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Joint Subcarrier Matching, Power Allocation and Bit Loading in OFDM Dual-Hop Systems

Hyung-Yun Kong · Jin-Hee Lee

Abstract

Orthogonal Frequency Division Multiplexing(OFDM) dual-hop systems can take full advantages of the techniques of both multi-hop communication and OFDM. To achievethis end, we propose a joint subcarrier matching, power allocation and bit loading algorithm operating under a total power constraint and the same Bit Error Rate(BER) threshold over all subcarriers. Simulation results demonstrated system throughput improvement compared to single-hop systems and dual-hop systems with different bit loading algorithms for each relay position, power constraint, and required BER.

Key words : Multi-Hop, Subcarrier Matching, Power Allocation, Bit Loading, OFDM.

Ⅰ. Introduction

Multi-hop communication has attracted a great deal of attention recently due to its capability to improve performance, increase system capacity, and extend coverage[1]. To mitigate the inter-symbol interference and frequency-selective fading, the orthogonal frequency division multiplexing(OFDM) modulation and a cyclic code are exploited to transmit data over multiple orthogonal narrowband subcarriers[2]. Therefore, multi-hop communication among OFDM terminals can take advantages of both techniques.

In this paper, information transmission from a source to a destination with the assistantance of a relay is considered. All OFDM terminals are assumed and the relay operates in the decode-and-forward mode. Since the source transmits its information on N subcarriers and the relay decodes and forwards the source information to the destination also on N subcarriers, some problems are arise. *These problems are 1, how to match the N subcarriers of the source with those of the relay in order to maximize system capacity(i.e., information on the subcarrier i of the source can be retransmitted on the subcarrier j of the relay), and, how to allocate power on subcarriers of the source and the relay appropriately under a total average power on the constraint.* These two problems are solved in [3], where the optimal subcarrier matching is to match subcarriers by the order of channel power gains. The optimal power allocation for the matched subcarrier pairs can be solved by applying the WF method. However, this water-filling solution requires an infinite-length codebook and continuous modulation and power levels, making it impossible to use directly in practice.

Different aspects of power allocation and bit loading algorithms in OFDM systems are extensively addressed in [4]~[6] under different aspects. Bit loading means that different discrete modulation levels can be employed on different subcarriers according to a certain measure(e.g., BER or SNR) and in some cases, some subcarriers can be nulled. In [4], an algorithm is proposed to minimize the total power for a given bit rate and probability of error. In [5], the algorithm maximizes the overall throughput while maintaining the mean BER below a prescribed threshold. In [6], constant power allocation and unconstrained BER are considered.

Bit loading algorithms are also proposed in cooperative OFDM systems[7] with a single source-destination pair and multiple relays. The objective is to allocate bits and power to each subcarrier to minimize the total transmission power. In addition, [8] considered two power allocation and bit loading algorithms for spatial multiplexing in MIMO systems, namely Rounding Off WF and Quality of Service(QoS)-based WF. However, we believe that the QoS-based WF requires five steps, not four as in [8]. Otherwise, the allocated total power exceeds the given total power. This work is closely relevant to ours and thus, we discuss it elaborately in Section Ⅳ.

This paper proposes a joint subcarrier matching, power allocation and bit loading algorithm in OFDM dual-hop systems under a total power constraint and the same BER threshold over all subcarriers. No reference has previously addressed this problem. Subcarrier matching

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Department of Electronic Engineering, University of Ulsan, Ulsan, Korea.

is performed in the same manner as [3] and power allocation and bit loading are processed in two stages. In the first stage, the Rounding Off WF is applied to distribute possible maximum modulation levels to all subcarriers. In the second stage, given these possible maximum modulation levels, compute the lowest power levels on subcarriers to just meet the required BER and then, use the residual power to increase modulation levels for those subcarriers which have not reached to available maximum modulation levels in the descending order of channel power gains. Simulation results show that the proposed algorithm significantly improves the system throughput over references.

The remainder of the paper is organized in the following manner. Section Ⅱ describes the channel model. The system model and optimal subcarrier matching are presented in detail in Section Ⅲ. Power allocation and bit loading are described in section Ⅳ. Simulation results are discussed in Section Ⅴ. Finally, the paper is closed in Section Ⅵ.

Ⅱ. Channel Model

We consider a multi-path fading channel called frequency-selective fading because the coherence bandwidth is smaller than the total bandwidth of the OFDM system. We also assume the fading process to be stationary and slowly varying compared with the OFDM symbol duration, i.e, it is approximately constant during one symbol duration T but independently changes to the next.

The complex equivalent low-pass time-variant impulse response of the channel between the transmitter z and receiver v can be written as

$$h_2(t) = \underbrace{\frac{K}{d_z^{3/2}}}_{F_1} \underbrace{\sum_{l=1}^{L} \alpha_{z,l}\delta(t-\tau_{z,l})}_{F_2} \tag{1}$$

where L is the number of resolvable paths, $\tau_{z,l}$ the time delay of the l_{th} path, $\alpha_{z,l}$ an independent complex Gaussian random variable tap weight, d_z the distance between the transmitter z and receiver v, β the path-loss exponent, K a constant that depends on the environment, $\delta(.)$ the Dirac delta function. We further assume that

$$\sum_{l=1}^{L} E[|\alpha_{z,l}|^2] = 1$$

Note that in (1), we captured the effect of path loss by assuming the channel is composed of long-term path-loss F_1 and short-term Rayleigh fading F_2[9]. Here $E[\,.\,]$ is the expectation.

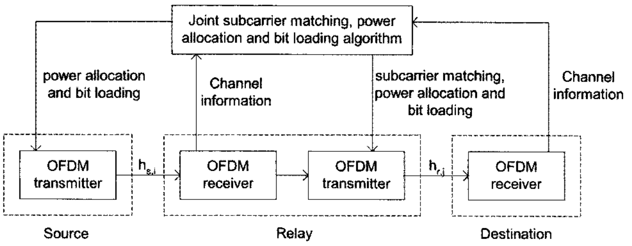

Fig. 1. An OFDM dual-hop system with joint subcarrier matching, power allocation and bit loading.

For OFDM systems, the channel power gain of the subcarrier n is given by

$$h_{z,n} = \frac{K^2}{d_z^{\beta}} \left| \sum_{l=1}^{L} \alpha_{z,l} e^{-j2\pi n\tau_{z,l}/T} \right|^2 \tag{2}$$

This section summarizes the results achieved in [3] The OFDM dual-hop system with a source-destination pair assisted by a relay as shown in Fig. 1. Denote $h_{s,i}$ and $h_{r,j}$ as source-relay and relay-destination channel power gains of subcarriers i and j, respectively; $i, j \in 1, \ldots, N$ where N is the total number of subcarriers. Without loss of generality, assuming that $h_{s,i}$ and $h_{r,j}$ are in descending order(i.e., $h_{s,1} \geq h_{s,2}, \ldots, h_{s,N}$ and $h_{r,1} \geq h_{r,2}, \ldots, h_{r,N}$) [1], the optimal matching between subcarriers at the source and subcarriers at the relay, it is best to match them by the order of channel power gains, i.e. $h_{s,i} \sim h_{r,i}$. Then, according to [3] as in equations (8) and (9), the channel capacity of the subcarrier pair 1) $(h_{s,i} \sim h_{r,i})$, R_{ii}, equals that of the subcarrier i over the source-relay channel($h_{s,i}$), $R_{s,i}(P_{s,i})$, and that of the subcarrier i over the relay-destination channel($h_{r,i}$), $R_{r,i}(P_{r,i})$, given an optimal power allocation between the corresponding subcarriers(*i.e.*, $h_{s,i} P_{s,i} = h_{r,i} P_{r,i}$):

$$\begin{aligned} R_{ii} &= R_{s,i}(P_{s,i}) = R_{r,i}(P_{r,i}) \\ &= \frac{B}{2N} \log_2\left(1 + \frac{h_i P_i}{\sigma^2}\right) \end{aligned} \tag{3}$$

where $P_{s,i}$ and $P_{r,i}$ are the power allocated to the subcarrier i at the source and the relay, respectively; $P_i = P_{s,i} + P_{r,i}$; B is the total available bandwidth; $\sigma^2 = N_0 B/N$ where N_0 is the power spectral density of AWGN; $h_i = h_{s,i} * h_{r,i}/(h_{s,i} + h_{r,i})$ is the equivalent channel power gain.

The total system channel capacity is given by

$$\sum_{i=1}^{N} \frac{B}{2N} \log_2\left(1 + \frac{h_i P_i}{\sigma^2}\right) \tag{4}$$

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[1] Otherwise, subcarriers at the source and the relay are firstly sorted by the permutations π and π'

Ⅲ. Power Allocation and Bit Loading

The total system channel capacity in (4) is maximized by allocating the power to subcarrier pairs according to WF[3]:

$$P_i = \max\left(\lambda - \frac{\sigma^2}{h_i}, 0\right) \tag{5}$$

where λ can be found based on a total system power constraint:

$$\sum_{i=1}^{N} P_i = P_{tot} \tag{6}$$

The next section presents three bit loading algorithms that distribute different discrete modulation levels on different subcarriers according to the estimated BER fot the purpose of maintaining-the 2-hop BER on each subcarrier over the source-relay-destination channel below the threshold, BERT. Due to (3), the 1-hop BERs on subcarrier i that are the BERs on subcarrier i over the source-relay channel or over the relay-destination channel are identical. Therefore, the 2-hop BER on each subcarrier pair is

$$\begin{aligned} &BER_{2hop,i} \\ &= 1-(1-BER_{1hop,i})^2 \le 2BER_{1hop,i} \le BER_T \\ &\Rightarrow BER_{1hop,i} \le \frac{BER_T}{2} \end{aligned} \tag{7}$$

where $BER_{1hop,i} = f(\gamma_i, M_i)$ is a function of SNR γ_i and modulation level M_i on subcarrier i. In simulations, we use same BER formulas as-in [5].

3-1 Rounding Off WF[8]

This algorithm rounds off the original non-constrained WF solution in two steps:

Step 1: Calculate the initial P_i in (5).

Step 2: Find the maximum usable M_i from the BER constraint (BER_T).

$$\gamma_i = \frac{h_i P_i}{\sigma^2}$$

$$f(\gamma_i, M_i) \le \frac{BER_T}{2} \Rightarrow M_i$$

3-2 QoS-based WF[8]

This algorithm only uses the minimum power to achieve BER_T on each subcarrier, in-four steps:

Step 1: Calculate the initial P_i in (5).

Step 2: α=0. Run the re-distribution routine for i=1, $\cdots$,N.

Step 3: α=1. Run the re-distribution routine for i=1, $\cdots$,N.

Step 4: Re-scale the power allocation for all subcarriers such as no final residual power is left

$$P_{i,new} = \frac{P_i}{\sum_{i=1}^{N} P_i}$$

There are four steps in the *re-distribution routine*:

Step 1: Calculate the current residual power:

$$P_{res} = \alpha\left(1 - \sum_{i=1}^{N} P_i\right)$$

Step 2: Find the maximum usable M_i from the BER constraint

$$\gamma_i = \frac{h_i(P_i + P_{res})}{\sigma^2}$$

$$f(\gamma_i, M_i) \le \frac{BER_T}{2} \Rightarrow M_i$$

Step 3: Find the necessary SNR for M_i:

$$f(\Gamma_i, M_i) = \frac{BER_T}{2} \Rightarrow \Gamma_i$$

Step 4: Calculate the necessary power level for this subcarrier:

$$P_i = \Gamma_i \frac{\sigma^2}{h_i}$$

Although the simulation results for [8] show that the QoS-based WF is better than the Rounding Off WF, the test gives contrary results in some cases. The reason is that the former is not complete. Another step should be added to find the maximum usable M_i from $P_{i,new}$ in Step 4. Then $P_{i,new}$ may be less or greater than P_i, making the overall throughput. If there are only four steps, it is observed that Step 3 used up to NP_{res}, making the allocated total power exceed the given power.

3-3 Proposed WF

The proposed algorithm consists of two stages. In the first stage(Step 1~2), the Rounding Off WF is applied to distribute the possible maximum modulation levels to all subcarriers. In the second stage(Step 3~6), given these possible maximum modulation levels, the lowest power levels on subcarriers are computed to just meet the required BER and then, use the residual power is used to increase modulation levels for those subcarriers that have not reached the available maximum modulation level in the descending order of channel power gains. The algorithm is summarized as follows:

Step 1: Calculate the initial P_i in (5).

Step 2: Find the maximum usable M_i from the BER constraint

$$\gamma_i = \frac{h_i P_i}{\sigma^2}$$

$$f(\gamma_i, M_i) \le \frac{BER_T}{2} \Rightarrow M_i$$

Step 3: Find the necessary SNR for M_i:

$$f(\Gamma_i, M_i) = \frac{BER_T}{2} \Rightarrow \Gamma_i$$

Step 4: Calculate the necessary power level for this subcarrier:

$$P_i = \Gamma_i \frac{\sigma^2}{h_i}$$

Step 5: Calculate the current residual power:

$$P_{res} = 1 - \sum_{i=1}^{N} P_i$$

Step 6: Consider each i from 1 to N in succession. If $M_i < M_{\max}$, do Step 2~5 with P_i in Step 2 replaced by $P_i + P_{res}$. Otherwise, no action is required.

Here $M_{\max}$ is the available maximum modulation level. In Step 6, the condition $M_i < M_{\max}$ is checked to see whether the residual power would be useful in increasing the current modulation level.

The proposed WF is clearly more complicated than the Rounding Off WF because of the additional Steps 3~6. It is less complex than the QoS-based WF since the proposed WF only corresponds to its Steps 1~2 and a part of Step 3.

At this point, we can summarize the proposed joint subcarrier matching, power allocation and bit loading algorithm as follows:

Step 1: Sort the subcarriers at the source and the relay in descending order to get $h_{s,1} \ge h_{s,2} \ldots, \ge h_{s,N}$ and $h_{r,1} \ge h_{r,2} \ldots, \ge h_{r,N}$.

Step 2: Match the subcarriers into pairs $h_{s,i} \sim h_{r,i}$

Step 3: Apply the proposed WF to achieve the overall system throughput $(1 - BER_T) \sum_{i=1}^{N} M_i$.

Ⅳ. Numerical Results

Consider the IEEE Std. 802.11a[5] with parameters: N =52 subcarriers, an operating frequency of 5.15~5.25 GHz, a signal bandwidth of 16.6 MHz, and the possible modulation schemes are BPSK, QPSK, 16-QAM, and 64-QAM. A two-path slowly varying channel model with equal power split between the paths and delays $\tau_{z,1} = 0$, $\tau_{z,2} = 0.1T$ is used for simulations. We also assume the source-relay and relay-destination channels follow this power delay profile for simplicity of exposition. A network geometry is examined where the relay lies on a straight line between the source and the destination[10]. The source-destination distance is normalized to be 1.

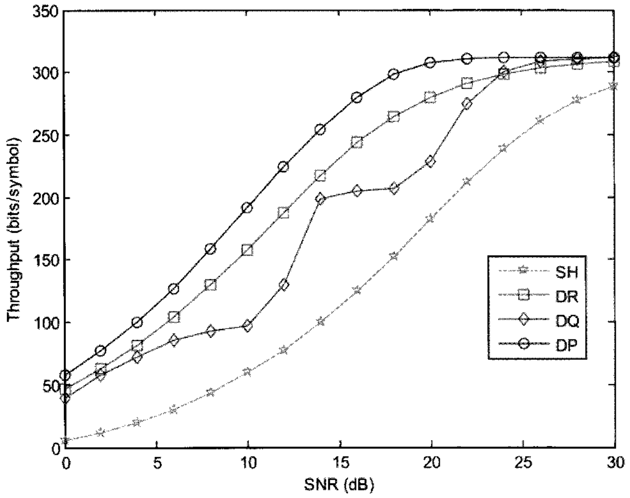

Fig. 2. Throughput versus SNR($BER_T = 10^{-3}$ and source-relay distance d=0.5).

We also denote d as the source-relay distance. Additionally, we only take an example of $K = 1$, $\beta = 4$ in (2). The SNR is defined as SNR= $P_{tot}/N_0 B$. To obtain the average overall system throughput, we have simu-lated 10,000 independent trials.

In this section, we compare three OFDM dual-hop systems, namely DR, DQ, and DP with an OFDM single-hop(SH). All three dual-hop systems perform joint subcarrier matching, power allocation and bit loading while the SH one only does power allocation and bit loading according to the Rounding Off WF. DR, DQ, and DP apply the Rounding Off WF, the QoS-based WF, and the Proposed WF, respectively. Therefore, DP is the proposed system while the others(DR, DQ, SH) are references.

Figs. 2~3 illustrate the overall throughput of the four examined systems operating at $BER_T = 10^{-3}$ and 10^{-5} and the relay is at the middle of the source and the destination, d=0.5. The proposed system DP is significantly better than the others over the whole range of SNR and any BER constraint. Specifically, at the throughput of 250 bits/OFDM symbol, the proposed system DP achieves an SNR gain of 3, 7, 11 dB over DR, DQ, and SH, respectively irrespective of the BER constraint. In addition, it is observed that the system throughput is dramatically reduced with the increase in QoS(i.e., reduced BER) at low SNRs and is negligibly decreased with the increase in QoS at high SNRs. This is reasonable since at low SNRs but high QoS, the system must operate at low modulation levels, dras-

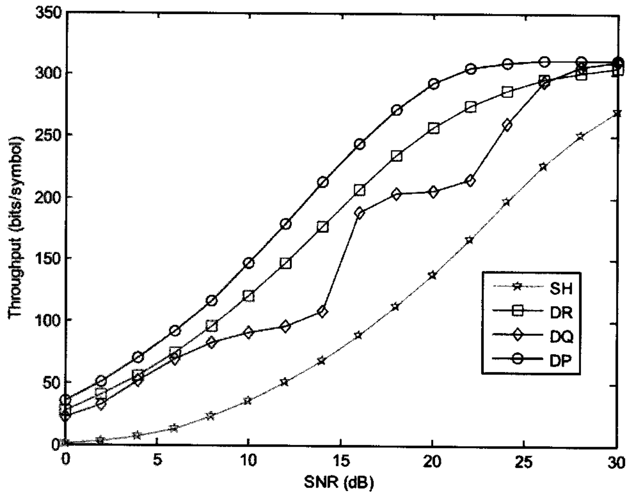

Fig. 3. Throughput versus SNR($BER_T = 10^{-5}$ and source-relay distance d=0.5).

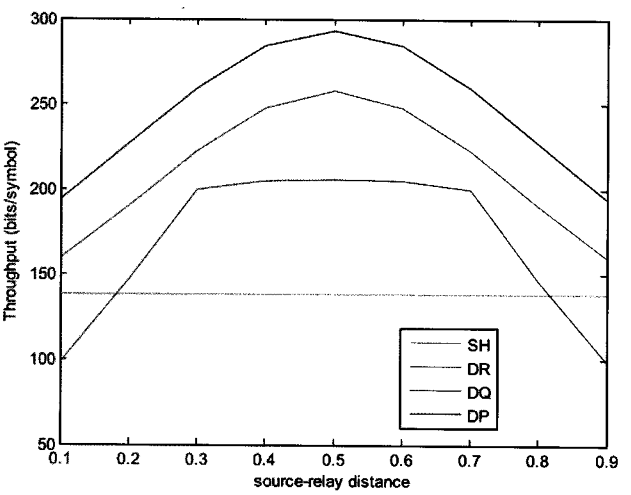

Fig. 4. Throughput versus source-relay distance(SNR=20 dB, $BER_T = 10^{-5}$).

tically reducing the throughput while at high SNRs, the system can still operate at high modulation levels to satisfy the high QoS, negligibly decreasing the through--put. However, all three dual-hop systems are affected by QoS less than the SH system. Fig. 4 shows the effect of relay position on the throughput at SNR=20 dB and $BER_T = 10^{-5}$. It is noted that the relay at the middle of the source and the destination results in the best throughput for the dual-hop systems since at this position, the path-loss trade-off between the source-relay link and the relay-destination link is the best. In addition, the proposed system DP always reaps a higher throughput than the others for any relay position.

V. Conclusion

Optimal joint subcarrier matching and power allocation in OFDM dual-hop systems in [3] can not be used directly in practice due to the disadvantages of non-constrained WF. We addressed this problem by proposing a joint subcarrier matching, power allocation and bit loading algorithm. A variety of simulation results demonstrated that the OFDM dual-hop system applying this algorithm brought a higher throughput than other dual-hop and single-hop systems for any SNR, relay position, and QoS constraint. These results are new and convincible in deploying OFDM dual-hop systems in the future.

Similar to [3], the extension of the joint subcarrier matching, power allocation and bit loading algorithm to OFDM multi-hop systems is straightforward. Although the simulation time is very short, thoretically analyzing the throughput is essential to verify the simulation results and to gain insight into the system parameters impacting the throughput. This is an interesting topic for future research.

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References

[1] F. H. P. Fitzek, M. D. Katz, *Cooperation in Wireless Networks: Principles and Applications*, Springer, 2006.

[2] R. V. Nee, R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House, 2000.

[3] W. Wang, S. Yan, and S. Yang, "Optimally joint subcarrier matching and power allocation in OFDM multihop system", *Eurasip Journal on Advances in Signal Processing*, vol. 2008, ArticleID 241378, doi: 10.1155/2008/241278.

[4] S. Nader-Esfahni, M. Afrasiabi, "Simple bit loading algorithm for OFDM-based systems", *IET on Commun.*, vol. 1, pp. 312-316, Jun. 2007.

[5] A. M. Wyglinski, F. Labeau, and P. Kabal, "Bit loading with BER-constraint for multicarrier systems", *IEEE Trans. Wire. Commun.*, vol. 4, pp. 1383-1387, Jul. 2005.

[6] Y. George, O. Amrani, "Bit loading algorithms for OFDM", *IEEE ISIT*, p. 391, 2004.

[7] B. Gui, L. J. Cimini, "Bit loading algorithms for cooperative OFDM Systems", *Eurasip Journal on Wireless Communications and Networking*, vol. 2008, ArticleID 476797, doi: 10.1155/2008/476797.

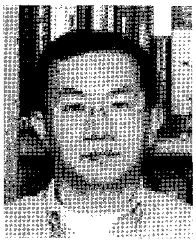
[8] X. Zang, B. Ottersten, "Power allocation and bit loading for spatial multiplexing in MIMO systems",

IEEE ICASSP, vol. 5, pp. V-53-56, 2003.

[9] H. Ochiai, P. Mitran, and V. Tarokh, "Design and analysis of collaborative diversity protocols for wireless sensor networks", *IEEE VTC2004-Fall*, vol. 7, pp. 4645-4649, Sep. 2004.

[10] N. Ahmed, M. A. Khojastepour, and B. Aazhang, "Outage minimization and optimal power control for the fading relay channel", *IEEE Information Theory Workshop*, pp. 458-462, Oct. 2004.

Hyung-Yun Kong



received M.E. and Ph.D. degrees in electrical engineering from Polytechnic University, Brooklyn, New York, USA, in 1991 and 1996, respectively, He received a B.E. in electrical engineering from New York Institute of Technology, New York, in 1989. Since 1996, he has been with LG electronics Co., Ltd., in the multimedia research lab developing PCS mobile phone systems, and from 1997 the LG chairman's office planning future satellite communication systems. Currently he is an Associate Professor in electrical engineering at the University of Ulsan, Korea. He works on several government projects supported by ITRC and the Korean Science and Engineering Foundation(KOSEF). His research area includes high data rate modulation, channel coding, detection and estimation, cooperative communications, and sensor networks. He is a member of IEEK, KICS, KIPS, and IEICE.

Jin-Hee Lee

received B.S. degrees in electrical engineering from University of Ulsan, Korea, in 2009. and is currently working toward the Master degree in the department of electrical engineering at Ulsan University. His current interests include Network-Coding, MARC(Multiple Access Relay Channel), Cooperative Communication.