교량의 생애주기비용 최적설계

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Optimal Life Cycle Cost Design of a Bridge

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Abstract: The importance of the life cycle cost (LCC) analysis for bridges has been recognized over the last decade. However, it is difficult to predict LCC precisely since the costs occurring throughout the service life of the bridge depend on various parameters such as design, construction, maintenance, and environmental conditions. This paper presents a methodology for the optimal life cycle cost design of a bridge. Total LCC for the service life is calculated as the sum of initial cost, damage cost, maintenance cost, repair and rehabilitation cost, user cost, and disposal cost. The optimization method is applied to design of a bridge structure with minimal cost, in which the objective function is set to LCC and constraints are formulated on the basis of Korean Bridge Design Code. Initial cost is calculated based on standard costs of the Korea Construction Price Index and damage cost on damage probabilities to consider the uncertainty of load and resistance. Repair and rehabilitation cost is determined using load carrying capacity curves and user cost includes traffic operation costs and time delay costs. The optimal life cycle cost design of a bridge is performed and the effects of parameters are investigated.

초 록 : 최근 들어 교량과 같은 구조물에 대한 생애주기비용(Life Cycle Cost, LCC) 분석의 중요성이 점차 커지고 있다. 그러나 교량의 공용수명 동안 발생할 수 있는 생애주기비용은 설계 및 시공조건 그리고 사용환경에 따라 많은 불확실성을 내포하고 있기 때문에 정확히 예측하기 힘들다. 본 논문에서는 교량의 생애주기비용 최적설계를 위한 설계방법을 제시하였다. 교량의 총생애주기비용은 초기비용, 손상비용, 유지관리비용, 보수/보강비용, 사용자비용, 해체/폐기비용의 합으로 산정하였다. 생애주기비용을 목적함수로 하고 도로교설계기준을 제약조건으로 최적화를 수행하였다. 초기비용은 종합물가정보 및 참고자료를 근거로 산출하였으며, 하중 및 부재에 대한 불확실성을 고려하기 위해 손상확률의 개념을 도입하여 손상비용을 산출하였다. 교량의 내하율곡선을 이용하여 교량의 보수/보강비용을 추정하였으며, 차량은행비용과 시간지연비용으로부터 사용자비용을 산정하였다. 이로부터 교량에 대한 생애주기비용 최적설계를 수행하고 주요인자들에 대한 영향을 살펴보았다.

Key Words: life cycle cost, repair and rehabilitation cost, user cost, load carrying capacity curve, optimal design

1. Introduction

Traditionally, the objective of structural design is to select member sizes within optimal proportioning of the overall structural geometry so as to achieve minimum initial cost design that meets the performance requirements specified in the conventional design code. A number of researchers have made efforts to develop optimization algorithms that are applicable to bridge structures. However, the concept of design objective is gradually changing from initial cost to life cycle cost¹⁻⁴⁾. Thus, the total cost does not only include initial construction cost but also other costs that may befall all along the service life of the structure such as maintenance cost, user cost, and disposal cost, etc. Accordingly, LCC became to occupy an important position in the design of structures and various cost models were developed to take LCC into account rationally³⁻⁶⁾. However, it is difficult to predict LCC precisely because the cost in the service life depends

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on various parameters such as design, construction, maintenance, and environmental conditions.

This paper presents a methodology for the optimal life cycle cost design of a bridge. Total LCC for the service life is calculated as the sum of initial cost, damage cost, maintenance cost, repair and rehabilitation cost, user cost, and disposal cost. The optimization method is applied to the design of a bridge structure with minimal cost, in which the objective function is set to LCC and constraints are formulated on the basis of Korean Standards. Initial cost is calculated based on standard costs of the Korea Construction Price Index and damage costs calculated from the damage probabilities to consider the uncertainty of load and resistance. Repair and rehabilitation cost is determined using load carrying capacity curves. The load carrying capacity curves together with the repair and rehabilitation histories are derived from bridge diagnostic results and condition grade curves suggested by the Korea Infrastructure Safety and Technology Corporation^{3,4,7,8)}. User cost is calculated based on the user cost model proposed by National Institute of Standard and Technology⁹⁾. The types of superstructure considered in this study are steel box girder, plate girder and PSC-I girder, and those of substructure are singlecolumn pier and double-column pier. The optimal life cycle cost design of a bridge was performed for various service lives and the effects of parameters were investigated.

Condition grade curves and load carrying capacity curves

2.1. Condition grade curves

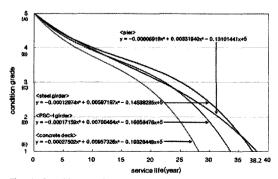


Fig. 1. Condition grade curves.

KISTC suggests condition grade curves as shown in Fig. 1 to estimate the level of damage^{7,8)}. In these curves, the conditions of the bridge members are classified into 5 grades. The curves were hypothetically developed through economic analysis in order to estimate the Life Cycle Profile of bridge member conditions and are the result of a regression analysis using a method similar to Delphi study.

2.2. Load carrying capacity curves

Load carrying capacity curves without repair and rehabilitation were derived using results of two consecutive bridge diagnostics and condition grade curves. The curves were derived on the assumption that, under high condition grade, the decline of load carrying capacity curve of the structure is slight and the slope of the load carrying capacity curve decreases rapidly as the condition grade decreases. The slope of the curve can be obtained with data of bridge diagnostics in high condition grade. However, in low condition grade, the slope cannot be obtained with data of bridge diagnostics alone, since the structure generally experiences repair and rehabilitation. Therefore, in low level, the slope of load carrying capacity curve was calculated by using a boundary condition at the end of service life^{3,4,10,11)}. Fig. 2 shows load carrying capacity curves proposed for superstructure members and pier without repair or rehabilitation. In general, the bridge composed of girder, deck and pier experiences several repairs and rehabilitations throughout its lifecycle. Therefore, the derivation of load carrying capacity curves considering repair and rehabilitation is complex and needs more maintenance data.

Condition grade below grade C means that the bridge member has general damage, defect, etc. When the condition grade reaches grade C, the structural member has to be repaired in order to prevent additional and rapid decrease of load carrying capacity. Proper repair increases its condition grade to grade B and makes the load carrying capacity and its slope recover those somewhat corresponding to grade B. However, the slope of the load carrying capacity after the repair is just assumed to be identical with the one at the beginning of grade B since it is difficult to predict the change in the structural load carrying

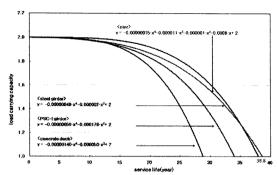


Fig. 2. Load carrying capacity curves.

capacity after repair. In the case of rehabilitation, the load carrying capacity itself is assumed to be increased and its slope to be restored to the required level of capacity. Many alternative scenarios can be made for maintenance of bridge superstructure including repair and rehabilitation. In this study, 2 repairs and 1 rehabilitation are decided to be the most appropriate plan in concrete slab, 2 repairs pertinent in steel girder, 1 repairs and 1 rehabilitation in PSC-I girder, and 1 rehabilitation adequate for pier^{3,4)}.

Formulation of optimal design of a bridge structure

The optimal design problem is formulated as a non-linear mathematical programming problem¹²⁾. In this paper, various types of bridge structure with span composition of 4@40m and width of 15.6 m (Fig. 3) are selected to analyze LCC and evaluate the economical efficiency. The types of superstructure considered in this study are steel box girder, plate girder and PSC-I girder, and those of substructure are single-column pier and double-column pier. Optimal design is carried out from concrete deck to pier sequentially.

3.1. Design variables

The first type of superstructure consists of steel box girders and a concrete deck. The design variables for the concrete deck are the slab height and amount of steel reinforcing bars. Those for steel box



Fig. 3, Bridge structure considered for optimal LCC design.

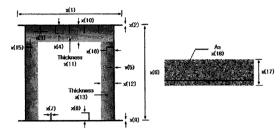


Fig. 4. Design variables of steel box girder.

girder are the thickness of flange and web, height of web, and dimensions of stiffeners. These are illustrated in Fig. 4.

The second type of superstructure consists of rationalized plate girders and a precast prestressed concrete deck. The design variables for the precast prestressed concrete deck are the slab height, amount of steel reinforcing bars and amount of prestressing strands. Those for the plate girder are the thickness of flange and web and, height of web as shown in Fig. 5.

The last type of superstructure consists of PSC-I girders and a concrete deck. The design variables for the concrete deck are the slab height and amount of steel reinforcing bars. Those for PSC-I girder are the thickness of flange and web, height of web, and area of prestressing strands as shown in Fig. 6.

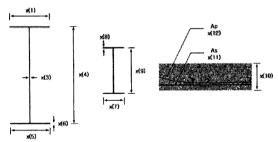


Fig. 5. Design variables of plate girder.

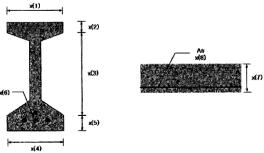


Fig. 6. Design variables of PSC-1 girder.

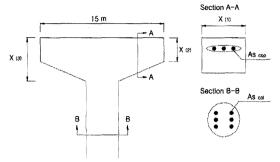


Fig. 7, Design variables of pier with single column.

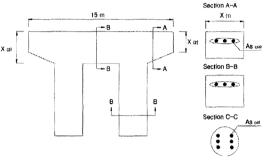


Fig. 8. Design variables of pier with double column.

Two types of substructure such as single-column pier and double-column pier are considered. The design variables of concrete pier with single column and double column are the coping thickness, amount of reinforcement in coping, diameter of column, amount of reinforcement in column (Fig. 7 and 8).

3.2. Constraints

The design constraints are formulated based on the ultimate strength design method for concrete members, and the allowable stress design method for steel members. For the optimization of bridge structure, the behavior and constraints are formulated on the basis of the Korean Bridge Design Code¹³⁾.

3.3. Life cycle cost function

The objective function is set to the total LCC including initial cost, damage cost, maintenance cost, repair and rehabilitation cost, user cost, and disposal cost for the whole service life^{3,4}. The total costs can be expressed as follows;

$$C_t = C_I + P_D \cdot C_D + C_M + n \cdot C_R + C_U + C_P \tag{1}$$

where C_I = total LCC; C_I = initial construction cost; $P_D \cdot C_D$ = damage cost; C_M = maintenance cost; $n \cdot C_R$ = repair and rehabilitation cost; C_U = user cost; and C_P = disposal cost, n means the number of repair or rehabilitation.

Initial cost

The initial cost includes cost for design, construction, management, and performance test before public use of the bridge. Construction cost is calculated by the standards of Korea Construction Price Index. Cost for design, management, and testing is usually estimated in terms of a percentage of construction cost on the basis of applicable engineering fees and experience.

Table 1. Construction cost

	steel	PC tendon	concrete	rebar
	(dollar/Ton)	(day/Ton)	(dollar/m³)	(day/Ton)
Construction cost	2859.6	6861.7	666.0	2457.5

Damage cost

The damage cost is estimated as the product of expected construction cost and damage probability¹⁴⁾. The damage probability is evaluated based on the limit state models for stress and strength. The limit state functions of bridges can be formulated as follows;

$$g(\cdot) = \sigma_{allow} - \sigma_{real} \tag{2}$$

$$g(\cdot) = \varphi \cdot M_n - M_u \tag{3}$$

where σ_{allow} = allowable stress; $j \cdot M_n$ = nominal ultimate strength specified in the code; σ_{real} = stress due to design load; M_u = ultimate strength. In the evaluation of element reliability, the Advanced First Order Second Moment method by Hasofer and Lind is used¹⁵).

Maintenance cost

The routine bridge maintenance cost consists of costs for usual diagnostics, cleaning, and minor repair. This cost associated with the routine bridge maintenance has been estimated by various methods. In this

study the maintenance cost is estimated in terms of a percentage of the initial cost.

Repair and rehabilitation cost

In this study, 2 repairs (19 year and 32 year) and 1 rehabilitation (45 year or 46 year) are assumed in concrete slab, 2 repairs (26 year and 45 year) in steel girder, 1 repairs (24 year) and 1 rehabilitation (46 year) in PSC-I girder, and 1 rehabilitation (32 year) adequate for pier. The repair and the rehabilitation costs are estimated from data such as dimension of members, cost per unit area, and expected period listed in Table 2.

User cost

The user cost is primarily attributable to the longtime restriction on the use of a bridge for repair, rehabilitation, exchange, and reconstruction. The restriction on the use of a bridge may cause additional vehicle-operating cost due to time delay, detours, raise of accident rate, etc., which should be included in the user cost. In this study, user cost is calculated

Table 2. Repair and rehabilitation cost

	Repair cost (dollar/m ²)	Rehabilitation cost (dollar/m ²)		
Concrete slab	159.8	265.3		
Steel box girder	161.6	-		
PSC-I girder	132.2	368.1		
Concrete pier	-	391.8		

Table 3. Ratio of Vehicle Types

Vehicle	National expressway		National h	ighway	Provincial road		
type	Traffic (vehicle/day)	Ratio (%)	Traffic (vehicle/day	Ratio y) (%)	Traffic (vehicle/day)	Ratio (%)	
Саг	25912	56.2	7610	66.5	2994	57.0	
Bus	4814	10.4	373	3.3	641	12.2	
Truck	15394	33.4	3449	30.2	161	30.8	

Table 4. Optimal design for each deck considering LCC

Service life (year)		60	65	70	75	80	85
Deck for steel box girder	Initial load carrying capacity Height of deck (cm) Amount of rebar (cm ²)	1.75 22.2 26.7	2.18 24.3 29.7	2.92 27.4 34.1	4.09 31.6 40.1	5.84 36.9 47.6	8.34 47.4 50.0
Deck for plate girder	Initial load carrying capacity Height of deck (cm) Amount of rebar (cm²)	1.74 39.5 526	1.93 41.2 548	2.31 44.3 590	2.98 49.3 657	4.05 56.4 752	5.67 65.8 876
Deck for PSC-I girder	Initial load carrying capacity Height of deck (cm) Amount of rebar (cm²)	1.76 22.0 16.2	2.18 22.3 20.4	2.92 22.7 26.9	4.09 26.1 31.7	5.84 30.4 37.8	8.34 35.5 45.1

Table 5. Optimal design for each girder considering LCC

Service	e life (year)	60	65	70	75	80	85
Steel box girder at max.	Initial load carrying capacity	1.76	1.96	2.29	2.80	3.54	4.58
	Upper flange width (mm)	2640	2640	2640	2640	2640	2640
	Upper flange thickness (mm)	9.6	9.7	9.9	10.2	10.8	12.0
	Height of web (mm)	2500	2500	2500	2500	2500	2500
positive moment	Thickness of web (mm)	12.0	12.0	12.0	12.0	12.0	12.0
	Lower flange width (mm)	2640	2640	2640	2640	2640	2640
	Lower flange thickness (mm)	10.3	11.4	13.1	16.9	23.2	33.3
	Initial load carrying capacity	1.82	2.15	2.66	3.40	4.44	5.83
	Upper flange width (mm)	500	500	500	500	500	500
Make adulance a commence at the	Upper flange thickness (mm)	15.2	15.2	15.2	15.2	15.2	15.2
Plate girder at max. positive	Height of web (mm)	3000	3000	3000	3000	3000	3000
moment	Thickness of web (mm)	14.4	14.4	14.4	14.4	14.4	14.4
	Lower flange width (mm)	628	689	783	849	850	1207
	Lower flange thickness (mm)	19.2	21.1	24.0	29.6	39.8	37.3
	Initial load carrying capacity	2.81	3.62	4.74	6.28	8.30	10.91
	Upper flange width (mm)	600	645	727	823	956	1128
DSC I -1-14	Upper flange thickness (mm)	196	254	331	346	462	614
PSC-I girder at max. positive moment	Height of web (mm)	1624	1558	1481	1301	1181	1051
	Thickness of web (mm)	200	200	200	200	200	200
	Lower flange width (mm)	610	731	894	925	1167	1485
	Lower flange thickness (mm)	178	187	187	350	356	334

Table 6. Optimal design for each pier considering LCC

Service	ce life (year)	60	65	70	75	80	85
	Coping thickness 1 (m)	4.17	4.18	4.18	4.18	4.16	4.13
Discretization of the	Coping thickness 2 (m)	2.09	2.09	2.09	2.09	2.08	2.06
Pier (1 col.) for steel box	Amount of rebar (coping, cm ²)	311	342	389	453	539	666
girder at both sides	Diameter of column (m)	1.75	1.80	1.88	1.98	2.12	2.29
	Amount of rebar (column, cm ²)	378	400	434	484	554	647
	Coping thickness 1 (m)	1.30	1.42	1.48	1.56	1.81	2.10
Dion (2 anl.) for stant how	Coping thickness 2 (m)	0.66	0.71	0.81	0.84	0.96	1.05
Pier (2 col.) for steel box girder at both sides	Amount of rebar (coping, cm ²)	183	187	209	234	257	321
girder at both sides	Diameter of column (m)	1.57	1.59	1.62	1.66	1.72	1.82
	Amount of rebar (column, cm ²)	432	442	459	484	521	584
	Coping thickness 1 (m)	4.72	4.73	4.73	4.73	4.72	4.71
Now (1 and) for whate minder	Coping thickness 2 (m)	2.36	2.36	2.37	2.37	2.36	2.36
rier (1 col.) for plate girder at both sides	Amount of rebar (coping, cm ²)	375	404	448	514	604	720
at both sides	Diameter of column (m)	1.84	1.89	1.96	2.07	2.21	2.34
	Amount of rebar (column, cm ²)	418	440	474	527	602	673
* ***	Coping thickness 1 (m)	1.69	1.85	1.92	2.03	2.35	2.73
No. (2 col.) for alote sinder	Coping thickness 2 (m)	0.86	0.92	1.05	1.09	1.25	37
Pier (2 col.) for plate girder at both sides	Amount of rebar (coping, cm ²)	237	244	271	304	335	417
at both sides	Diameter of column (m)	1.60	1.62	1.65	1.70	1.77	1.86
	Amount of rebar (column, cm ²)	451	461	479	507	549	608
	Coping thickness 1 (m)	3.67	3.66	3.78	3.63	3.59	3.55
Pier (1 col.) for PSC-I	Coping thickness 2 (m)	1.84	1.83	1.89	1.81	1.79	1.77
girder at both sides	Amount of rebar (coping, cm ²)	406	440	478	584	702	852
girder at both sides	Diameter of column (m)	1.79	1.83	1.90	2.01	2.16	2.30
	Amount of rebar (column, cm ²)	393	412	442	496	572	650
Pier (2 col.) for PSC-I girder at both sides	Coping thickness 1 (m)	1.22	1.34	1.39	1.47	1.70	1.97
	Coping thickness 2 (m)	0.62	0.67	0.76	0.79	0.90	0.99
	Amount of rebar (coping, cm ²)	172	176	196	220	242	301
	Diameter of column (m)	1.58	1.60	1.62	1.67	1.74	1.83
	Amount of rebar (column, cm ²)	439	448	463	491	532	588

based on the user cost model proposed by National Institute of Standard and Technology⁹⁾. The user cost expected during the period of repair, rehabilitation, and exchange is considered in the form of time delay costs and vehicle-operation cost. However, the user cost during regular maintenance and management is ignored since it is small portion of total user cost. The time values for vehicle types are estimated as 77.3dollar/hr for a bus, 10.3dollar/hr for a car and 10.6dollar/hr for a truck. The used average daily traffic of each vehicle for road types is shown in Table 3.

Disposal cost

Disposal cost is a typical cost generated when a bridge has reached the end of its service life. This cost consists of costs for dismantlement, disposal, and recycling.

4. Result of optimal life cycle cost design

The optimal life cycle cost design of a bridge

structure was performed for various target service lives. The results obtained from optimization process are summarized in Tables 4, 5 and 6.

It is seen in Tables 4 and 5 that for increased target service life, the initial load carrying capacity as well as the structural dimension increases. LCC also increases with larger dimensions of the section. Since the service life and the LCC increase at the same time, it is needed to calculate the annual cost for the evaluation of economical efficiency regard to the service life. The annual cost is obtained by dividing the LCC by the service life.

4.1. Effects of service life on LCC and annual cost

Life cycle costs for superstructures and substructures are calculated by sum of initial cost, damage cost, maintenance cost, repair and rehabilitation cost, user cost, and disposal cost for the whole service life. Fig. 9 shows the trend of costs with respect to the service life for the considered three bridge superstructures. The annual cost decreases monotonically to reach a minimum at the intended service life but increases for longer period of service, while the total life cycle cost increases gradually. The annual cost for PSC-I girder has the smallest value at the small service lives and ones for steel box girder has at the large service lives. Fig. 10 shows the trend of annual costs with respect to the service life for the substructures corresponding to the considered three superstructures. As shown in Fig. 9 and 10, the annual cost with respect to the service life is exhibiting concave variation from which an optimal service life can be decided.

4.2. Effects of superstructure on pier

It is seen in Fig. 10 that the double-column pier appears to be more economical or cost-effective than the single-column pier since the thickness of coping in the single-column pier is thicker than in double-column pier. Therefore, double-column pier can be selected for optimal design.

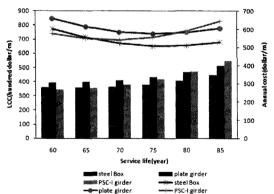


Fig. 9. LCC and annual cost for superstructures

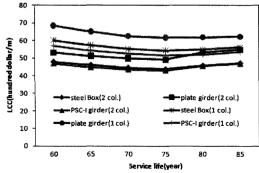


Fig. 10. Annual cost for substructures,

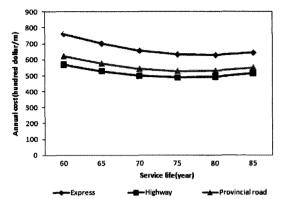


Fig. 11. Annual cost of steel box girder bridge with doublecolumn piers for different road systems,

4.3. Effects of user costs on LCC

Fig. 11 plots the annual cost of the bridge structure, which consists of concrete deck, steel box girder, and double-column pier, for different road system. The considered road systems are provincial road, national expressway and national highway systems, which determine the bridge classifications and subsequently its traffic capacity and service life. The user costs differ for each road type and decrease monotonically while other costs are same for each road type. These result shows that the user cost is the important factor in the optimal design of the bridge considering life cycle cost. Therefore, the user cost should be considered more appropriately.

Conclusion

This study was performed on optimal LCC design of a 4@40 m bridge with a number of different types of superstructure and substructure. The types of superstructure considered in this study were steel box girder, plate girder, and PSC-I girder, and those of substructure were single-column and double-column pier. The analysis results make it possible to derive the following conclusions for the design of a bridge considering LCC.

- The annual cost for PSC-I girder has the smallest value at the small service lives and ones for steel box girder has at the large service lives in the example.
 - 2) Piers being influenced by the dead load of the

superstructure, the primary factors considered in design were the span length as a factor influencing the self-weight of the superstructure, and the distance between the girders as a factor affecting the thickness of the deck.

- 3) Optimal design results shows that as the service life increases, economical efficiency of steel girders increase compared to PSC-I girder. Since the weight of PSC-I girder increases rapidly as the service life increases, initial cost of PSC-I girder also increases rapidly compared to one of steel girders.
- 4) To obtain more reliable results for the optimal design considering life cycle cost, the load carrying capacity curves should be updated by collecting more diagnostic data and maintenance histories including repairs and rehabilitations.

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