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레이저 도플러의 진동에 대한 분석과 3차원 예측연구

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요 약

본 연구는 레이저 도플러의 진동모드에 대한 분석연구이다. 도플러에서 생성되는 진동모드를 측정하여 각 성분을 분석하여 2차원 연구로부터 3차원을 예측 할 수 있는 성질들을 연구하고자 한다. 진동모드는 범위 탐지기(거리 측정 센서)에 의존하고 있다. 즉, 측정거리에 의해 결정되고 있으므로, 생성되는 변수들로부터 2차원에서부터 3차원에서의 진동모드가 어떻게 생성되는지, 어떤 특성의 패턴으로 나타나는지를 연구함과 더불어, 진동모드와 거리와의 관계도 아울러 연구한다.

An Analysis and a 3D Prediction of vibration modes in a Laser Doppler

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Abstract

This is a study on the analysis of vibration mode of a laser doppler. We measure the vibration mode of a doppler and analyze each component, and want to estimate three dimensional properties from 2-dimensional data. The vibration mode relies on a range detector that uses a distance sensor. Since the outputs are determined by the measured distance, we want to study how 3-dimensional vibration mode is generated from 2-dimensional ones. The study will include the patterns of generating a 3-dimensional vibration mode as well as the relationship between the distance and the vibration mode.

Keywords : Laser Doppler, object coordinates, laser coordinate system, orthogonal components, eigenvalue, eigenvector

I. INTRODUCTION

A Laser Doppler(LD) is a velocity transducer with scanning capability. The scanner aims the laser beam at desired measurement points on the object. The LD offers many advantages over the conventional accelerometer in dynamic measurements. It has found many applications in the research laboratory and industry for dynamic testing, modal analysis, noise control and damage identification.

(Figure 1) shows the definitions of the coordinate systems used in this paper.

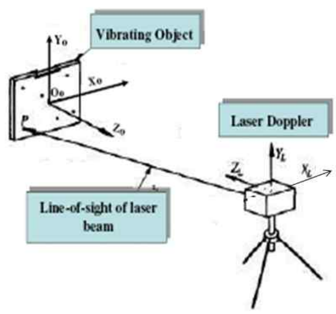

Figure 1. A Scanning Laser Doppler and Coordinate System

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A Cartesian coordinate system called the object coordinate system $O_oX_oY_oZ_o$, (global coordinate system) is established. The points of interest on the object are defined with respect to this coordinate system, which means the object coordinates (x_o,y_o,z_o) of the points of interest are known. Although the object coordinate system is placed on the object in Figure 1, it can be put anywhere in space. Another Cartesian coordinate system called the laser coordinate system $O_LX_LY_LZ_L$ is also established. It is placed inside the laser head. [5, 6, 7]

In our laser-based six degree-of-freedom mobility measurement system, the LD is used for the velocity measurement. The measured velocity is a vector along the line-of-sight of the laser beam. The vector direction of the velocity should be described with respect to the object coordinate system. The direction of the line-of-sight cannot be directly measured by the LD, however, this direction can be obtained once the pose of the laser coordinate system is obtained with respect to the object coordinate system. [8, 9, 10]

Furthermore, in engineering there is a need to be able to measure the vibrations of three-dimensional bodies, and experimental techniques which can measure motion in only a single direction are not always satisfactory.

Optical techniques have been shown to have certain practical advantages for vibration analysis. Generally, this involves obtaining a minimum of three sets of readings, each measuring components in a particular direction known as the sensitivity vector. The principle disadvantage of this method is that it is not possible to measure orthogonal components independently; hence, to calculate a resultant vector displacement it is necessary to make a minimum of three sets of readings and solve them simultaneously. This can be computationally intensive and generally requires extra redundant data to avoid large computational errors. What is needed is an optical technique that is capable of measuring orthogonal components of vibration independently. In our work, it will show how to approach to measure orthogonal components independently and try to find what is the relationship with the components.

II. Main Principles of the Vibration Analysis

2.1. An analysis of components of the measured velocity

In 3-dimension case, we want to measure the velocity from 3 LDs and obtain each components of the measured velocity. If it is obtained the vibrating components of each axis, it is calculated the characteristic of the vibration using the given algorithms, Peak picking method, quadrature picking method [1, 2, 4]. However, to calculate the vibrating mode and operating Deflection Shape in the 3-dimension, we need an analysis of how is related with each component of three axes at each measured point.

Now, at the first step it is considered to measure the each component of the axes, it is concrete to understand it. This outline of the concept is as follows (Figure 2).

Figure 2. A direction component of two axes using two sensors

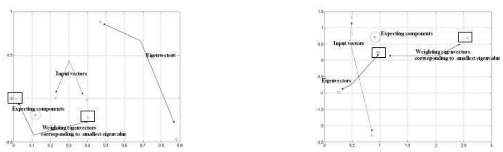

Figure 3. Distribution of vectors when the angle is same

From two cases, it is obtained the components of axes on 2-dimension with 2 LDs. The input vectors is considered as characteristic vectors.

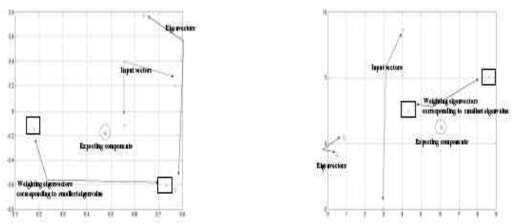

Figure 4. Distribution of vectors when the angle is different

The covariance matrix Σ is obtained from the characteristic vectors. (Figure 3, 4) show the distribution of the expecting each components and some related vectors from input measured velocity from LDs.

The eigenvalues of the covariance matrix Σ give an approach the components on the vibrating object. It is expected some criterion of the component of axes from the eigenvectors of the given covariance matrix Σ without the angle of the LD. It is shown some criterion to approach the components on the vibrating object with the velocity of LD (random velocities and random angles)

The aim here is to extend the technique to examine three dimensional vibrating structures, develop the theory to support the analysis, and present the pattern study of the vibration mode. Finally, the areas of application of this technique are discussed, together with its potential for development and its limitations.

2.2. Coordinate transformation

A spatial point P can be fully determined by its object coordinates $(x_{P,O}, y_{P,O}, z_{P,O})$. It can also be fully determined by its laser coordinates $(x_{P,L}, y_{P,L}, z_{P,L})$. The relationship between the object coordinates and the laser coordinates is the coordinate transformation.

$$\begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix}_O = \{T\}_L^O + \{R\}_L^O \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix}_L \qquad (1)$$

where $\{T\}_L^O$ is the translation vector. It is the coordinates of the origin O_L, measured in the object coordinate system, i.e. the position of the laser coordinate system. $\{R\}_O^L$ is the rotation matrix, i.e. the orientation of the laser coordinate system. Note that the rotation matrix is an orthogonal matrix and its determinant is positive one. Thus, the rotation matrix is defined by three independent variables. The pose of the LD is defined as the position and orientation of the laser coordinate system, i.e. $\{T\}_L^O$ and $\{R\}_L^O$.

2.3. Transformation of the characteristic space and pattern study

The characteristic vectors is considered as components with the velocity that is measured by each LD. On the 3-dimension, denote the distance of each group by $d(y, Y)$ where a pattern is $y=(x_1,x_2,x_3)^T$ and a class is $Y=\{y_1,y_2,\cdots,y_n\}$. Define the distance such that

$$d^2(y,Y)=\frac{1}{n}\sum_{i=1}^{n} d^2(y,y_i)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{3}(x_j-x_{ij}).$$

The classification is determined where is located in LD and measured the velocity of the characteristic vectors. To visualize the characteristic vectors, it is to transform the characteristic space. The transformation of the characteristic space is to generating mapping from characteristic space to another new space. A vector x of a characteristic space is transformed to y where $y=Ax$. The matrix A is dependent on the size of x and y. It is called a transformation with respect to x and y. It is a need to find the transformation for our problem. To extract the characteristic pattern, it must be located close each other in the similar characteristic vectors. Furthermore, what it is distinguished among them as expected needs to transform the characteristic space to minimize the distance of each vectors in a group. Define the distance from any two vectors in a group by $D(X)$ such that

$$D^2(X) = 2\sum_{i=1}^{3}(\sigma_k)^2$$

A distance from any two vectors in a group is to calculate by sum of variance from characteristic pattern that composed a group. The larger the sum of variance is, the longer the distance from any two vectors in a group is. This information give a standard basis to transform from a space to the other space to distinguish easily. It is a outline by definition for distance from any two vectors in a group as follows.

$$D^2(X) = 2\sum_{i=1}^{3}(\omega_{ii}\sigma_k)^2 \qquad (2)$$

It is obtained ω_{ii} under the condition $\sum_{i=1}^{3}\omega_{ii} = 1$ to minimize the distance.

This restricted condition is preserved uniformly to a normalized unit cubic of the characteristic space. It is obtained to minimize it from (1) under the equation (2) by the method of Lagrange undetermined multiplicity.

$$L = 2\sum_{i=1}^{3}(\omega_{ii}\sigma_k)^2 - \alpha\left(\sum_{i=1}^{3}\omega_{ii} - 1\right) \qquad (3)$$

Therefore, the equation (3) can be partially derivative with respect to ω_{ii} to obtain the extreme values of L and the result can be equal to zero.

$$\frac{\partial L}{\partial \omega_{ii}} = 4\omega_{ii}\sigma_i^2 - \alpha = 0,\ 1 \le i \le 3$$

From above the result, it is obtained $\omega_{ii} = \frac{\alpha}{\sigma_i^2}$ by ω_{ii}. If the pattern x is composed of n -characteristic such that $x_1, x_2, \cdots, x_n$, it is a representation as $x = (x_1, x_2, \cdots, x_n)^T \in R^n$. Let $\nu = \{v_1, v_2, \cdots, v_n\}$ be a normalized orthogonal basis and a matrix A is composed by $v_1, v_2, \cdots, v_n$ where the size of A is $n \times n$. Then it conclude that $A^T A = AA^T = I$ and each characteristic of the normalized orthogonal basis is brought out

$$\begin{cases} v_i^T v_j = \delta_{ij} \\ v_1 v_1^T + \cdots + v_n v_n^T = n \end{cases}.$$

By these properties, arbitrary pattern x can be expressed as a unique linear combination of normalized or the orthogonal basis. The eigenvalues of the covariance of the pattern, $\Sigma = E(xx^T)$, is expressed $\lambda_1 \ge \lambda_2 \ge \cdots \lambda_n$ by ordering and each normalized eigen vector μ_i corresponding to each eigen value λ_i has the properties such that $\begin{cases} \sum \mu_i = \lambda_i \mu_i \\ \mu_i^T \mu_i = 1 \end{cases}$. Therefore, a set of the eigen vectors, $\mu = \{\mu_1, \mu_2, \cdots, \mu_n\}$ is composed of a normalized orthogonal basis and any pattern x can be expanded as $x = z_1\mu_1 + z_2\mu_2 + \cdots, + z_n\mu_n$ with μ. In our work, it is considered $n = 3$.

III. AN EXTENSION OF THE THREE DIMENSIONAL VIBRATION MEASUREMENTS

There are many situations in vibration analysis where it is necessary or desirable to make three-dimensional measurements. By using three different illumination geometries around a single imaging system, LD can be used to measure the orthogonal components of vibration amplitude independently.

These can be combined to determine the three-dimensional amplitude and mode shape. Examples of experimental results are presented for volume vibrations of a thick cylinder and observation of in-plane modes in a thin plate.

It is considered 3 different velocities and different angles as follows.

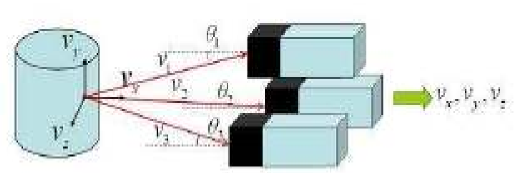

Figure 5. Components of three axes using three laser beams

If a LD is located on the x-axes, the input vector can be presented by $(0,\ v_1\cos\theta_1,\ v_1\sin\theta_1)$. When the same concept applies to both y-axes and z-axes, it will be obtained $(v_2\cos\theta_1,\ 0,\ v_2\sin\theta_2)$, $(v_3\cos\theta_3,\ v_3\sin\theta_3, 0)$.
From these input velocities, some criterion region of the components is obtained. Figure 6 is shown all eigenvectors and some expected region of the criterion of the components.

From those experiments of two dimensional and three dimensional cases, we apply the modified newton method as follows [3]:

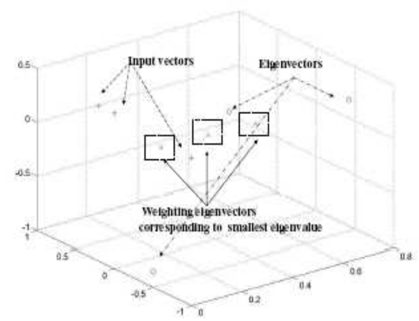

Figure 6. Distribution of vectors when each angle and velocity are different

Modified Newton's Algorithm

Set $\alpha_k^{(1)}$ and $x_k^{(1)}$. $(\alpha_k^{(1)}, x_k^{(1)})$ is an initial eigenpairs where $\alpha_k^{(1)}$ and $x_k^{(1)}$, $k=1,\cdots,n$ are the diagonal entries of the covariance matrix Σ that give an approach the components on the vibrating object and $\{e_1, e_2, \cdots, e_n\}$

For $k=1,\ldots,n$ do (in parallel)

For $j=1,\ldots$ do until convergence

(i) Solve for $y_k^{(j)}$, $(\alpha_k^{(j)} I - \Sigma) y_k^{(j)} = x_k^{(j)}$

(ii) Compute $\beta_k^{(j)} = x_k^{(j)T} \cdot y_k^{(j)}$

(iii) Compute $\widehat{\beta_k^{(j)}} = |y_k^{(j)}|_2$.

(iv) Compute $x_k^{(j+1)} = \dfrac{1}{\widehat{\beta}_k^{(j)}} y_k^{(j)}$

(v) Compute $\alpha_k^{(j+1)} = \alpha_k^{(j)} - \dfrac{1}{\beta_k^{(j)}}$

It is an advantage to get all eigenpairs simultaneously without consecutive computing.

IV. CONCLUSIONS

W conclude three main result. First we analyze the vibration mode and we study the relation between the eigenvalues of the covariance matrix Σ that give an approach the components on the vibrating object and vibration velocity in section 2. It is estimated the bound criterion for a vibration measurement. It can be shown that the eigenvector corresponding to the smallest eigenvalue of Σ is a approaching to the component of the vibrating object. From these experiments, it is obtained some bound criterion for error from the eigenvector. Suppose v_1, v_2 are input vectors, $\|v_1\| \le \|v_2\|$ and $\mu_{\min}$ is the eigenvector corresponding the smallest eigenvalue then it is the expecting component vector cmp such that $\|v_1\| \le \|cmp\| \le \|v_2\|$. Let the angle of the component be θ and the angle of the eigen vector $\mu_{\min}$ be ρ, then $\|\theta-\rho\| \le \frac{\pi}{6}$. These are four different experiments [Figure 7 and 8] for Figure 3 and Figure 4 in section 2.1 with input vectors $n=100$. Thirdly, we extend and predict 3-dimensional case through

2-dimensional case's properties in section 3.

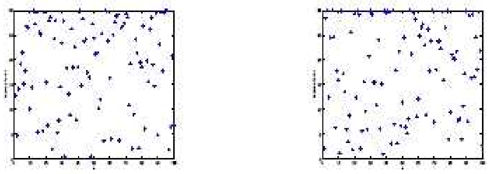

Figure 7. Distributions of error of degree between the component and the approaching components

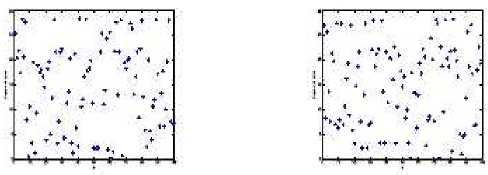

Figure 8. Distributions of error of degree between the component and the approaching components

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