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SOMEWHAT FUZZY ALMOST α-IRRESOLUTE FUNCTIONS

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Abstract. In this paper the concepts of somewhat fuzzy almost α-irresolute functions and strongly somewhat fuzzy β-open functions are introduced and studied. Besides giving characterizations of these functions, some interesting properties of these functions are also given.

1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L. A. Zadeh [11]. Fuzzy sets have applications in many fields such as information [7] and control [8]. The theory of fuzzy topological spaces was introduced and developed by C. L. Chang [3] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concepts of somewhat fuzzy continuous functions and somewhat fuzzy β-continuous functions was introduced and studied by G.Thangaraj and G.Balasubramanian in [9] and [10] respectively. The concept of fuzzy almost α-irresolute functions was introduced and studied in [6]. In this paper we introduce the concepts of somewhat fuzzy almost α-irresolute functions and strongly somewhat fuzzy β-open functions and study their properties.

2. Preliminaries

By a fuzzy topological space we shall mean a non-empty set X together with fuzzy topology T [3] and we shall denote it by (X, T). A fuzzy point in X with support $x \in X$ and value $p(0<p \leq 1)$ is denoted by x_p. The complement μ' of a fuzzy set μ is $1-\mu$, defined by $\mu'(x) = (1_X - \mu)(x) = 1 - \mu(x)$ for all $x \in X$ [3]. If λ is a fuzzy set in X and μ is a fuzzy set in Y, then $\lambda \times \mu$ is a fuzzy set in $X \times Y$, defined by $(\lambda \times \mu)(x, y) = \min(\lambda(x), \mu(y))$, for every (x, y) in $X \times Y$ [1]. A fuzzy topological space X is product related to a fuzzy topological space Y [1] if for fuzzy sets γ in X and ξ in Y whenever $\lambda'(=1-\lambda) \ngeq \gamma$ and $\mu'(=1-\mu) \ngeq \xi$ (in which case $(\lambda' \times 1) \vee (1 \times \mu') \geq (\gamma \times \xi)$) where λ is fuzzy open set in X and μ is a fuzzy open set in Y, there exists a fuzzy open set λ_1 in X and a fuzzy open set μ_1 in Y such that $\lambda_1' \geq \gamma$ or $\mu_1' \geq \xi$ and $(\lambda_1' \times 1) \vee$

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$(1 \times \mu_1') = (\lambda' \times 1) \vee (1 \times \mu')$. Let $f : X \to Y$ be a mapping from X to Y. If λ is a fuzzy set of $X, f(\lambda)$ is defined by

$$f(\lambda)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda(x), & \text{if } f^{-1}(y) \neq \varnothing \\ 0, & \text{otherwise} \end{cases}$$

for each mapping $y \in Y$; and if μ is fuzzy set of $Y, f^{-1}(\mu)$ is defined by $f^{-1}(\mu)(x) = \mu(f(x))$, for each $x \in X$ [3]. Let f be a mapping from X to Y. Then the graph g of f is mapping from X to $X \times Y$ sending x in X to $(x, f(x))$. For two mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$, we define the product $f_1 \times f_2$ of f_1 and f_2 to be a mapping from $X_1 \times X_2$ to $Y_1 \times Y_2$ sending (x_1, x_2) in $X_1 \times X_2$ to $[f_1(x_1), f_2(x_2)]$. For any fuzzy set λ in a fuzzy topological space, it is shown in [1] that (i)$1-$ cl $\lambda = \text{int}(1-\lambda)$, (ii) $\text{cl}(1-\lambda) = 1-$ int λ. A fuzzy set λ in fuzzy topological space (X, T) is called fuzzy α-dense if there exists no fuzzy α-closed set μ such that $\lambda < \mu < 1$.

Definition 2.1. [5] A function $f : (X, T) \to (Y, S)$ is said to be **fuzzy α-irresolute** if $f^{-1}(\lambda)$ is fuzzy α-open set in (X, T) for every fuzzy α-open set λ of (Y, S).

Definition 2.2. [6] A function f from a fuzzy topological space (X, T) to a fuzzy topological space (Y, S) is said to be **fuzzy almost α-irresolute** if $f^{-1}(\lambda)$ is fuzzy β-open in (X, T) for each fuzzy α-open set λ in (Y, S).

Definition 2.3. [10] Let (X, T) and (Y, S) be fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called **somewhat fuzzy β-open function** if and only if for all $\lambda \in T, \lambda \neq 0$ there exists a fuzzy β-open set μ of Y such that $\mu \neq 0$ and $\mu \leq f(\lambda)$.

Definition 2.4. Let (X, T) be a fuzzy topological space and λ be any fuzzy set in X.

(1) λ is called fuzzy α-open set [2] if $\lambda \leq int\ cl\ int\lambda$
(2) λ is called fuzzy β-open set [4] if $\lambda \leq cl\ int\ cl\lambda$

The complement of fuzzy α-open (fuzzy β-open) set is called fuzzy α-closed (fuzzy β-closed) set.

3. Somewhat fuzzy almost α-irresolute functions

The concept of fuzzy almost α-irresolute functions was introduced and studied in [6]. In this section we shall introduce the concept of somewhat fuzzy almost α-irresolute functions and study their properties.

Definition 3.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is said to be **somewhat fuzzy almost α-irresolute** if for any non-zero fuzzy α-open set λ in Y and $f^{-1}(\lambda) \neq 0$ then there exists a fuzzy β-open set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$.

Clearly every fuzzy almost α-irresolute function is somewhat fuzzy almost α-irresolute, but the converse is not true as the following example shows:-

Example 3.1. Let μ_1, μ_2 and μ_3 be fuzzy sets on $I = [0, 1]$.

$$\mu_1(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{2}, \\ 2x - 1, & \frac{1}{2} \le x \le 1; \end{cases}$$
$$\mu_2(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{4}, \\ -4x + 2, & \frac{1}{4} \le x \le \frac{1}{2}, \\ 0, & \frac{1}{2} \le x \le 1; \end{cases}$$
$$\mu_3(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{4}, \\ \frac{1}{3}(4x - 1), & \frac{1}{4} \le x \le 1. \end{cases}$$

Clearly $S_1 = \{0,\ \mu_2,\ (\mu_2 \vee \mu_3),\ (\mu_2 \wedge \mu_3),\ 1\}$ and $S_2 = \{0,\ \mu_3,\ \mu_2',\ (\mu_2 \wedge \mu_3)', 1\}$ are fuzzy topologies on I. Let $f : (I, S_1) \to (I, S_2)$ be defined by $f(x) = x$ for each $x \in I$. Let λ, ρ and σ be fuzzy sets such that $0 < \lambda < \mu_3, \mu_3 < \rho < \mu_2'$ and $\mu_2' < \sigma < (\mu_2 \wedge \mu_3)'$. Then λ, ρ and σ are not fuzzy α-open sets in (I, S_2). Now the only non-zero fuzzy α-open sets in (I, S_2) are $1, \mu_3, \mu_2', (\mu_2 \wedge \mu_3)'$and fuzzy sets δ such that $(\mu_2 \wedge \mu_3)' < \delta < 1$. Now $f^{-1}(1) = 1$; $f^{-1}(\mu_3) = \mu_3$; $f^{-1}(\mu_2') = \mu_2'$; $f^{-1}((\mu_2 \wedge \mu_3)') = (\mu_2 \wedge \mu_3)'$ and $f^{-1}(\delta) = 1$. The fuzzy β-open set $(\mu_2 \wedge \mu_3)$ in (I, S_1) is contained in $f^{-1}(1)$, $f^{-1}(\mu_3), f^{-1}(\mu_2'), f^{-1}((\mu_2 \wedge \mu_3)')$ and $f^{-1}(\delta)$. Hence f **is somewhat fuzzy almost α-irresolute function** from (I, S_1) to (I, S_2). It can be easily seen that int $\mu_2' = (\mu_2 \wedge \mu_3)$; cl $\mu_3 = \mu_2'$; cl $(\mu_2 \wedge \mu_3) = (\mu_2 \vee \mu_3)'$ in (I, S_1). We obtain $\mu_3 \not\le$ cl int cl μ_3 and μ_3 is not a fuzzy β-open set in (I, S_1). Now μ_3 is a fuzzy α-open set in (I, S_2) and $f^{-1}(\mu_3) = \mu_3$, which is not a fuzzy β-open set in (I, S_1). Hence f **is not fuzzy almost α-irresolute.**

Theorem 3.1. *Let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, Q)$ be any two functions. If f is somewhat fuzzy almost α-irresolute and g is fuzzy α-irresolute, then $g \circ f$ is somewhat fuzzy almost α-irresolute.*

Proof. Let λ be non-zero fuzzy α-open set in (Z, Q). Since g is fuzzy α-irresolute, $g^{-1}(\lambda) \neq 0$ is fuzzy α-open in (Y, S). Now$(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda)) \neq 0$. Since $g^{-1}(\lambda)$ is fuzzy α-open in (Y, S) and f is somewhat fuzzy almost α-irresolute, then there exists a fuzzy β-open set $\mu \neq 0$ in (X, T) such that $\mu \le f^{-1}(\ g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$. Hence $g \circ f$ is somewhat fuzzy almost α-irresolute. □

In above Theorem 3.1 if f either is fuzzy almost α-irresolute or f is fuzzy α-irresolute and g is somewhat fuzzy almost α-irresolute, then it is not necessarily true that $g \circ f$ is somewhat fuzzy almost α-irresolute as the following example shows:-

Example 3.2. Let μ_1, μ_2 and μ_3 be fuzzy sets in I defined in Example 3.1. Let $T_1 = \{0, \mu_1, 1\}$; $T_2 = \{0, \mu_3, 1\}$ and $T_3 = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$. Then T_1,

T_2 and T_3 are fuzzy topologies on I. Let $f : (I, T_1) \to (I, T_2)$ be defined by $f(x) = x$ for each $x \in I$. It can be easily seen that int $\mu_3 = \mu_1$; int $\mu_2' = \mu_1$; cl $\mu_3 = 1$; cl $\mu_2' = 1$ and cl$\mu_1 = 1$ in (I, T_1). Let λ be fuzzy set such that $0 < \lambda < \mu_3$. Then λ is not a fuzzy α-open set in (I, T_2). Now the only fuzzy α-open set in (I, T_2) are $0, 1, \mu_3$ and fuzzy sets μ such that $\mu_3 < \mu < 1$. Now $f^{-1}(0) = 0$; $f^{-1}(1) = 1$; $f^{-1}(\mu_3) = \mu_3$ and $f^{-1}(\mu) = \mu_2'$ are fuzzy α-open set and also fuzzy β-open set in (I, T_1). Then f **is fuzzy α-irresolute** and also f **is fuzzy almost α-irresolute** from (I, T_1) to (I, T_2). Let $g : (I, T_2) \to (I, T_3)$ be defined by $g(x) = \frac{x}{2}$ for each $x \in I$. Let λ, ρ and δ be fuzzy sets such that $0 < \lambda < \mu_1, \mu_1 < \rho < \mu_2$ and $\mu_2 < \delta < (\mu_1 \vee \mu_2)$. Then λ, ρ and δ are not fuzzy α-open sets in (I, T_3). Now the only fuzzy α-open sets in (I, T_3) are $0, 1, \mu_1, \mu_2, \mu_1 \vee \mu_2$ and fuzzy sets μ such that $(\mu_1 \vee \mu_2) < \mu < 1$. Now $g^{-1}(1) = 1$; $g^{-1}(\mu_1) = 0$; $g^{-1}(\mu_2) = \mu_1' = g^{-1}(\mu_1 \vee \mu_2)$ and $g^{-1}(\mu) = 1$. It can be easily seen that cl $\mu_1' = 1$ and int $\mu_1' = 0$ in (I, T_2). We have $\mu_1' \leq$ cl int cl μ_1' and μ_1' is a fuzzy β-open set in (I, T_2). Now μ_1' is a fuzzy β-open set in (I, T_2) such that $\mu_1' \leq g^{-1}(1)$, $\mu_1' \leq g^{-1}(\mu_2)$ and $\mu_1' \leq g^{-1}(\mu_1 \vee \mu_2)$. This proves that g **is somewhat fuzzy almost α-irresolute function.**

Now consider the functions $(g \circ f)$: $(I, T_1) \to (I, T_3)$. Then $(g \circ f)^{-1}(1) = 1$; $(g \circ f)^{-1}(\mu_1) = f^{-1}(g^{-1}(\mu_1)) = 0$; $(g \circ f)^{-1}(\mu_2) = f^{-1}(g^{-1}(\mu_2)) = f^{-1}(\mu_1') = \mu_1'$ and $(g \circ f)^{-1}(\mu_1 \vee \mu_2) = f^{-1}(g^{-1}(\mu_1 \vee \mu_2)) = f^{-1}(\mu_1') = \mu_1'$. But $(g \circ f)^{-1}(\mu_2) = \mu_1'$ and if λ is a fuzzy set such that $0 < \lambda \leq \mu_1'$, then λ is not a fuzzy β-open set in (I, T_1). Hence there is no non-zero fuzzy β-open set in (I, T_1) such that it is contained in $(g \circ f)^{-1}(\mu_2) = \mu_1'$. This shows that $(g \circ f)$ **is not somewhat fuzzy almost α-irresolute function.**

Definition 3.2. [10] A fuzzy set λ in fuzzy topological space (X, T) is called fuzzy β-dense if there exists no fuzzy β-closed set μ such that $\lambda < \mu < 1$.

Theorem 3.2. *Let (X, T) and (Y, S) be any two fuzzy topological spaces and $f : (X, T) \to (Y, S)$ be a function. Then the following assertions are equivalent.*

1. *f is somewhat fuzzy almost α-irresolute.*
2. *If λ is a fuzzy α-closed set in Y such that $f^{-1}(\lambda) \neq 1$, then there exists a fuzzy β -closed set $\mu \neq 1$ in X such that $\mu \geq f^{-1}(\lambda)$.*
3. *If λ is a fuzzy β-dense set in X, then $f(\lambda)$ is fuzzy α-dense set in Y.*

Proof. **(1) ⇒(2):** Suppose f is somewhat fuzzy almost α-irresolute and λ is a fuzzy α-closed set in Y such that $f^{-1}(\lambda) \neq 1$. Therefore clearly $1 - \lambda$ is fuzzy α-open set in (Y, S) and $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda) \neq 0$(since $f^{-1}(\lambda) \neq 1$). By (1), there exists a fuzzy β-open set η in X such that $\eta \leq f^{-1}(1 - \lambda)$. That is, $\eta \leq 1 - f^{-1}(\lambda)$ which implies that $f^{-1}(\lambda) \leq 1 - \eta$. Clearly $1 - \eta$ is fuzzy β-closed set and taking $\mu = 1 - \eta$, we have therefore $f^{-1}(\lambda) \leq \mu$. Thus we find that (1) ⇒(2) is proved.

(2) ⇒(3): Let λ be a fuzzy β-dense set in X and suppose $f(\lambda)$ is not fuzzy α-dense in Y. Then there exists a fuzzy α-closed set η(say) in Y such that $f(\lambda) < \eta < 1$ ....(A). Since $\eta < 1, f^{-1}(\eta) \neq 1$ and so by (2) there exists a fuzzy β-closed set δ ($\delta \neq 1$) such that $\delta \geq f^{-1}(\eta) > f^{-1}(f(\lambda)) \geq \lambda$(from(A)). That is, there exists a fuzzy β-closed set δ such that $\delta > \lambda$ which is contradiction to the assumption on λ. Therefore (2) $\Rightarrow$(3) is proved.

(3) $\Rightarrow$(1): Suppose λ is fuzzy α-open set in Y and $f^{-1}(\lambda) \neq 0$ and therefore $\lambda \neq 0$. Suppose there exists no fuzzy β-open set μ in X such that $\mu \leq f^{-1}(\lambda)$. Then $1 - f^{-1}(\lambda)$ is fuzzy set in X such that there is no fuzzy β-closed set δ in X with $1 - f^{-1}(\lambda) < \delta < 1$(otherwise $1 - f^{-1}(\lambda) < \delta \Rightarrow 1 - \delta \leq f^{-1}(\lambda)$ and $1 - \delta$ is fuzzy β-open, a contradiction). This means $1 - f^{-1}(\lambda)$ is fuzzy β-dense in X. Then by (3), $f(1 - f^{-1}(\lambda))$ is fuzzy α-dense in Y. But $f(1 - f^{-1}(\lambda)) = f$ $(f^{-1}(1 - \lambda)) < 1 - \lambda < 1$(since $\lambda \neq 0$). This is a contradiction to the fact that f $(1 - f^{-1}(\lambda))$ is fuzzy α-dense. Therefore, there exists a fuzzy β-open set μ in X such that $\mu \leq f^{-1}(\lambda)$. Hence f is somewhat fuzzy almost α-irresolute function. □

Theorem 3.3. *Let $(X_1, T_1), (X_2, T_2), (Y_1, S_1)$ and (Y_2, S_2) be fuzzy topological spaces such that X_1 is product related to X_2 and Y_1 is product related to Y_2. Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be somewhat fuzzy almost α-irresolute functions. Then $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is somewhat fuzzy almost α-irresolute function.*

Proof. Let $\lambda = \bigvee_{i,j}(\lambda_i \times \mu_j)$ be fuzzy α-open set in $Y_1 \times Y_2$(where λ_i and μ_j are fuzzy α-open sets in Y_1 and Y_2 respectively). We can assume that λ_i's and μ_j's are not all zeros. If any one is zero, that factor can be omitted. Now $(f_1 \times f_2)^{-1}(\lambda) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\lambda_i \times \mu_j)) = \bigvee_{i,j}(f_1 \times f_2)^{-1}(\lambda_i \times \mu_j) = \bigvee_{i,j}(f_1^{-1}(\lambda_i) \times f_2^{-1}(\mu_j))$. Since $f_1 : X_1 \to Y_1$ is somewhat fuzzy almost α-irresolute and λ_i is fuzzy α-open set in Y_1 and $f_1^{-1}(\lambda_i) \neq 0$, there exists a fuzzy β-open set δ_i in X_1 such that $\delta_i \leq f_1^{-1}(\lambda_i)$. Also since $f_2 : X_2 \to Y_2$ is somewhat fuzzy almost α-irresolute and μ_j is fuzzy α-open set in Y_2 and $f_2^{-1}(\mu_j) \neq 0$, there exists a fuzzy β-open set η_j in X_2 such that $\eta_j \leq f_2^{-1}(\mu_j)$. Therefore $\delta_i \times \eta_j \leq f_1^{-1}(\lambda_i) \times f_2^{-1}(\mu_j) = (f_1 \times f_2)^{-1}(\lambda_i \times \mu_j)$. Then by Theorem 4.7 and Theorem 4.12 in [4] $\bigvee_{i,j}(\delta_i \times \eta_j)$ is a fuzzy β-open set and $\bigvee_{i,j}(\delta_i \times \eta_j) \leq \bigvee_{i,j}(f_1 \times f_2)^{-1}(\lambda_i \times \mu_j) = (f_1 \times f_2)^{-1}$ $(\bigvee_{i,j}(\lambda_i \times \mu_j)) = (f_1 \times f_2)^{-1}(\lambda)$. This proves $f_1 \times f_2$ is somewhat fuzzy almost α-irresolute. □

The following lemma which is establish in [1] is required to prove the Theorem 3.4.

Lemma 3.1. [1] *Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. If λ is fuzzy set X and μ is a fuzzy set of Y, then $g^{-1}(\lambda \times \mu) = \lambda \wedge f^{-1}(\mu)$.*

Theorem 3.4. *Let $f : (X, T) \to (Y, S)$ be a function from fuzzy topological space (X, T) to another fuzzy topological space (Y, S). If the graph $g : X \to$*

$X \times Y$ of f is somewhat fuzzy almost α-irresolute, then f is somewhat fuzzy almost α-irresolute.

Proof. Let λ be a non zero fuzzy α-open set in Y. Then, by Lemma 3.1, we have $f^{-1}(\lambda) = 1 \wedge f^{-1}(\lambda) = g^{-1}(1 \times \lambda)$. Since g is somewhat fuzzy almost α-irresolute and $1 \times \lambda \neq 0$ is a fuzzy α-open set in $X \times Y$, there exists a fuzzy β-open $\mu(\neq 0)$(say) of X such that $\mu \leq g^{-1}(1 \times \lambda) = f^{-1}(\lambda)$. This proves that f is somewhat fuzzy almost α-irresolute function. □

4. Strongly somewhat fuzzy β-open functions.

The concept of somewhat fuzzy β-open function was introduced in [10]. In this section we shall introduce the strongly notion as follows:

Definition 4.1. Let (X, T) and (Y, S) be fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called **strongly somewhat fuzzy β-open** if and only if for each non-zero fuzzy α-open set λ of (X, T), there exists a fuzzy β-open set μ in (Y, S) such that $\mu \neq 0$ and $\mu < f(\lambda)$.

Clearly every strongly somewhat fuzzy β-open function is somewhat fuzzy β-open function. However the converse is not true as the following example shows:-

Example 4.1. Let μ_1, μ_2 and μ_3 be fuzzy sets in I defined in Example 3.1. Clearly $T_1 = \{0, \mu_1, 1\}$ and $T_2 = \{0, \mu_2, 1\}$ are fuzzy topologies on I. Let $f : (I, T_1) \to (I, T_2)$ be defined by $f(x) = \min\{2x, 1\}$ for each $x \in I$. It can be easily seen that int $\mu_3 = \mu_1$; cl $\mu_1 = 1$ in (I, T_1). Simple computations gives $f(0) = 0$; $f(1) = 1$; $f(\mu_1) = 0$. Thus f **is somewhat fuzzy β-open function.** Since $\mu_3 \leq$ int cl int μ_3 in $(I, T_1), \mu_3$ is a fuzzy α-open set in (I, T_1). But

$$f(\mu_3)(y) = \begin{cases} 0, & 0 \leq y \leq \frac{1}{2}, \\ \frac{1}{3}, & \frac{1}{2} \leq y \leq 1. \end{cases}$$

Let λ be any non-zero fuzzy set such that $\lambda \leq f(\mu_3)$ in (I, T_2). Then cl int cl $\lambda = 0$, this shows that λ is not fuzzy β-open set. Thus there is no non-zero fuzzy β-open set such that it is contained in $f(\mu_3)$. Hence f **strongly somewhat fuzzy β-open** functions.

Theorem 4.1. *Suppose (X, T) and (Y, S) be fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an onto function. If f is strongly somewhat fuzzy β-open and λ is a fuzzy β-dense set in Y, then $f^{-1}(\lambda)$ is fuzzy α-dense in X.*

Proof. Suppose λ is a fuzzy β-dense set in Y. We want to show that $f^{-1}(\lambda)$ is a fuzzy α-dense set in X. Suppose not. Then there exists a fuzzy α-closed set μ in X such that $f^{-1}(\lambda) < \mu < 1$. Then $1 - f^{-1}(\lambda) > 1 - \mu > 0$. $f(1 - f^{-1}(\lambda)) > f(1 - \mu)$ which implies that $f(f^{-1}(1 - \lambda)) > f(1 - \mu)$. That is, $f(1 - \mu) < ff^{-1}(1 - \lambda) = 1 - \lambda$. Now μ is fuzzy α-closed $\Rightarrow 1 - \mu$ is fuzzy α-open in X. Since f is strongly somewhat fuzzy β-open, $1 - \mu$ is fuzzy α-open in $X \Rightarrow$ there exists a fuzzy β-open set $\delta \neq 0$ in (Y, S) such that $\delta < f(1 - \mu)$.

Therefore $\delta < f(1-\mu) < 1-\lambda \Rightarrow \delta < 1-\lambda \Rightarrow \lambda < 1-\delta$. Now $1-\delta$ is fuzzy β-closed set and $\lambda < 1-\delta \Rightarrow \lambda$ is not a fuzzy β-dense set in Y, which is a contradiction to our hypothesis. Therefore $f^{-1}(\lambda)$ must be a fuzzy α-dense in (X, T). □

Theorem 4.2. *Suppose (X, T) and (Y, S) be fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be a 1-1 and onto function. Then the following conditions are equivalent.*

(1) *f is strongly somewhat fuzzy β-open.*
(2) *If λ is a fuzzy α-closed set in X such that $f(\lambda) \neq 1$, then there exists a fuzzy β-closed set μ in Y such that $\mu \neq 1$ and $f(\lambda) < \mu$.*

Proof. (1)⇒(2). Let λ be a fuzzy α-closed set in X such that $f(\lambda) \neq 1$. Then $1-\lambda$ is fuzzy α-open and since f is 1-1 and onto $f(1-\lambda) = 1 - f(\lambda) \neq 0$ [3]. As f is strongly somewhat fuzzy β-open, there exists a fuzzy β-open set η in Y such that $\eta \neq 0$ and $\eta < f\ (1-\lambda) = 1 - f(\lambda)$. That is $f(\lambda) < 1 - \eta = \mu($ say) and μ is a fuzzy β-closed set. This proves (1)⇒(2).

(2)⇒(1). Let λ be a fuzzy α-open set in X such that $\lambda \neq 0$. Then $1-\lambda$ is fuzzy α -closed and $1-\lambda \neq 1$. Now $f(1-\lambda) = 1 - f(\lambda) \neq 1$(for, if $1 - f(\lambda) = 1$, then $f(\lambda) = 0 \Rightarrow \lambda = 0$). Hence by (2) there exists a fuzzy β-closed set μ in Y such that $\mu > f(1-\lambda)$. Then $\mu > 1 - f(\lambda)$. That is $f(\lambda) > 1 - \mu = \delta$(say). Clearly δ is fuzzy β-open set in Y such that $\delta < f(\lambda)$ and $\delta \neq 0$(since $\mu \neq 1$). This completes the proof of (2)⇒(1). □

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