Study on the Chirped Waveform of the USPR Pulse using the Impulse Response of a Waveguide

Young-Su Roh*

Abstract

In ultrashort-pulse reflectometry (USPR), a chirped waveform transformed from the USPR source impulse signal via waveguide makes it possible to employ millimeter-wave mixers for the frequency up-conversion process. Consequently, the frequency bandwidth of the USPR system is sufficiently wide to cover a large portion of the electron density profile of the plasma. Some physical aspects of the chirped waveform, such as maximum amplitude and length, are critical factors to determine the performance of the system. In this paper, the propagation of the USPR impulse signal through a rectangular waveguide is numerically studied to derive the chirped waveform using the impulse response of the waveguide. The results of numerical computation show that the chirped waveform significantly depends on the waveguide cutoff frequency as well as the waveguide length.

Key Words : USPR, Chirped Waveform, Waveguide, Frequency Bandwidth, Impulse Response

1. Introduction

In the USPR system utilized to measure the electron density profile on the Sustained Spheromak Physics Experiment (SSPX), a 5[V], 65[ps] duration full width half maximum (FWHM) impulse signal is dispersed via a waveguide to form a chirped waveform [1-4]. This technique makes it possible to up-convert a lower frequency impulse signal to higher frequencies for propagation into the plasma using broadband millimeter-wave mixers. Some physical aspects of the chirped waveform, such as maximum amplitude, length and power spectrum, are critical factors to determine the performance of the USPR system. For the theoretical and quantitative examination of the characteristics of the chirped waveform, in this paper, the propagation of the USPR impulse signal through a rectangular waveguide is numerically studied by means of the impulse response of the waveguide. As numerous authors have demonstrated, the impulse response of a waveguide can be expressed in closed-form in terms of Bessel functions of the first kind [5-7].

Before proceeding to the discussion of details regarding the derivation of the impulse response in closed-form and the chirped waveform, a brief description of the USPR system and the frequency

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bandwidth of an impulse signal will be presented in the following section to understand the importance of the chirped waveform in the system.

2. USPR System on SSPX

Figure 1 shows a schematic diagram of the USPR system installed on SSPX [8]. The basic principle of USPR has been discussed in Ref. [8] along with a detailed description of the SSPX USPR system. In SSPX, the envisioned range of plasma density is $0.5 \sim 3 \times 10^{14}$ [cm^{-3}] which is equivalent to 33~158[GHz] in the frequency domain of the O-mode cutoff layers [9]. As will be discussed in Sec. 3, extremely short duration (<5[ps]) impulse sources are required to achieve such wide frequency bandwidth. Currently, however, such ultrashort duration and high power impulse generators are not commercially available. In order to overcome this practical problem, millimeter-wave mixers are utilized to expand frequency bandwidth. This technique permits the use of a commercially available impulse generator. Figure 2 shows the waveform of the USPR impulse source (Model: Picosecond Pulse Labs 3500c) which provides a 5 [V], 65[ps] duration (FWHM) pulse.

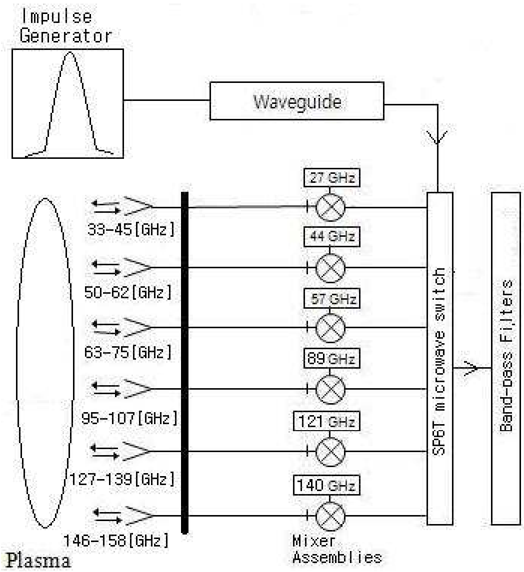

Fig. 1. Schematic diagram of the USPR system

3. Frequency Bandwidth of a Gaussian Pulse

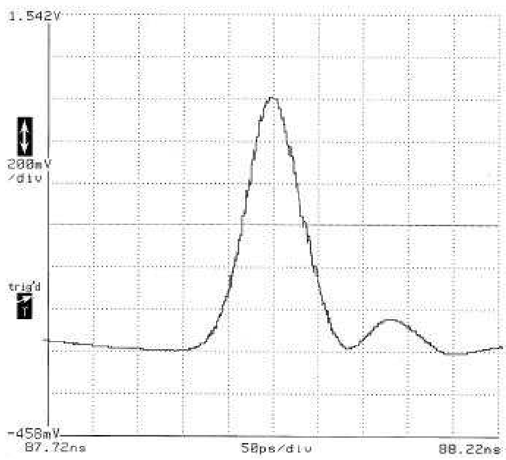

Fig. 2. The waveform of the USPR source

For the determination of the optimum pulse duration of the USPR impulse source to completely probe the plasma density profile, it is necessary to consider the relation between the pulse duration in the time domain and its bandwidth in the frequency domain. If an impulse signal is assumed to be a Gaussian pulse, it can be expressed as a function of time;

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} \tag{1}$$

Here, $1/\sqrt{2\pi}\sigma$ is the normalization factor. By applying the following integral to Eq. (1), it is straightforward to obtain the Fourier transform of $f(t)$ as a function of angular frequency, ω;

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt = e^{-\frac{\sigma^2 \omega^2}{2}} \tag{2}$$

Equation (2) demonstrates that the Fourier transform of a Gaussian pulse is also Gaussian. The pulse duration is conventionally expressed in terms of FWHM. FWHM is defined as the time interval between the leading edge and trailing edge of at a time where the amplitude is 50[%] of the peak value. As shown in Fig. 3, the FWHM of a Gaussian pulse is given by

$$FWHM = \Delta t = 2\sigma\sqrt{2\ln 2} \tag{3}$$

The frequency bandwidth is assumed to be the interval between the center frequency of and a frequency where the amplitude is 10[%] of the peak value. That is,

$$\Delta\omega = 2\pi\Delta f = \frac{\sqrt{2\ln 10}}{\sigma} \tag{4}$$

Hence, the relation between the pulse duration and the frequency bandwidth becomes

$$\Delta t\Delta f = \frac{2\sqrt{\ln 2 \ln 10}}{\pi} \tag{5}$$

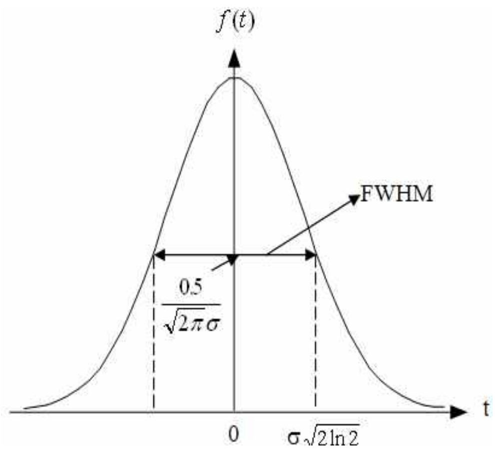

Fig. 3. The pulse duration of a Gaussian pulse

Figure 4 illustrates the dependence of the frequency bandwidth of a Gaussian pulse on the pulse duration. It is obvious that the frequency bandwidth of a pulse is directly related to the pulse duration. As the pulse duration becomes shorter, the frequency bandwidth becomes wider. Particularly, the frequency bandwidth is very sensitive to the pulse duration in the range from 5 to 20[GHz]. It should be noted that the pulse duration of the USPR source must be at least 5[ps] to satisfy the SSPX diagnostic requirement for frequency bandwidth.

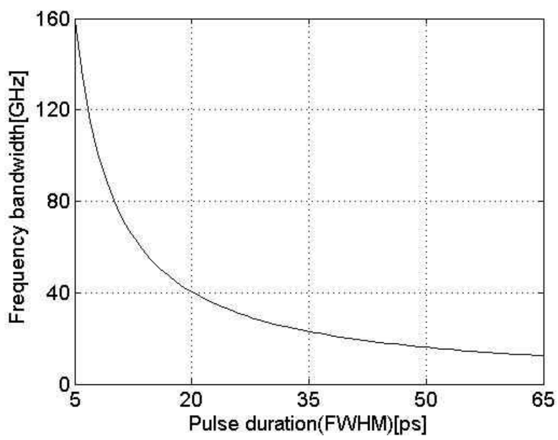

Fig. 4. Frequency bandwidth of a Gaussian pulse as a function of pulse duration (FWHM)

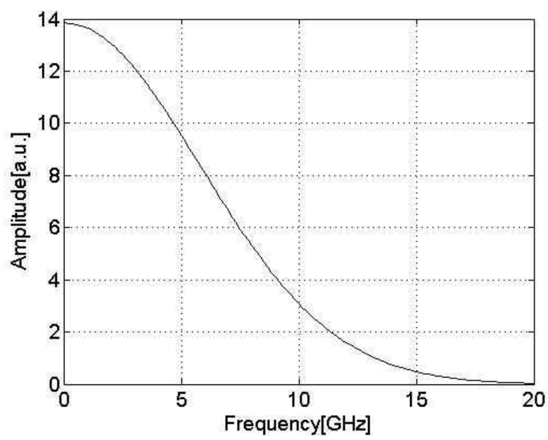

Fig. 5. Fourier transform of the Gaussian pulse whose pulse duration is 65[ps]

As mentioned previously, the pulse duration of the USPR impulse source is 65[ps] (FWHM). Figure 5 shows the Fourier transform of the Gaussian pulse whose pulse duration is 65[ps]. As can be seen, the frequency bandwidth of this pulse is approximately 12[GHz] which is too narrow to cover the whole range of the SSPX plasma density profile. It is inevitable, therefore, to use broadband millimeter-wave mixers to expand the frequency bandwidth of the USPR system.

4. Chirped Waveform

As discussed in the previous section, broadband millimeter-wave mixers are utilized to up-convert the USPR source radiation to higher frequencies. However, these mixers are limited to IF signals below a level of 500[mV]. Therefore, it is necessary to disperse the 5[V] impulse signal into a monotonically increasing frequency chirp via waveguide. In this section, the propagation of a Gaussian pulse through a rectangular waveguide is numerically studied to examine the property of a chirped waveform.

Figure 6 depicts the simplest form of the propagation of a Gaussian pulse through a rectangular waveguide. Here, the impulse response of the waveguide is given by

$$h(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j(\omega t - kz)} d\omega \tag{6}$$

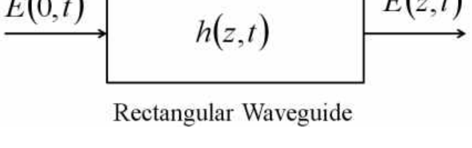

Fig. 6. Propagation of a Gaussian pulse through a rectangular waveguide

The wave number for the waveguide is

$$k(\omega) = \frac{1}{c}\sqrt{\omega^2 - \omega_c^2} \tag{7}$$

where ω_c is the waveguide cutoff frequency and c is the speed of light in vacuum. The output signal of the waveguide, $E(z,t)$ can be obtained by taking the convolution of the impulse response, $h(z,t)$ and the input signal of the waveguide, $E(0,t)$. That is

$$E(z,t) = \int_{-\infty}^{\infty} h(z,t')E(0,t-t')dt' \tag{8}$$

It is well known that the impulse response of a waveguide, $h(z,t)$ can be expressed in closed form in terms of Bessel functions as follows [5-7]. For $t \ge t_f = L/c$ (L is the waveguide length),

$$h(z,t) = \delta(t - t_f) - \frac{J_1\left(\omega_c\sqrt{t^2 - t_f^2}\right)}{\sqrt{t^2 - t_f^2}}\omega_c t_f u(t - t_f) \tag{9}$$

where J1 is the ordinary Bessel function. According to the causality condition, $h(z,t) = 0$ for $t < t_f$. By substituting Eq. (9) into Eq. (8), it is possible to obtain the output signal of the waveguide;

$$E(z,t) = E(0,t - t_f) - \int_{t_f}^{t} \frac{J_1\left(\omega_c\sqrt{t'^2 - t_f^2}\right)}{\sqrt{t'^2 - t_f^2}}\omega_c t_f E(0,t-t')dt' \tag{10}$$

Assuming that $E(0,t)$ is a Gaussian pulse of which pulse duration is 65[ps] as shown in Fig. 7,

$E(z,t)$ can be numerically computed using Eq. (10). Figure 8 shows typical output signals of the waveguide after propagating through the waveguide. As can be seen, the waveform is chirped with a gradually decreasing frequency that settles out to the waveguide cutoff. The basic principle of transforming a pulse signal into a chirped waveform lies in the dispersive property of a waveguide. Due to the dependence of the group velocity on frequency in a dispersive medium, the higher frequency component of the Gaussian pulse propagates through the waveguide faster than the lower frequency component. In other words, the higher frequency component comes out of the waveguide earlier than the lower frequency component. Consequently, the input pulse spreads out to become a chirped waveform. Since the lower frequency components below the waveguide cutoff frequency cannot propagate through the waveguide and most of the power of the Gaussian pulse resides in the lower frequencies, the amplitude of the chirped waveform becomes extremely smaller than that of the Gaussian pulse.

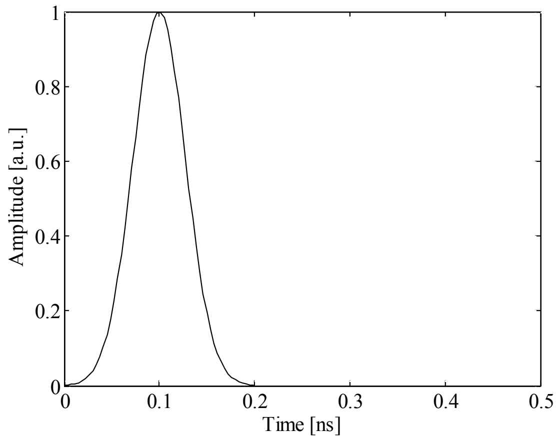

Fig. 7. The Gaussian pulse launched into the waveguide at z=0[m]

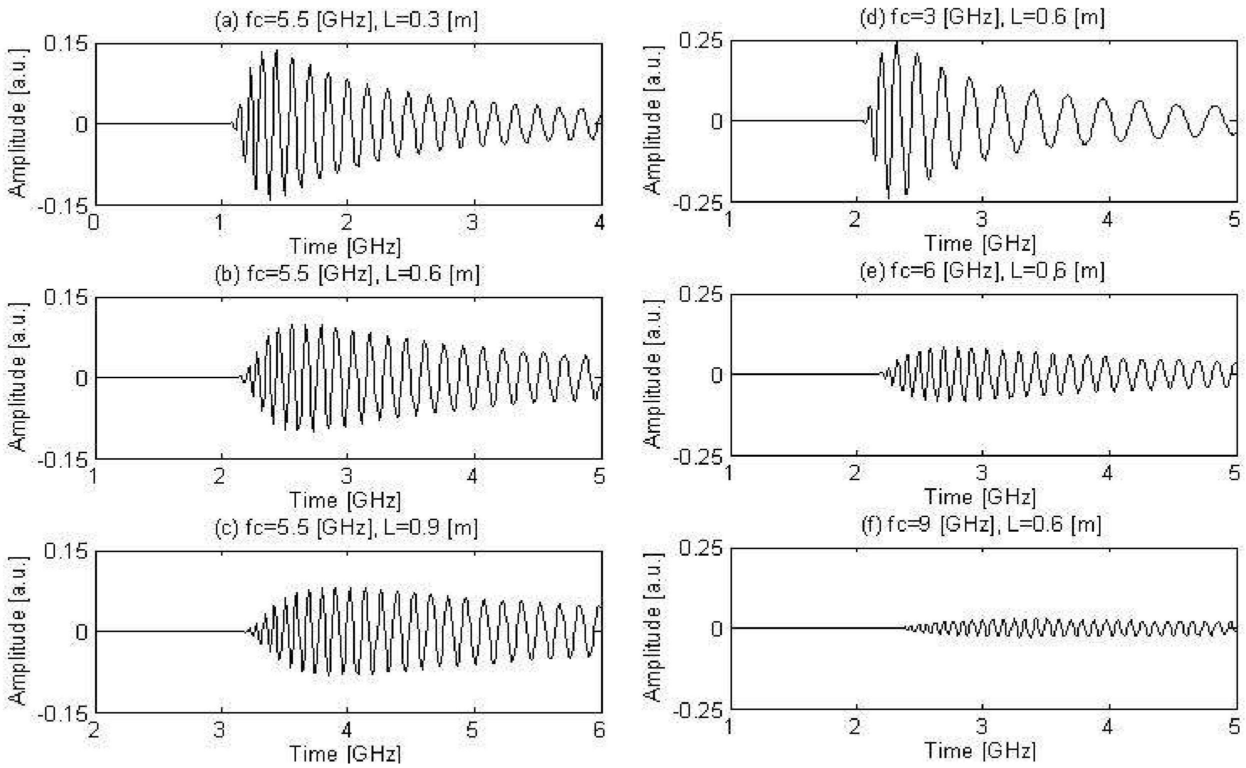

Fig. 8. Various chirped waveforms transformed from the Gaussian pulse via waveguide

As shown in Figs. 8 (a), (b) and (c), the length of the waveguide has a significant effect on the maximum absolute amplitude of the chirped waveform, $|E(L,t)|_{\max}$. Figure 9 depicts the maximum amplitude ratio of input and output signals of the waveguide, $1/|E(L,t)|_{\max}$ as a function of waveguide length. Here, the dotted line indicates 0.1 on the y axis. Considering that the 5[V] USPR source signal is assumed to be a normalized Gaussian pulse, i.e., $|E(0,t)|_{\max}=1$, the dotted line is equivalent to 500[mV]. Consequently, the y-axis range above the dotted line cannot be used;

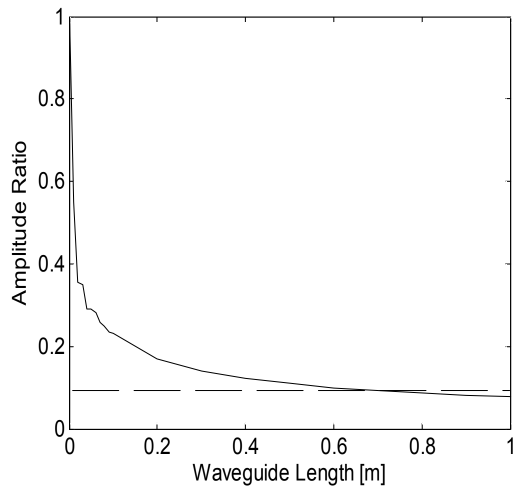

Fig. 9. The dependence of $1/|E(L,t)|_{\max}$ on the waveguide length at fc=5.5[GHz]

otherwise, the chirp peak may damage the mixer. In other words, the minimum length of the waveguide of which the cutoff frequency is 5.5[GHz] should be approximately 0.6[m] if no additional lossy component is involved before the waveguide.

In Figs. 8 (d), (e) and (f), it is found that the waveguide cutoff frequency is also a crucial factor to determine the property of the chirped waveform. Figure 10 shows the dependence of $1/|E(L,t)|_{\max}$ on the waveguide cutoff frequency. For the same reason mentioned in Fig. 9, the minimum cutoff frequency of the waveguide of which the length is 0.6[m] should be approximately 5.5[GHz].

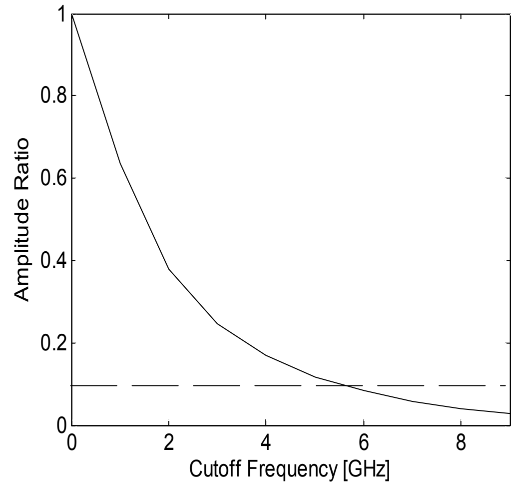

Fig. 10. The dependence of $1/|E(L,t)|_{\max}$ on the waveguide cutoff frequency at L=0.6[m]

5. Conclusion

In this paper, the propagation of the USPR impulse signal through a rectangular waveguide has been numerically studied using the waveguide impulse response expressed in terms of the ordinary Bessel function. The result of computation shows that the output signal of the waveguide is chirped with a gradually decreasing frequency that settles out to the waveguide cutoff frequency. It is also demonstrated that the chirped waveform significantly depends on the waveguide cutoff frequency as well as the waveguide length.

Acknowledgements

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Biography

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