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A Profile Analysis about Thermal Life Data of Electrical insulating materials at Accelerated Life Test

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ABSTRACT

Since 1987, when statistical analyzing guide for thermal life test of Accelerated Life Test(ALT) was proposed as ANSI/IEEE Std 101, this guide has been used widely for many experiment data. Shim(2004) had done Monte Carlo simulation to compare life of two different systems or materials, based on statistic values obtained from ANSI/IEEE Std 101 data. In this study, a profile analysis is proposed for comparing life of two different systems or materials, and some examples using pre-existing data are given.

Key words: Arrhenius model, Accelerated life data, Electrical insulating materials, profile analysis, thermal life data

1. INTRODUCTION

Reliability is a possibility or an ability of a product to perform a certain function under a given condition for an intended period. To evaluate an accurate reliability, functions and environment related to products should be regulated. Accelerated Life Test(ALT), a method for detecting a product's life or fault rate, accelerates degradation cause both in the sense of time and physical aspects. The main purpose of ALT is to obtain the data of a product's life, and to statistically estimate life when it is used.

If the life and stress are in a linear relation, estimate parameters are needed to find out regression line. However, if there are some problems in the data, estimated parameters and confidence intervals can be affected seriously. Shim(2004) has proposed that for the data following log-normal life distribution or whose numbers are small, using simulation for can be estimating parameters and confidence intervals more accurate than using method proposed in ANSI/IEEE Std 101.

In this study, based on Shim(2004) and ANSI/IEEE Std 101, profile analysis is used to compare life of two different systems or materials for the case of small number of data following lognormal life distribution. For the life-stress relation, Arrhenius Model is used, and data about life is created based on statistics from ANSI/IEEE Std 101 data. In chapter 2, Arrhenius Lognormal Model ,which is widely used for thermal life data model, is explained, in chapter 3, the regression model between life and temperature is estimated, the profile analysis for comparing groups is applied in chapter 4, and in chapter 5, cases on practical data are analyzed.

2. Arrhenius Lognormal Model

The most general environmental stress for ALT on electronic device is temperature. Arrhenius Model is used when the product's life is a function of temperature. This equation shows the relation between temperature and life of an insulator.

$$K = S' \exp(-E/kT) \qquad (1)$$

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K is Chemical reaction rate, E is an activation energy of the reaction which varies values with respect to failure mechanism, and its unit is eV(electron Volt). K is Boltzmann Constant $k = 8.617\times10^{-5} eV/^{\circ}K$, T is Kelvin Temperature $℃ + 273.16$, and S' is a constant which shows a product's characteristic or testing condition. Arrhenius Model is an application of Arrhenius Equation to ALT, and it assumes that when the reaction reaches the critical amount, fault occurs.

Therefore, the critical value can be obtained as shown below,

Critical value = reaction rate×time to failure and the time to failure t can be expressed like below.

$$t = \frac{critical\ value}{reaction\ rate} = S \cdot \exp\left(\frac{E}{kT}\right)$$

By the log transformation of the model, median life of the insulator data becomes proportional to 1/t, and this relation is shown below. Let $\log(L)$ the log value of equation K, the equation (1) can be written in below form.

$$\log(L) = \log(S) + (E/kT) \tag{2}$$

Here, log is a common logarithm. Equation (2) can be expressed in algebraic form.

$$Y = A + BX \tag{3}$$

In the equation, $X = 1/T, A = \log(S)$, $B = E/k$, and Y is $\log(L)$ which can be expressed as a log linear equation. In the equation (3), Coefficients A and B can be estimated from the experiment data, and both a and b are sample estimators.

L_{ij} is sample j's life under temperature i, and if its log value is Y_{ij}, $i = 1, \cdots, n$, $j = 1, \cdots, m$,

$$Y_{ij} = \log(L_{ij})$$

then $Y_{ij} = \log(L_{ij})$, and under the arbitrary temperature i, the mean of Y_{ij} is $\overline{Y_i}$, and standard deviation is s_i.

3. An estimation between life and temperature

Let's apply Arrhenius Model to analyzing the thermal life data, and make several assumption to estimate the change of life with respect to temperature. Here are assumptions.

(1) The relation between log life and inverse Kelvin temperatures are linear relation to the temperature range we are interested in.

(2) Sample data are statistically independent.

(3) Sample is selected arbitrarily from population which is under interest.

(4) Random variation of log life follows a normal distribution which has equal standard deviations under every temperature of interest.

Following above assumptions, change each Celsius temperature T to inverse Kelvin temperatures like below.

$$X_i = 1/(T_i + 273),\ i = 1, \cdots, n \tag{4}$$

Since each failure life in sample L is converted to Y ($Y = \log(L)$), sample estimator from population A, B in the equation (3) can be expressed as below.

$$a = \overline{Y} - b\overline{X} \tag{5}$$

$$b = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{X}\,\overline{Y}}{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2} \tag{6}$$

If $m(T_i)$ is the sample estimator of population mean log life from selected temperature T_i, the $m(T_i)$ can be calculated by using below equation.

$$m(T_i) = a + b[(1/(T_i + 273)] \tag{7}$$

The antilog of $m(T_i)$ is a median life estimated value for time unit under the temperature T_i. Here is a sample standard deviation of log life.

$$s = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(Y_i - (a + bX_i))^2} \tag{8}$$

For a variance of selected temperature T_i, following equation can be obtained.

$$V(T_i) = \frac{(X_i - \overline{X})^2}{\sum_{i=1}^{n}(X_i - \overline{X})^2} \tag{9}$$

Here, X_i is an inverse Kelvin temperature, $X_i = 1/(T_i + 273)$.

Confidence upper limit $m_U(T_i)$ and confidence lower limit $m_L(T_i)$ for mean log life $m(T_i)$ can be calculated as below by using equation (8) and (9).

$$m_U(T_i) = (a + bX_i) + t_{n-2}(\alpha)s\sqrt{(1/n) + V(T_i)} \tag{10}$$

$$m_L(T_i) = (a + bX_i) - t_{n-2}(\alpha)s\sqrt{(1/n) + V(T_i)} \tag{11}$$

4. Profile analysis of two group data

For an Electrical insulating materials, thermal resistance is one of the most important properties. They can be classified to 7 groups by the upper limit of temperature, and all 7 groups of insulator are composed of different materials.

Therefore, if there is some difference in the mean log life between two equal kinds of insulators, or if there is little difference between two different insulators, one of them can be a faulty one.

To compare the life of two different kinds of systems or materials, thermal life test can be applied. Once the data about both insulating materials are obtained, whether one material is clearly better or not can be observed by using Arrhenius line. Observed differences can be originated from an unexpected change. To assess if there is any reasonable difference, two sets of data should be compared, and this is similar to the method of comparing two means.

Comparison between lines can be done under more than one temperature, and sometimes, comparison under a certain range of temperatures can be more useful.

Profile Analysis is an analyzing method, examining whether the effect is equal or not when p processes(test, question, etc.) are done for more than two groups. It is similar to analysis of variance in the circumstance of agreement analysis of average vectors, but the difference is that profile Analysis includes several steps.

IF $\mu_1^T = [\mu_{11}, \mu_{12}, \cdots, \mu_{1p}]$ and $\mu_2^T = [\mu_{21}, \mu_{22}, \cdots, \mu_{2p}]$ are the mean log life of response value for two populations, null hypothesis $H_o : \mu_1 = \mu_2$ can be reconstituted with three steps shown below.

First, are profile of two groups parallel to each other?

Second, assuming that the profiles are parallel, are the profiles coincidental?

Third, assuming that the profiles are coincidental, are all means equal to the same constant?

Let $\overline{x}_1$, $\overline{x}_2$ be the sample mean of two samples with sizes n_1 and n_2, and independent to each other. C is the contrast matrix.

$$C = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

1) Test of parallel of two population profiles.

For testing whether two population's profiles are parallel to each other, null hypothesis can be written below.

$$H_{01} : C\mu_1 = C\mu_2 \tag{12}$$

This null hypothesis can be examined by modified observations Cx_{1i}, $i = 1, \cdots, n_1$ and

Table 1. Seven types of heat resistant at electrical insulating material

Types	Y	A	E	B	F	H	C
Limit of temperature rise (℃)	90 and less	91~105	106~120	121~130	131~155	156~180	181 and more

Cx_{2j} , $j=1,\cdots,n_2$. Sample mean vector of modified observations are $C\bar{x}_1$, $C\bar{x}_2$ each, and covariance matrix is $CS_{pooled}C^T$. Therefore, null hypothesis (12) is rejected when below conditions are satisfied.

$$T^2 = (\bar{x}_1 - \bar{x}_2)^T C^T \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) CS_{pooled} C^T \right]^{-1} C(\bar{x}_1 - \bar{x}_2) > c^2 \quad (13)$$

and,

$$c^2 = \frac{(n_1+n_2-2)(p-1)}{n_1+n_2-p} F_{p-1,n_1+n_2-p}(\alpha).$$

2) Test for coincident of the values of two population's profiles.

If null hypothesis H_{01} was accepted and satisfies the parallel-condition, then test for coincident should be done. Since profiles are coincident only if $\mu_{11}+\mu_{12}+\cdots+\mu_{1p}=1^T\mu_1$ and $\mu_{21}+\mu_{22}+\cdots+\mu_{2p}=1^T\mu_2$ are equal, null hypothesis should be examined.

$$H_{02} : 1^T\mu_1 = 1^T\mu_2 \quad (14)$$

Null hypothesis (14) is rejected under significance level α when below condition is satisfied.

$$T^2 = 1^T(\bar{x}_1 - \bar{x}_2)^T \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) 1 S_{pooled} 1^T \right]^{-1} \times 1^T(\bar{x}_1 - \bar{x}_2) > F_{1,n_1+n_2-2}(\alpha) \quad (15)$$

3) Test for equality of all means equal to the same constant.

When null hypothesis H_{01} and H_{02} are accepted, then test for equality of all means should be done. Common mean vector μ for two groups can be estimated as shown below by using n_1+n_2.

$$\bar{x} = \frac{\sum_{j=1}^{n_1} x_{1j} + \sum_{j=1}^{n_2} x_{2j}}{n_1+n_2} = \frac{n_1}{n_1+n_2}\bar{x}_1 + \frac{n_2}{n_1+n_2}\bar{x}_2 \quad (16)$$

If average values are equal, null hypothesis can be written as below since $\mu_1=\mu_2=\cdots=\mu_p$.

$$H_{03} : C\mu = 0 \quad (17)$$

Null hypothesis (17) is dismissed under significance level α when below condition is satisfied.

$$(n_1+n_2)\bar{x}^T C^T [CS_{pooled}C^T]^{-1} C\bar{x} > F_{p-1,n_1+n_2-p}(\alpha) \quad (18)$$

5. Examples

The table shown below is Nelson's(2004) data from two different electrical insulator. This data is the C type in Table 1, and it shows the life under testing temperatures of 200℃, 225℃, 250℃. Life time is the mean value obtained during exam period. Since the application to the atmosphere and the space is sometimes related to 250℃, the purpose of life test is comparing the mean log life in the whole range of testing temperatures and design temperature at 200℃.

Table 2. Nelson(2004)'s data from two types electrical insulators

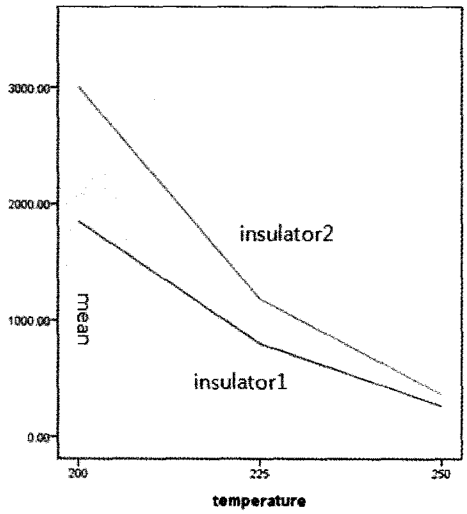
reference period	insulator 1			insulator 2		
	200℃	225℃	250℃	200℃	225℃	250℃
1st	1176	624	204	2520	816	300
2nd	1512	624	228	2856	912	324
3rd	1512	624	252	3192	1296	372
4th	1512	816	300	3192	1392	372
5th	3528	1296	324	3258	1488	444

5.1 Profile analysis

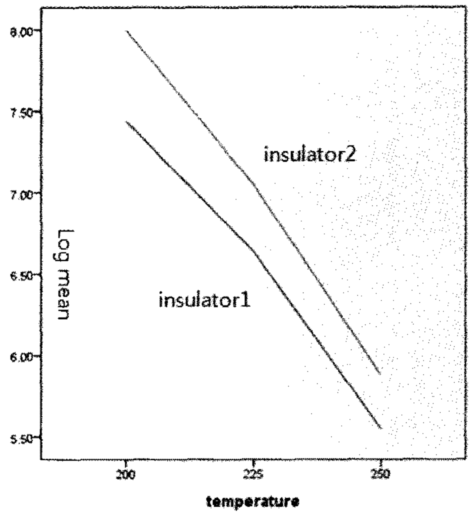
This is the result of 3-step profile analysis for hypotheses (12), (14), and (16) by using equation (13), (15), and (18). It is for profile analysis for log life of insulator 1 and 2.

Table 3. Profile analysis result of two groups' data

hypothesis	test statistics	critical value ($\alpha=0.05$)
H_{01} : $C\mu_1 = C\mu_2$	11.6343	6.9282
H_{02} : $1^T\mu_1 = 1^T\mu_2$	19.5541	4.1961
H_{03} : $C\mu = 0$	18.9099	3.3404



(a) profile diagram using mean life



(b) profile diagram using mean log life

Figure a. profile diagram of mean life and mean log life.

From the result that all hypotheses under significance level 5% are rejected, we can find out that parallel of profile under given temperature is not valid and both profile values and mean values are not coincident. These are profile diagrams of mean log life.

5.2 Simulation

Considering given temperature for both insulator 1 and 2, we have done 1000 times of Monte Carlo Simulations to estimate the relation between inverse Kelvin temperature and the log life. The estimation result for equation (3) is shown below.

1) An estimated formula of insulator 1

$$Y = -4.4504 + 3615.104X$$

2) An estimated formula of insulator 2

$$Y = -4.9549 + 3785.861X$$

For selected temperatures, value of appropriate point can be obtained from equation (7). It can be done by calculating corresponding estimated value of mean log life. From the antilog of this calculated value, estimated value of mean log life in the unit of time is obtained.

For example, for 200℃,

$$m(200) = -4.4504 + (3615.104)\times [1/(273+200] = 3.1925$$

Antilog of this is the estimated value of mean log life, and it is 1558 hours.

Table 4. An estimates of mean log life

T_i	insulator 1	insulator 2
200℃	3.1926	3.4720
225℃	2.8092	3.0490
250℃	2.4618	2.5662

5. Conclusion

Based on ANSI/IEEE Std 101 data, Shim(2004) had estimated regression model about inverse Kelvin temperature and log life with 10 measured values under 150℃, 6 values under 175℃, and 10 values under 200℃. However, from that simulation, Shim(2004) only found out that there are some differences between estimated values, and failed to prove by the statistical analysis because of limited data.

In this study, profile analysis for the log life of thermal life test data in ALT is done, to compare data of one type from <table 1>. Since getting much data from thermal life test is difficult, as mentioned, it is considered to be more accurate for estimation to do simulation based on information of the empirical data.

However, setting reasonable conditions for simulation is still difficult because of limitation of using experimental data proposed in ANSI/IEEE Std 101. Therefore, we are leaving this accurate simulation problem as our next challenge.

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