

# 멀티홉 셀룰러 네트워크에서 릴레이 위치에 따른 하향링크 SINR 분석

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## Downlink SINR Analysis of Multihop Cellular Networks according to Relay Positions

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### 요 약

본 논문은 멀티홉 셀룰러 네트워크에서 릴레이 노드의 배치 위치가 하향링크 SINR에 미치는 영향을 분석한다. 분석 모델에서는 릴레이 노드가 셀 안쪽으로 배치되는 것과 인접한 두 셀의 경계지역에 배치되는 두 가지 시나리오를 가정한다. 본 논문에서는 위 두 가지 시나리오의 하향링크 SINR을 비교 분석하기 위한 수학적 모델을 도출한다. 제안하는 수학적 모델에서는 멀티-셀 구조 및 셀 간 간섭 등을 고려한다. 수학적 분석 결과는 릴레이 노드들을 셀 안쪽에 배치하는 경우에 셀 간 간섭의 증가에도 불구하고 수신 신호 세기의 증가로 인해 릴레이 노드들을 셀 경계에 배치하는 경우에 비해 SINR이 증가함을 보인다.

**Key Words** : Multihop Cellular Networks, relay, Relay Positions, SINR, Interference

### ABSTRACT

This paper studies the effect of the deployment position of the relay stations on the downlink signal-to-interference-noise-ratio (SINR) in multihop cellular networks. Two different relay deployment scenarios are considered where relay stations are located either inside cells or on the boundary among adjacent cells. The fundamental contribution is to compare fairly the average SINR between two scenarios with the proposed relay modeling framework that includes multi-cell geometries and inter-cell interferences. The mathematical results show that the SINR increases when relay stations are located inside cells because of higher received signal power.

### I. Introduction

In multihop cellular networks, technologies sometimes called relay technologies are applied to the current cellular networks to increase high-data-rate regions or to compensate for shadowing effect, which is mainly caused by large

obstacles between transceivers<sup>[1,2]</sup>. Recently, the capacity of multihop cellular networks has been studied from the various points of view<sup>[3-5]</sup>. The performance of time-division multihop cellular networks, where a base station (BS) and relay stations (RSs) are multiplexed in the time domain, has been investigated in [3]. Reference [4] gives

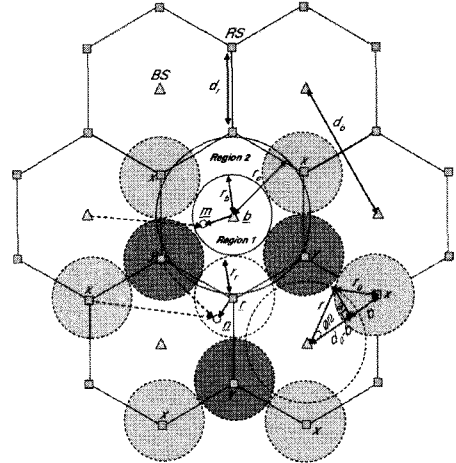
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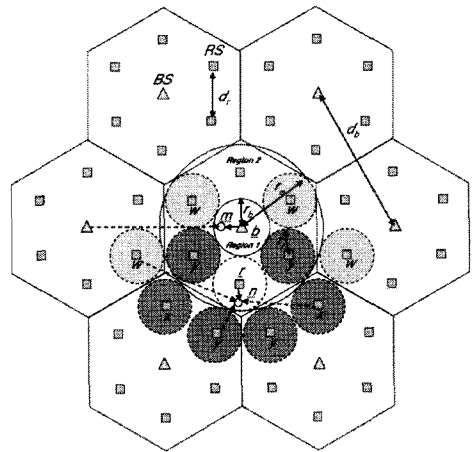
upper bounds for multihop cellular networks with uniform placement of relays within a cell and evaluates the capacity from relays according to the number of hops. Reference [5] presents a relay-station-placement algorithm in high speed downlink packet access (HSDPA). This algorithm is an attempt to determine an optimal number of relay stations for a given data-rate threshold and determine appropriate locations of relay stations. The previous works well define the capacity of multihop cellular networks compared with single-hop networks. However, a SINR or capacity model of multihop cellular networks according to the position of relay stations has not been clearly defined, even though the placement of relay stations highly affects the SINR and capacity of multihop cellular networks. Therefore, this paper defines the typical relay station deployment scenarios and compares downlink SINR values between them by mathematical models.

## II. System Model

Figures 1(a) and 1(b) illustrate the considering relay deployment *Scenario 1* and *Scenario 2*, respectively. RSs are supposed to be located on the boundary among adjacent cells and controlled by the adjacent BSs as in *Scenario 1* that is better at reducing the total deployment number of RSs. Meanwhile, RSs are located inside cells and controlled by a base station (BS), as in *Scenario 2*. Many previous works<sup>[3,4]</sup> assume that RSs are located inside cells and controlled by a base station (BS), as in *Scenario 2*, because it is favorable to increase cell radiuses or high-data-rate regions. The frequency-reuse factor (FRF) of both scenarios is assumed to be one, which means that all BSs and RSs share the same spectrum band. Therefore, the resources of a BS and RSs should be coordinated for multiplexing. This paper mainly considers time division multiplexing (TDM) between a BS and RSs. It is assumed that the transparent-mode frame structure defined in IEEE 802.16<sup>[6]</sup> is used in the proposed mathematical models. The timing of every frame is assumed to be synchronized between



(a) Relay deployment scenario 1



(b) Relay deployment scenario 2

Fig. 1. Relay deployment scenarios

adjacent BSs and the duplexing between downlink and uplink is based on time division duplexing.

## III. The SINR Analysis

This section proposes an analysis of the received SINR values from mathematical models for *Scenarios 1* and *2*. The model assumes that a perfect equalization and an OFDM transmission completely compensate for fast fading and intra-cell interferences, respectively. Large-scale fadings such as propagation loss and shadow fading are considered with uniformly distributed mobile stations (MSs) in seven cells.

### 3.1 Relay Deployment Scenario 1

As shown in Fig.1(a), two regions are defined in each cell. The first region defined by an equivalent radius of  $r_b$  is a circled area where MSs are served by a BS. The second one is a doughnut-shaped region which is defined by two radiuses,  $r_b$  and  $r_e$ . In the second region, all MSs have communications with relay stations. In Region 1, the received signal power at an MS  $m$  from a BS  $b$ ,  $P_{b \rightarrow m}^{(r)}$  is calculated as:

$$P_{b \rightarrow m}^{(r)} = \alpha_m P_{BS}^{(t)} \cdot r_{b \rightarrow m}^{-\gamma} \cdot 10^{-\zeta_{b \rightarrow m}/10}, \quad (1)$$

where  $P_{BS}^{(t)}$ ,  $\alpha_m$ ,  $\gamma$ , and  $\zeta_{b \rightarrow m}$  denote the maximum transmission power of a BS, the proportion of the allocated resources for the MS  $m$  to the whole downlink resources, an attenuation constant, and a log-normal shadow fading, respectively.

The received interference power at MS  $m$ ,  $I_m^{(r)}$  is derived as [7]:

$$I_m^{(r)} = \frac{\theta_b}{360^\circ} \sum_z \left\{ \sum_{j \in z} (\alpha_j P_{BS}^{(t)}) \cdot r_{z \rightarrow m}^{-\gamma} \cdot 10^{-\zeta_{z \rightarrow m}/10} \right\}, \quad (2)$$

where  $\theta_b$ ,  $z$ , and  $j$  represent a main lobe width of sectored cells, an index of adjacent BSs around BS  $b$ , and an index of MSs, respectively. From Eqs. (1) and (2), the received SINR for MS  $m$  in Region 1 can be defined as  $E[P_{b \rightarrow m}^{(r)} / (I_m^{(r)} + N_0)]$ . Since  $P_{b \rightarrow m}^{(r)}$  and  $I_m^{(r)}$  are i.i.d in the proposed model, the lower bound of the received SINR for MS  $m$  in Region 1 can be derived by Jensen's inequality as follows:

$$\begin{aligned} I_{b \rightarrow m}^{(r)} &= \frac{E[P_{b \rightarrow m}^{(r)}]}{E[I_m^{(r)}] + N_0} \\ &\approx \frac{\overline{\alpha_b P_{BS}^{(t)}} \cdot \mathbf{F}_{-\gamma}(r_b, \epsilon_b) \cdot \mathbf{S}(\zeta_{b \rightarrow m})}{\frac{\theta_b}{360^\circ} \sum_z (\overline{\alpha_b P_{BS}^{(t)}} \cdot \overline{\delta_b} \cdot \mathbf{G}_{-\gamma}(r_b, d_b) \cdot \mathbf{S}(\zeta_{z \rightarrow m})) + N_0} \\ &\approx \frac{\overline{\alpha_r P_{BS}^{(t)}} \cdot \mathbf{F}_{-\gamma}(r_b, \epsilon_b) \cdot \mathbf{S}(\zeta_{b \rightarrow m})}{\frac{\alpha_b P_{BS}^{(t)} \cdot \theta_b \cdot 6}{360^\circ} \cdot \frac{\overline{\delta r_b^2}}{r_e^2} \cdot \mathbf{G}_{-\gamma}(r_b, d_b) \cdot \mathbf{S}(\zeta_{z \rightarrow m}) + N_0} \quad (3) \end{aligned}$$

where  $\overline{\alpha_b}$ ,  $\overline{\delta_b}$ , and  $\overline{\delta}$  denote the mean resource allocation ratio, the average number of active MSs within a radius of  $r_b$ , and the average number of active MSs in an entire cell, respectively. Moreover, functions  $\mathbf{F}_n(r, d)$  and  $\mathbf{G}_n(r, \epsilon)$  represent the expectations of propagation loss terms, and  $\mathbf{S}(\zeta)$  is the expectation of shadow fading  $\zeta$ . The detailed derivations of these functions are presented in Appendix.

In Region 2, an MS  $n$  is served by RS  $r$ , and is interfered by the three nearest neighbor RSs indexed by  $y$  and the six second nearest neighbor RSs indexed by  $x$ , which are located at a distance of  $\sqrt{3}d_r$  from RS  $r$ . The radius of RS cell,  $r_r$  is equal to  $(r_e - r_b)$  in this scenario. Thus, the lower bound of the received SINR of MS  $n$  can be expressed as:

$$\begin{aligned} I_{r \rightarrow n}^{(r)} &= \frac{E[P_{r \rightarrow n}^{(r)}]}{E[I_n^{(r)}] + N_0} \\ &\approx \frac{\overline{\alpha_r P_{RS}^{(t)}} \cdot \mathbf{F}_{-\gamma}(r_r, \epsilon_r) \cdot \mathbf{S}(\zeta_{r \rightarrow n})}{\left\{ \frac{\overline{\alpha_r P_{RS}^{(t)}} \cdot \theta_r}{360^\circ} \cdot \frac{\overline{\delta r_r^2}}{r_e^2} \cdot (3\mathbf{G}_{-\gamma}(r_r, d_r) \cdot \mathbf{S}(\zeta_{y \rightarrow n})) \right.} \\ &\quad \left. + 6\mathbf{G}_{-\gamma}(r_r, \sqrt{3}d_r) \cdot \mathbf{S}(\zeta_{x \rightarrow n}) \right\} + N_0} \quad (4) \end{aligned}$$

where  $P_{RS}^{(t)}$ ,  $\overline{\alpha_r}$ , and  $\theta_r$  denote the maximum transmission power of a RS, the mean resource allocation ratio of an MS served by a RS, and a main lobe width of sectored RSs, respectively.

### 3.2 Relay Deployment Scenario 2

Like Scenario 1, two regions are defined as shown in Fig. 1(b). In Region 1, all derivations are the same as the relay deployment Scenario 1 because the equivalent radius of Region 1 is also set to  $r_b$ . Thus, the lower bound of the received SINR for MS  $m$  in Region 1 is the same as Eq. (3).

In Region 2, there are three types of major interferers for a given MS  $n$  served by RS  $r$ . The most significant interferers are the four nearest RSs indexed by  $y$ . The second group of interferers indexed by  $x$  are two neighbor RSs at a distance of  $\sqrt{2}d_r$  from RS  $r$ . There exist the four third nearest neighbor RSs indexed by  $w$  at a distance of  $\sqrt{3}d_r$ .

The other RSs located farther can be ignored like relay deployment *Scenario 1*. Therefore, the lower bound of the received SINR for MS  $\underline{n}$  in Region 2 can be derived as:

$$\Gamma_{\underline{r} \rightarrow \underline{n}}^{(r)} = \frac{E[P_{\underline{r} \rightarrow \underline{n}}^{(r)}]}{E[I_{\underline{n}}^{(r)}] + N_0} \approx \overline{\alpha_r} P_{RS}^{(t)} \cdot \mathbf{F}_{-\gamma}(r_r, \epsilon_r) \cdot \mathbf{S}(\zeta_{r \rightarrow \underline{n}}) / \left\{ \begin{aligned} & \left[ \frac{\overline{\alpha_r} P_{RS}^{(t)} \cdot \theta_r}{360^\circ} \cdot \frac{\delta r_r^2}{r_e^2} \cdot (4\mathbf{G}_{-\gamma}(r_r, d_r) \cdot \mathbf{S}(\zeta_{y \rightarrow \underline{n}})) \right. \\ & \left. + 2\mathbf{G}_{-\gamma}(r_r, \sqrt{2}d_r) \cdot \mathbf{S}(\zeta_{x \rightarrow \underline{n}}) \right. \\ & \left. + 4\mathbf{G}_{-\gamma}(r_r, \sqrt{3}d_r) \cdot \mathbf{S}(\zeta_{w \rightarrow \underline{n}}) \right] + N_0 \end{aligned} \right\}, \quad (5)$$

where  $r_r$  and  $d_r$  are  $(r_e - r_b)/2$  and  $(r_b + r_r)$ , respectively, in this scenario.

### 3.3 Numerical example and discussion

For a fair comparison, the average SINR  $\overline{\Gamma}^{(r)}$  can be defined as the weighted sum of SINR values for two regions, and can be expressed as:

$$\overline{\Gamma}^{(r)} = \frac{r_b^2}{r_e^2} \cdot I_{b \rightarrow m}^{(r)} + \left(1 - \frac{r_b^2}{r_e^2}\right) \cdot \Gamma_{\underline{r} \rightarrow \underline{n}}^{(r)} \quad (6)$$

All shadow fading are assumed to be identical to  $\zeta_0$ . Table 1 shows the detailed parameter set for a numerical example. Fig. 2 shows the average SINR of *Scenarios 1, 2*, and single-hop cellular network. The offered load becomes 100% when the number of MSs per cell is 100 because  $\overline{\alpha_b}$  and  $\overline{\alpha_r}$  are set to 0.01. As the number of MSs per cell increases, the average SINR values decrease due to the increased

Table 1. Parameters for numerical example

Parameter	Value	Parameter	Value
$\overline{\alpha_b}$	0.01	$\overline{\alpha_r}$	0.01
$P_{max,BS}$	42 [dBm]	$P_{max,RS}$	30 [dBm]
$\theta_b$	360°	$\theta_r$	360°
$\epsilon_b$	10 [m]	$\epsilon_r$	10 [m]
$r_e$	1000 [m]	$d_b$	$\sqrt{3} r_e$
$\gamma$	4	$N_0$	-174 [dBm]
$\sigma_{\zeta_0}$	8 [dB]		

interference. As shown in the result, the average SINR in multihop cellular network is larger than in single-hop cellular network. In addition, as it can be expected from Eqs. (4) and (5), the average SINR in *Scenario 2* is larger than in *Scenario 1* for every cases due to higher signal power strength in Region 2. Especially, the SINR difference between *Scenarios 1* and 2 increases as the relay-cell radius becomes larger.

## IV. Conclusions

This paper proposes the average SINR models for two different relay deployment scenarios taking into consideration multi-cell geometries and inter-cell interferences. The proposed mathematical models prove that *Scenario 2* has better performance than *Scenario 1* in terms of SINR. However, there exist a clear trade-off between the SINR gain and the deployment cost because *Scenario 2* requires larger number of RSs per cell even though it has better SINR. In addition, if we consider the implementation overhead of relay systems such as resource sharing and signaling overhead between BSs and RSs, the system capacities may not exactly correspond to the SINR models. Therefore, it is for further study to extend this work to analyze the capacities of multihop cellular networks according to relay positions, and to find optimal relay deployment positions for maximizing the capacities with due regard to implementation overhead.

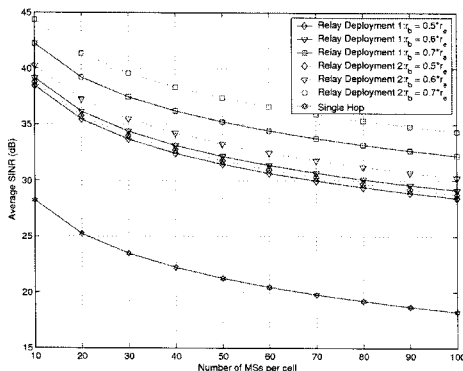


Fig. 2. Average downlink SINR

Appendix

1. Expectations of pathloss terms and shadow fading

With a given cell with a radius of  $r_0$ , we can define two types of random variables, which represent specific distances. First, a random variable  $R_1$  represents a distance from the center of the cell to an MS located within a radius of  $r_0$ . If MSs are uniformly distributed in the cell, the minus  $\gamma$ -th moment of  $R_1$  can be derived as:

$$\begin{aligned}
 F_{-\gamma}(r_0, \epsilon_0) &= \int_{\epsilon_0}^{r_0} r^{-\gamma} f_{R_1}(r) dr \\
 &= \int_{\epsilon_0}^{r_0} \frac{2r^{-\gamma+1}}{r_0^2 - \epsilon_0^2} dr = \frac{2 \cdot (r_0^{-\gamma+2} - \epsilon_0^{-\gamma+2})}{(r_0^2 - \epsilon_0^2) \cdot (-\gamma+2)}, \quad (7)
 \end{aligned}$$

where  $\epsilon_0$  denotes the minimum distance between a BS and an MS.

Second, we can define a random variable  $R_2$  which represents a distance from a point outside of the given cell to an MS within the cell. The probability density function  $f_{R_2}(r)$  can be approximated from Fig. 3, where  $a$  and  $b$  denote the length of a perpendicular line from the cross point to the base line and the distance from the center of the cell to a contact point between the perpendicular line and the base line, respectively. From trigonometrical functions, we can obtain the following equations:

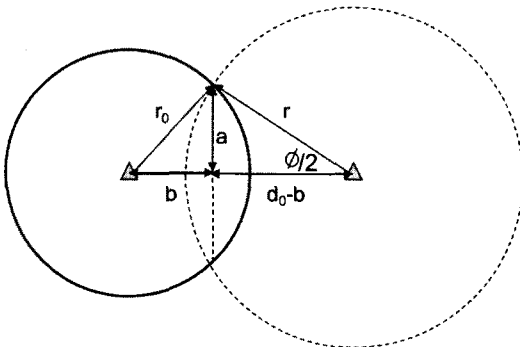


Fig. 3. Probability density function for  $R_2$

$$a^2 + b^2 = r_0^2, \quad (8)$$

$$r \sin(\phi/2) = a, \quad (9)$$

$$r \cos(\phi/2) = d_0 - b. \quad (10)$$

Using above three equations, we can obtain  $\phi$  and thus, the minus  $\gamma$ -th moment of  $R_2$  can be derived as:

$$\begin{aligned}
 G_{-\gamma}(r_0, d_0) &= \int_{d_0-r_0}^{d_0+r_0} r^{-\gamma} \cdot f_{R_2}(r) dr \\
 &= \int_{d_0-r_0}^{d_0+r_0} r^{-\gamma} \cdot \frac{r \phi}{\pi r_0^2} dr \\
 &= \int_{d_0-r_0}^{d_0+r_0} \frac{2r^{-\gamma+1}}{\pi r_0^2} \cdot \cos^{-1} \left( \frac{r^2 + d_0^2 - r_0^2}{2rd_0} \right) dr, \quad (11)
 \end{aligned}$$

where  $d_0$  denotes a distance from the center of the cell to a specific point outside of the cell.

Typically, shadowing in a wireless channel is modelled as a log-normal distribution with zero mean. Thus, the expectation of a log-normally distributed random variable with a mean of  $m_\zeta = 0$  and a variance of  $\sigma_\zeta^2$  is written as:

$$\begin{aligned}
 S &= 10^{-m_\zeta/10} \cdot \exp \left\{ \frac{\left( \frac{\ln 10}{10} \sigma_\zeta \right)^2}{2} \right\} \\
 &= \exp \left\{ \frac{\left( \frac{\ln 10}{10} \sigma_\zeta \right)^2}{2} \right\}. \quad (12)
 \end{aligned}$$

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