | |

## III. SUBSPACE TRACKING

Consider the sampled received vector, $\mathbf{r}$ and sampling index $t$, then an unconstrained cost function [6],

$$\begin{aligned} J(\mathbf{W}(t)) &= E\{\|\mathbf{r}(t) - \mathbf{W}(t)\mathbf{W}^H(t)\mathbf{r}(t)\|\}^2 \\ &= tr(\mathbf{R}(t)) - 2tr(\mathbf{W}^H(t)\mathbf{R}(t)\mathbf{W})(t) \\ &\quad + tr(\mathbf{W}^H(t)\mathbf{R}(t)\mathbf{W}(t).\mathbf{W}^H(t)\mathbf{W}(t)) \end{aligned} \quad (7)$$

will have a stationary point $\mathbf{W}$, such that $\mathbf{W} = \mathbf{U}_s\mathbf{Q}$, where $\mathbf{W} \in \mathbf{C}^{K \times K}$ contains $K$ dominant eigenvectors of correlation matrix of the received vector $\mathbf{r}$. Furthermore by iterative minimization of $J(\mathbf{W})$, its global minimum, $\mathbf{W}_{\min}$, contains $K$ distinct dominant eigenvectors of correlation matrix, $\mathbf{R}$. Replacing the expectation in the cost function with exponentially weighted modified sum modifies the cost function as

$$J'(\mathbf{W}(t)) = \sum_{j=1}^{t} \beta^{t-j} \|\mathbf{r}(t) - \mathbf{W}(t)\mathbf{W}^H\mathbf{y}(t)\|^2 \quad (8)$$

where $\beta$ is forgetting factor, defined as $0 < \beta < 1$. Here, the authors [6] make an approximation that $\mathbf{y}(t) := \mathbf{W}^H(t)\mathbf{r}(t) \cong \mathbf{W}^H(t-1)\mathbf{r}(t)$, such that equation (9) becomes

$$J'(\mathbf{W}(t)) = \sum_{j=1}^{t} \beta^{t-j} \|\mathbf{r}(t) - \mathbf{W}(t)\mathbf{y}(t)\|^2. \quad (9)$$

Above approximated exponentially weighted modified cost function can be solved for $\mathbf{W}(t)$ using RLS algorithm which results in the PAST and its deflation version PASTd algorithm, which is shown in Table 1.

In the PASTd algorithm, after updating $k$th eigenvector, it is subtracted away from the received signal and resultant quantity is treated as received vector for updating next eigenvector (step (VI), Table 1). This is done to ensure that next eigenvector spans the null space of previous eigenvector and hence the orthonormality can be achieved. However, this method of orthonormalization is not effective, as addressed in [13].

### A. Proposed Subspace Tracking Scheme

Now consider any general coupled-learning algorithm, where innovation introduced to update each eigenvector is denoted as $\mathbf{m}_k(t)$, so that we can write

$$\mathbf{u}_k(t) = \mathbf{u}_k(t-1) + \mathbf{m}_k(t). \tag{10}$$

Once this eigenvector is updated, the standard Gram-Schmidt procedure for orthonormalization is given by two steps

$$\mathbf{u}_k^*(t) = \mathbf{u}_k(t) - \sum_{j=1}^{k-1}\{\mathbf{u}_j^H(t)\mathbf{u}_k(t)\}\mathbf{u}_j(t) \tag{11}$$

$$\mathbf{u}_k^o(t) = \frac{\mathbf{u}_j^*(t)}{\|\mathbf{u}_j^*(t)\|}. \tag{12}$$

As can be seen, the summation in (11) contains both index $j$ and $k$, which means for each $k$th eigenvector, the $j$ must be started from zero all over again, which is the main reason that the complexity of standard Gram-Schmidt procedure is considered to be high. The complexity can be reduced if index $k$ is taken out of summation, which can not be done in this form of equation (11) since the index $k$ is involved in vector product inside the summation.

Now if we introduce (11) and (12) in (10) and considering that, since $\mathbf{m}(t)$ is very small its quadratic terms can be considered to be zero, then we can write the approximated form of Gram-Schmidt procedure [16], [17], as given in equation (13), where $\hat{\mathbf{u}}(t)$ is approximately orthogonalized form of $\mathbf{u}(t)$ and normalization step in (12) is also absorbed in (13)

$$\begin{aligned}\hat{\mathbf{u}}(t) =& \{\sum_{j=1}^{k-1}(\mathbf{u}_j(t-1)\mathbf{m}_k(t) + \mathbf{u}_k^H(t-1)\mathbf{m}_j(t))\mathbf{u}_j(t-1)\}\\ &- (\mathbf{u}_k^H(t-1)\mathbf{m}_k(t))\mathbf{u}_k(t-1).\end{aligned} \tag{13}$$

The orthonormalization step (13) in its current form will reduce the complexity since summation still involves both index $j$ and $k$. However, as we will show now, if (13) is applied for the orthonormalization of PASTd algorithm, the index $k$ can be taken out of summation, resulting in a new orthonormalization expression with linear complexity. Consider only the summation term of (13) for now, which can be broken into two parts as

$$\begin{aligned}\mathbf{sum} =& \sum_{j=1}^{k-1}(\mathbf{u}_j^H(t-1)\mathbf{m}_k(t))\mathbf{u}_j(t-1)\\ &+ \sum_{j=1}^{k-1}(\mathbf{u}_k^H(t-1)\mathbf{m}_j(t))\mathbf{u}_j(t-1).\end{aligned} \tag{14}$$

Now here, if we want to use (13) to update eigenvector in PASTd algorithm to get orthonormalized eigenvectors, we can remove the step (VI) in Table 1. In such case, we can see that, $\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_{k-1}(t)$ and let us denote it as $\mathbf{x}(t)$. Then the innovation can $\mathbf{m}_k(t)$ in the step (IV) can be written as

$$\mathbf{m}_k(t) = [\mathbf{x}(t) - \mathbf{u}_k(t-1)y_k(t)]\tau_k(t). \tag{15}$$

Consider the first summation in (14) and put the value of $\mathbf{m}_k(t)$ from (15),

$$\mathbf{sum1} = \sum_{j=1}^{k-1}\{\mathbf{u}_j^H(t-1)[\mathbf{x}(t) - \mathbf{u}_k(t-1)y_k(t)]\tau_k(t)\}\mathbf{u}_j(t-1). \tag{16}$$

Now, we can see from Table 1 that $y_j(t) = \mathbf{u}_j^H(t)\mathbf{x}(t)$ as $\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_{k-1}(t) = \mathbf{x}(t)$. Furthermore, we assume that eigenvectors were orthogonal in previous iteration (and this assumption becomes reality if we initialize PASTd algorithm by applying SVD on first 50 data vectors, as in [15]) and since the index $j$ goes upto $k-1$ only, the product $\mathbf{u}_j^H(t-1)\mathbf{u}_k(t-1) = 0$. Introducing these two facts in (16), we get

$$\begin{aligned}\mathbf{sum1} &= \sum_{j=1}^{k-1} y_j(t)\tau_k(t)\mathbf{u}_j(t-1)\\ &= \tau_k(t)\sum_{j=1}^{k-1} y_j(t)\mathbf{u}_j(t-1)\end{aligned} \tag{17}$$

where the second equality follows from the fact that $\tau_k(t)$ is a scalar independent of index $j$ and can be taken out of summation. Thus, we successfully removed the index $k$ from the first summation. Now consider the second summation in (15) and put the value of $\mathbf{m}_j(t)$ from (15),

$$\mathbf{sum2} = \sum_{j=1}^{k-1}\{\mathbf{u}_k^H(t-1)[\mathbf{x}(t) - \mathbf{u}_j(t-1)y_j(t)]\tau_j(t)\}\mathbf{u}_j(t-1). \tag{18}$$

Using the same reasoning as that for the first summation, we can say $y_k(t) = \mathbf{u}_k^H(t-1)\mathbf{x}(t)$ and $\mathbf{u}_j^H(t-1)\mathbf{u}_k(t-1) = 0$, so that we can modify (18) as

$$\begin{aligned}\mathbf{sum2} &= \sum_{j=1}^{k-1} y_k(t)\tau_j(t)\mathbf{u}_j(t-1)\\ &= y_k(t)\sum_{j=1}^{k-1} \tau_j(t)\mathbf{u}_j(t-1)\end{aligned} \tag{19}$$

where the second equality frollows from the fact that $y_k(t)$ is scalar and independent of index $j$, so it can be taken out of the summation. Thus, using (17), (18), and (19) in (13), the approximated form Gram-Schmidt algorithm can be written as

$$\begin{aligned}\hat{\mathbf{u}}_k(t) =& \mathbf{u}_k(t) - \tau_k(t)(\sum_{j=1}^{k-1} y_j(t)\mathbf{u}_j(t-1))\\ &- y_k(t)(\sum_{j=1}^{k-1}\tau_j(t)\mathbf{u}(t-1))\\ &- (\mathbf{u}(t-1)\mathbf{m}(t)\mathbf{u}(t-1)).\end{aligned} \tag{20}$$

Table 2. Proposed modified PASTd algorithm.

$\mathbf{x}_1(t) = \mathbf{r}(t), \mathbf{sum1} = \mathbf{0}, \mathbf{sum2} = \mathbf{0}$

FOR $k = 1 : K$

$y_k(t) = \mathbf{u}_k^H(t-1)\mathbf{x}_k(t)$ (I)

$\lambda_k(t) = \beta\lambda_k(t-1) + |y_k(t)|^2$ (II)

$\tau_k(t) = (\frac{y_k^H(t)}{\lambda_k(t)})$ (III)

$\mathbf{m}_k(t) = [\mathbf{x}_k(t) - \mathbf{u}_k(t-1)y_k(t)]\tau_k(t)$ (IV)

IF $k \geq 2$

$\mathbf{sum1} = \mathbf{sum1} + (y_{k-1}(t)\mathbf{u}_{k-1}(t-1))$ (V)

$\mathbf{sum2} = \mathbf{sum2} + (\tau_{k-1}(t)\mathbf{u}_{k-1}(t-1)$ (VI)

$\mathbf{sum3} = \{\mathbf{u}_k^H(t-1)\mathbf{m}_k(t)\}\mathbf{u}_k(t-1))$ (VII)

END

$\mathbf{u}_k'(t) = \mathbf{u}_k(t-1) + \mathbf{m}_k(t) - \tau_k(t)\mathbf{sum1}$

$-y_k(t)\mathbf{sum2} - \mathbf{sum3}$ (VIII)

$\mathbf{u}_k(t) = \frac{\mathbf{u}_k'(t)}{\|\mathbf{u}_k'(t)\|}$ (IX)

END

Table 3. Detailed account of complexity for proposed scheme.

| Step | Additions | Multiplications |
|---|---|---|
| I | $NK$ | $NK - K$ |
| II | $2K$ | $K$ |
| III | $2K$ | 0 |
| IV | $2NK$ | $NK$ |
| V | $NK$ | $NK$ |
| VI | $NK$ | $NK$ |
| VII | $2NK$ | $NK - K$ |
| VIII | $2NK$ | $4NK$ |
| IX | $N - 1$ | $N + 1$ |
| Total | $9NK + N + 4K - 1$ | $8NK + N - K - 1$ |
| Order | $O(NK)$ | $O(NK)$ |

Table 4. Comparison of subspace tracking complexity algorithms.

| Step | Additions | Multiplications |
|---|---|---|
| PASTd | $3NK + K$ | $4NK + 2K$ |
| Proposed | $9NK + 4K$ | $9NK - K$ |
| NOOja | $NK + 5N + K + 9$ | $4NK + 6N + 3K + 3$ |
| OPAST | $4NK + 2K - 1$ | $4NK + 4N + 4K^2$ $+5K + 11$ |
| FAPI | $3NK + N + 4K^2$ $+4K$ | $3NK + 2N + 6K^2$ $+11K + 14$ |

Apparently, (20) looks very complicated, however, as shown in the Table 2, it has much less complexity since the index $j$ needs not be started from zero for all the eigenvectors. Moreover, since eigenvalues' update is kept unchanged from PASTd algorithm's approach, they are tracked efficiently, unlike the OPAST.

### *B. Complexity Comparison*

As can be seen from the Table 3, the total computational complexity is linear with respect to both $N$ and $K$, though it is higher than PASTd and OPAST. Thus, we derived an approximation of Gram-Schmidt procedure for PASTd algorithm, which reduced the complexity from $O(NK^2)$ to $O(NK)$. This difference will become significant when the number of subspace components to be extracted is high.

Table 4 compares the computational complexity of subspace tracking algorithms. It should be noted, however, that normalized orthogonal Oja (NOOja) [20], OPAST, and FAPI algorithms do not track the eigenvalues of the principle subspace, so they cannot be used directly to compute subspace multiuser detector. To compute eigenvalues from eigenvectors, the Rayleigh quotient [21]

$$\lambda_k(t) = \frac{\mathbf{u}_k^H(t)\mathbf{R}(t)\mathbf{u}_k(t)}{\mathbf{u}_k^H(t)\mathbf{u}_k(t)} \quad (21)$$

is used and for that we additionally need to keep an updated correlation matrix of received vector by

$$\mathbf{R}(t) = \beta\mathbf{R}(t-1) + \mathbf{r}(t)\mathbf{r}^H(t). \quad (22)$$

This additional computation requires $N^2K + 2NK$ multiplications and $N^2K+2NK$ additions. Thus, for the multiuser detection application, complexity of NOOja, OPAST and FAPI algorithms turn outs to be $O(N^2K)$. Contrary to that, the proposed algorithm keeps track of eigenvalues and can be used to implement multiuser detection within linear computational complexity.

## IV. SIMULATION

In this section, we will present simulation results to demonstrate the performance of proposed scheme. Consider a synchronous CDMA cell with $K = 10$ active users, transmitting BPSK modulated data, spread by randomly generated code with spreading gain of $N = 31$. Let $k = 1$ is the desired user, with signal to noise ratio of $SNR = 20$ (unless otherwise notified). Five of the interfering users are 10 dB, three are 20 dB and one user is 30 dB stronger than desired user, e.g., $A_1^2 = 1, A_2^2 = A_3^2 = \cdots = A_6^2 = 10, A_7^2 = A_8^2 = A_9^2 = 100$, and $A_{10}^2 = 1000$. A non-stationary frequency selective channel is used with $L = 5$ channel paths is used, where the Doppler frequency of desired user is 100 Hz and that for interfering users is uniformly distributed in the interval of $[1, 100)$. Under the Rayleigh channel assumption, the channel gains are generated as Gaussian random numbers. A total of 1000 bits were transmitted in all examples except the Example 3 where 10,000 bits were used to calculate BER results. The results for PASTd have not been included since it could not produce comparable results in most examples. Where applicable, the eigenvalues for OPAST and FAPI algorithms are obtained by Rayleigh quotient.

*Example 1 (Subspace error)*: In the first example, we will demonstrate the subspace estimation efficiency of the proposed scheme. Since, for a certain matrix, eigen components are unique, so we can demonstrate the tracking capability by measuring the relative difference between eigen components ob-

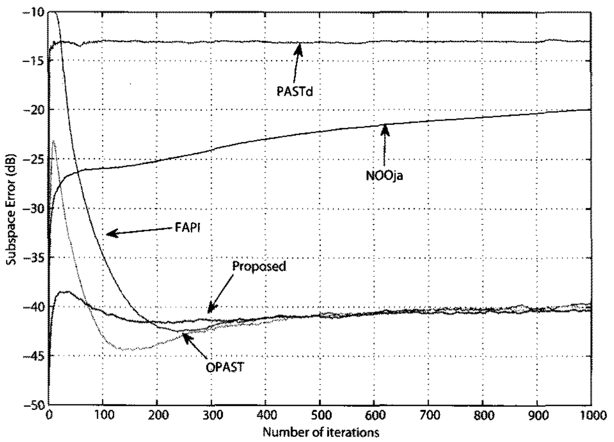


Fig. 1. Comparison of subspace error of proposed algorithm with different subspace tracking algorithms.

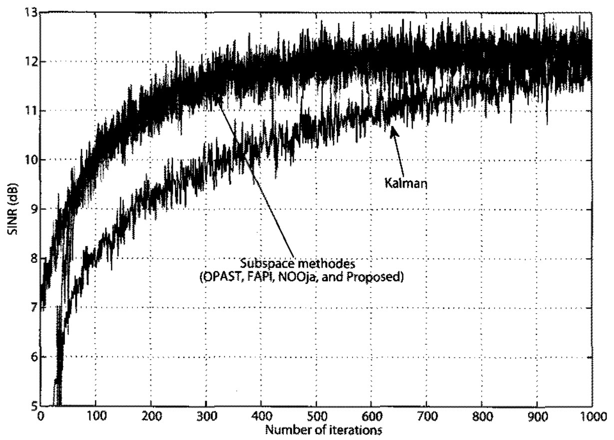


Fig. 2. BER performance comparison of different subspace based and Kalman filter based multiuser detectors.

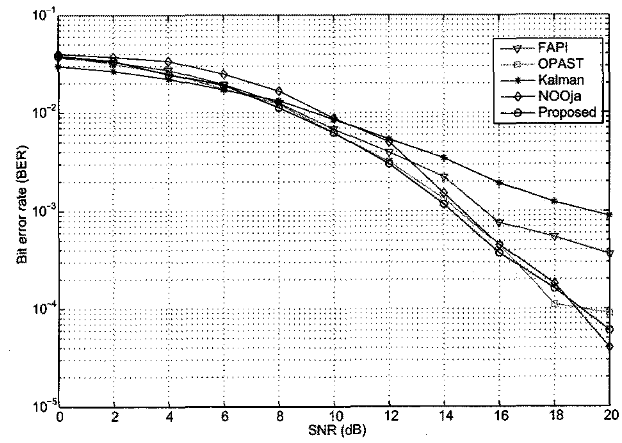


Fig. 3. BER performance comparison of subspace detector with Kalman filter based multiuser detector.

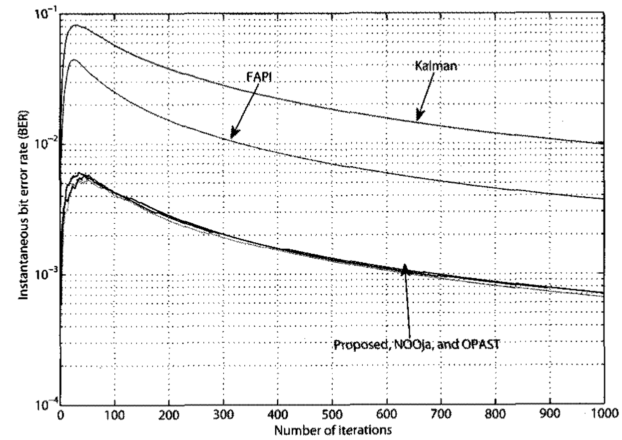


Fig. 4. Instantaneous BER tracking comparison of different subspace based and Kalman filter based multiuser detectors.

tained by tracking and those by directly applying EVD on matrix. Subspace error can thus be given as [13]

$$\mathrm{SE}(t) = \frac{\|\{\mathbf{I}_N - \mathbf{U}_s(\mathbf{U}_s^H\mathbf{U}_s)^{-1}\mathbf{U}_s^H\}\mathbf{U}_E\mathbf{U}_E^H\|}{\sqrt{K}} \tag{23}$$

where $\mathbf{U}_E$ and $\mathbf{U}_S$ are obtained by EVD and any tracking algorithm respectively. Fig. 1 plots and compares the subspace error performance of proposed scheme, OPAST, and FAPI algorithms in units of dB. As can be seen, the proposed scheme maintain $-40$ dB error right from the start; however, OPAST and FAPI reach the same result in mature state.

*Example 2 (SINR performance)*: The output signal to noise and interference ratio (SINR) is given by

$$\mathrm{SINR}(t) = 10\log_{10}\left(\frac{\{\mathbf{m}_1^H(t)A_1\mathbf{d}_1(t)\}^2}{\{\mathbf{m}_1^H(t)(\mathbf{r}(t) - \widehat{b}_1(t)A_1\mathbf{d}_1(t))\}^2}\right). \tag{24}$$

Fig. 2 plots the output SINR averaged over 500 simulation runs for different subspace tracking methods. For the sake of comparison, we have plotted SINR performance of recently proposed Kalman filter detector [22], applied to same set of system parameters. As can be seen, subspace based methods outperforms the Kalman filter detector significantly, where as the proposed schemes needs least computation for this performance.

*Example 3 (BER performance)*: The bit error rate, (BER) performance of subspace multiuser detector is compared with Kalman filter in Fig. 3. A total of $100,000$ bits were inputted to each algorithm at SNR ranging from 5 dB to 20 dB. The numbers of errors considered for each SNR value were averaged over 25 simulation runs.

*Example 4 (BER tracking performance)*: An adaptive algorithm improves its performance as it tends towards its convergence. An important measure of performance can thus be the instantaneous bit error rate at each iteration, which shows how much it is probable for a detector to receive a bit in error towards its way to convergence. Fig. 4 compares the BER tracking performance of subspace detectors and Kalman filter.

BER performance comparison of subspace detector with Kalman filter based multiuser detector.

## V. CONCLUSION

In this paper, we proposed a new subspace tracking algorithm by approximately orthonormalizing the PASTd algorithm with an approximation of Gram-Schmidt procedure. We showed that, although the complexity of proposed tracking scheme was higher than the PASTd algorithm itself, but it was still linear with respect to both number of subspace components to be tracked as well the dimension of noisy data. We showed that some of the recently proposed well known subspace tracking schemes cannot implement multiuser detector directly as they cannot track eigenvalues. We implemented the subspace multiuser detection using the subspace components tracked by the proposed scheme and showed using several performance measurement criteria that it outperforms the well know Kalman filter based multiuser detector.

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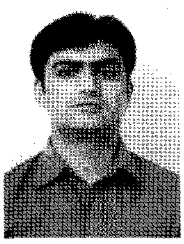

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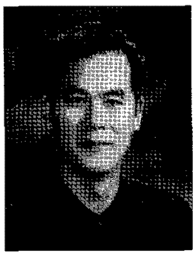

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