

On the Relationship Between the Performance Criteria of Unitary Space-Time Codes with Noncoherent and Coherent Decoding

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ABSTRACT

Hochwald *et al.* introduced unitary space-time codes for quasi-static Rayleigh fading channels which allows for noncoherent decoding when the channel response is not known at the receiver. However, when reliable information on the channel response is available, coherent decoding is preferable for improved performance. Here, we study the relationship between the performance criteria on the diversity and coding advantages provided by unitary space-time codes with noncoherent and coherent decoding. We show that when a unitary space-time code achieves full spatial diversity with noncoherent decoding, full spatial diversity is also guaranteed with coherent decoding.

Key Words : Unitary Space-Time Codes (STC), Noncoherent, Coherent, Diversity, Transmit Antennas

1. Introduction

A recent approach to obtaining spatial diversity over multipath fading channels is to employ coding techniques appropriate for multiple transmit antennas, namely, space-time coding^{[1]-[7]}. Tarokh *et al.* in [1] developed design criteria for space-time codes under the assumption that the channel is known at the receiver. Specifically, the rank and determinant criteria for quasi-static Rayleigh fading channels quantify the diversity and the coding gains of space-time codes, respectively. Hochwald *et al.* in [2], introduced unitary space-time codes for quasi-static Rayleigh fading channels which allows for noncoherent decoding for the case when the channel response is not available at the receiver.

Since the channel environment for a receiver

may change depending on time and place, the assumption that the channel information is always either known or unknown at the receiver is unrealistic. If reliable channel response becomes available at the receiver, coherent decoding may be desirable for improved performance. In this case, we would at least like to be guaranteed that the unitary space-time code in use achieving full spatial diversity and maximizing coding advantage with noncoherent decoding also achieves full spatial diversity and maximizes coding advantage with coherent decoding. In this paper, we study the relationship between the performance criteria on the diversity and coding advantages provided by unitary space-time codes under noncoherent and coherent decoding. We show that when a unitary space-time code achieves full spatial diversity with noncoherent decoding, full spatial

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diversity is also guaranteed with coherent decoding.

The remainder of the paper is organized as follows. In Section II, the signal model is presented and in Section III, we briefly review the performance criteria of unitary space-time codes for the coherent and noncoherent decoding. In Section IV, we show that a unitary space-time code achieving full spatial diversity with noncoherent decoding also achieves full spatial diversity with coherent decoding and conclusions are drawn in Section V.

II. Signal Model

We consider a communication system with M transmit and N receive antennas under quasi-static, frequency flat Rayleigh fading channels. Each receive antenna responds to each of the transmit antennas through a statistically independent channel response, assumed to be constant for T symbol periods. Using the complex baseband representation, the lowpass equivalent channel input-output relationship for such channels can be written as

$$Y = \sqrt{\frac{\rho}{M}} SH + W \tag{1}$$

Here, $Y = \{y_{tn}\}$ is the $T \times N$ received signal matrix with y_{tn} being the signal received by the n th receive antenna at time t , $S = \{s_{tm}\}$ is the $T \times M$ transmitted signal matrix with s_{tm} being the signal transmitted on the m th transmit antenna at time t . The channel coefficient matrix $H = \{h_{mn}\}$ is an $M \times N$ matrix of independent and identically distributed (i.i.d.) Rayleigh fading coefficients with h_{mn} being the fading coefficient between the m th transmit antenna and the n th receive antenna. The noise matrix, $W = \{w_{tn}\}$ is the $T \times N$ matrix representing the i.i.d. additive receiver thermal noise with w_{tn} being the thermal noise observed by the n th receive antenna at time t . Let $CN(0,1/2)$ denote a complex Gaussian

random variable with independent real and imaginary parts, each with zero mean and variance $1/2$. The fading coefficients h_{mn} are assumed to be $CN(0,1/2)$ and each element w_{tn} of W are also $CN(0,1/2)$ distributed. The average energy of the symbols transmitted from each antenna is normalized to be one, i.e., $\frac{1}{M} \sum_{m=1}^M E\{|s_{tm}|^2\} = 1$. Therefore, the average received SNR at each of the receive antennas is ρ .

In this paper, we consider unitary space-time codes^[3] of size L consisting of codewords $S_l = \sqrt{T} \Phi_l$, $l = 1, \dots, L$, where Φ_l are $T \times M$ complex matrices satisfying $\Phi_l^H \Phi_l = I_M$. Here, A^H denotes the conjugate transpose of A and I_M denotes the $M \times M$ identity matrix. In the sequel, we will identify S_l with Φ_l and use them interchangeably.

III. Review of the Design Criteria of Unitary Space-Time Codes for Coherent and Noncoherent Decoding

First, let us briefly review the design criteria for unitary space-time codes for the cases when the channel coefficient matrix H is either known or unknown at the decoder. For simplicity, we will refer to the two decoding operations, as coherent and noncoherent decoding, respectively.

3.1 Coherent decoding

Most of the previous works on space-time codes assume that the channel coefficient matrix H is known at the receiver. The following provides a quick review of optimal receivers, pairwise error probability and the design criteria for coherent decoding of unitary space-time codes^{[2],[3]}.

When H is known to the decoder, the maximum-likelihood (ML) decoder is given as^[2]

$$\Phi_{ML} = \arg \min_{\Phi \in U} \text{Tr} \left\{ \left(Y - \sqrt{\frac{\rho T}{M}} \Phi H \right) \left(Y - \sqrt{\frac{\rho T}{M}} \Phi H \right)^H \right\} \tag{2}$$

where $U = \{\Phi_1, \dots, \Phi_L\}$ and $\text{Tr}\{A\}$ denotes the trace of the matrix A . The Chernoff bound on the pairwise error probability between Φ_l and $\Phi_{l'}$ takes the form^[3]

$$P_{l,l'} \leq \left| I_M + \frac{\rho T}{4M} (\Phi_l - \Phi_{l'})^H (\Phi_l - \Phi_{l'}) \right|^{-N} \quad (3)$$

$$\approx \left(\Lambda_c(\Phi_l, \Phi_{l'}) \frac{\rho T}{4M} \right)^{-\nu_c(\Phi_l, \Phi_{l'})N}, \quad \rho \gg 1$$

where $\nu_c(\Phi_l, \Phi_{l'})$ denotes the rank of the difference matrix $(\Phi_l - \Phi_{l'})$ and $|A|$ denotes the determinant of A . The quantity $\nu_c(\Phi_l, \Phi_{l'})$ can be interpreted as the *diversity advantage* of the corresponding codeword pair. The quantity $\Lambda_c(\Phi_l, \Phi_{l'})$, on the other hand, can be interpreted as the *coding advantage* of the corresponding codeword pair and is given by^[11]

$$\Lambda_c(\Phi_l, \Phi_{l'}) = |(\Phi_l - \Phi_{l'})^H (\Phi_l - \Phi_{l'})|_+^{1/\nu_c(\Phi_l, \Phi_{l'})} \quad (4)$$

where $|A|_+$ denotes the product of the nonzero eigenvalues of A . Hence, for large values of ρ , the performance of a unitary space-time code under coherent decoding is determined primarily by the minimum diversity advantage ν_c^m given as

$$\nu_c^m = \min_{1 \leq l \leq L, l \neq l'} \nu_c(\Phi_l, \Phi_{l'}) \quad (5)$$

and the minimum coding advantage Λ_c^m given by,

$$\Lambda_c^m = \min_{1 \leq l \leq L, l \neq l', \nu_c(\Phi_l, \Phi_{l'}) = \nu_c^m} \Lambda_c(\Phi_l, \Phi_{l'}) \quad (6)$$

3.2 Noncoherent decoding

Assuming that neither the transmitter nor the receiver knows the channel coefficient matrix H , the ML decoding rule for unitary space-time codes is given by^[2]

$$\hat{\Phi}_{ML} = \arg \max_{\Phi \in U} \text{Tr}\{Y^H \Phi \Phi^H Y\} \quad (7)$$

The Chernoff bound on the pairwise error probability between codewords Φ_l and $\Phi_{l'}$ is then given by^[3]

$$P_{l,l'} \leq \left| I_M + \frac{(\rho T/M)^2}{4(1 + \rho T/M)} (I_M - \Phi_l^H \Phi_{l'} \Phi_l^H \Phi_{l'}) \right|^{-N} \quad (8)$$

$$\approx \left(\Lambda_n(\Phi_l, \Phi_{l'}) \frac{\rho T}{4M} \right)^{-\nu_n(\Phi_l, \Phi_{l'})N}, \quad \rho \gg 1.$$

The diversity advantage, $\nu_n(\Phi_l, \Phi_{l'})$ is equal to the rank of matrix $(I_M - \Phi_l^H \Phi_{l'} \Phi_l^H \Phi_{l'})$. Hochwald and Marzetta^[2] noted that the maximum value of $\nu_n(\Phi_l, \Phi_{l'})$ is M , which is achieved when 1 is not a singular value of $\Phi_l^H \Phi_{l'}$. The coding advantage $\Lambda_n(\Phi_l, \Phi_{l'})$ is given by^[3]

$$\Lambda_n(\Phi_l, \Phi_{l'}) = |I_M - \Phi_l^H \Phi_{l'} \Phi_l^H \Phi_{l'}|_+^{1/\nu_n(\Phi_l, \Phi_{l'})} \quad (9)$$

For large values of ρ , the performance of a unitary space-time code under noncoherent decoding is again determined mainly by the minimum diversity advantage given as

$$\nu_n^m = \min_{1 \leq l \leq L, l \neq l'} \nu_n(\Phi_l, \Phi_{l'}) \quad (10)$$

and the minimum coding advantage given by

$$\Lambda_n^m = \min_{1 \leq l \leq L, l \neq l', \nu_n(\Phi_l, \Phi_{l'}) = \nu_n^m} \Lambda_n(\Phi_l, \Phi_{l'}) \quad (11)$$

Hence, for both coherent and noncoherent decoding, the important design criteria are to maximize the minimum diversity advantages ν_c^m and ν_n^m .

IV. Relationship of the Performance Criteria for Coherent and Noncoherent Decoding

Even when a unitary space-time code is employed in order to allow noncoherent decoding, we may wish to perform coherent decoding at the receiver when the channel variation is slow

enough to allow accurate channel estimation for an extra performance boost. Hence, we need to address some basic questions regarding the performance of unitary space-time codes optimized for noncoherent decoding under coherent decoding. We will provide the answer to the most important question, i.e., does a unitary space-time code achieving full spatial diversity and maximizing coding advantage with noncoherent decoding guarantee full spatial diversity with coherent decoding? We first establish a *Lemma* that is crucial in relating the minimum diversity advantages of unitary space-time codes under coherent and noncoherent decoding.

Lemma 1: The minimum diversity advantage of unitary space-time codes under noncoherent decoding can also be written as

$$\nu_c^m = \min_{1 \leq l \leq L, l \neq l'} \text{rank}([\Phi_l \Phi_{l'}]) - M. \quad (12)$$

Proof: Let us first consider the following $2M \times 2M$ Hermitian matrix

$$P = \begin{bmatrix} I_M & \Phi_l^H \Phi_{l'} \\ (\Phi_l^H \Phi_{l'})^H & I_M \end{bmatrix}, \quad l \neq l'.$$

We know from^{[8]1)} that

$$\text{rank}(P) = M + \text{rank}(I_M - \Phi_l^H \Phi_{l'} \Phi_{l'}^H \Phi_l). \quad (13)$$

The matrix P can also be written as

$$\begin{bmatrix} I_M & \Phi_l^H \Phi_{l'} \\ (\Phi_l^H \Phi_{l'})^H & I_M \end{bmatrix} = [\Phi_l \Phi_{l'}]^H [\Phi_l \Phi_{l'}]. \quad (14)$$

Moreover, since matrices A and $A^H A$ have identical rank ^[6], we have

1) Suppose that a Hermitian matrix G is partitioned as $G = \begin{bmatrix} A & B \\ B^H & C \end{bmatrix}$ where A and C are $M \times M$ positive definite matrices. Then, this matrix G is positive definite and $\text{rank}(G) = M + \text{rank}(A - BC^{-1}B^H)$.

$$\text{rank}(P) = \text{rank}([\Phi_l \Phi_{l'}]). \quad (15)$$

Combining this with (13), we may rewrite the rank of the matrix $(I_M - \Phi_l^H \Phi_{l'} \Phi_{l'}^H \Phi_l)$ as

$$\text{rank}(I_M - \Phi_l^H \Phi_{l'} \Phi_{l'}^H \Phi_l) = \text{rank}([\Phi_l \Phi_{l'}]) - M. \quad (16)$$

Therefore, the minimum diversity advantage of unitary space-time codes with noncoherent decoding is given by

$$\nu_n^m = \min_{1 \leq l \leq L, l \neq l'} \text{rank}([\Phi_l \Phi_{l'}]) - M. \quad (17)$$

From *Lemma 1*, it is required that $\text{rank}([\Phi_l \Phi_{l'}]) = 2M$, for all $l \neq l'$, in order to achieve full spatial diversity of M , which also requires $T \geq 2M$. Assuming that $T \geq 2M$, and armed with this result, the following theorem relating the minimum diversity advantages of unitary space-time codes under coherent and noncoherent decoding is easily shown.

Theorem 1: If a unitary space-time code with noncoherent decoding achieves full spatial diversity, i.e., $\nu_n^m = M$, then it also guarantees full spatial diversity under coherent decoding, i.e., $\nu_c^m = M$, when the channel response is available at the receiver.

Proof: Assume that a given unitary space-time code U consisting of L codewords under noncoherent decoding achieves full spatial diversity. Then, $\text{rank}([\Phi_l \Phi_{l'}]) = 2M$, for all $l \neq l'$, which requires that the columns of Φ_l and $\Phi_{l'}$ are linearly independent for $l \neq l'$. Full spatial diversity under coherent decoding, on the other hand, is guaranteed if the columns of the matrix $\Phi_l - \Phi_{l'} = [\phi_{l1} - \phi_{l'1} \dots \phi_{lM} - \phi_{l'M}]$ are linearly independent which is trivially satisfied if $[\Phi_l \Phi_{l'}]$ has full rank. Hence, the given unitary space-time code achieves full spatial diversity under coherent decoding when the channel response is available

at the receiver. ■

An alternate proof of *Theorem 1* is possible following the results of [5].

Alternate proof: With noncoherent decoding, the diversity advantage is given by $rN^{[5]}$, where

$$r = \min_{1 \leq l \leq L, l \neq l'} \left(\dim(W_{\Phi_l}) - \dim(W_{\Phi_l} \cap W_{\Phi_{l'}}) \right). \quad (18)$$

Here, W_{Φ_l} denotes the subspace spanned by the columns of Φ_l and $\dim(W_{\Phi_l})$ denotes the dimension of W_{Φ_l} . Since $\dim(W_{\Phi_l}) = M$, we need $\dim(W_{\Phi_l} \cap W_{\Phi_{l'}}) = 0$ for r to equal M . This in turn requires the columns of Φ_l and $\Phi_{l'}$ to be linearly independent which again implies linear independent of the columns of $\Phi_l - \Phi_{l'}$. ■

Next, we relate the coding advantage of a unitary space-time code under coherent and noncoherent decoding. Suppose that a given unitary space-time code U with L codewords under noncoherent decoding achieves full spatial diversity. Then, the coding advantage between codewords Φ_l and $\Phi_{l'}$ under noncoherent decoding is given by

$$|I_M - \Phi_l^H \Phi_l \Phi_{l'}^H \Phi_{l'}|^{1/M} = \left\{ \prod_{m=1}^M (1 - d_{l'm}^2) \right\}^{1/M} \quad (19)$$

where $0 \leq d_{l'M} \leq \dots \leq d_{l'1} \leq 1$ are the singular values of the $\Phi_l^H \Phi_{l'}^{[2]}$. Hence, in order to maximize the coding advantage, we need to minimize the singular values $d_{l'm}$, $m = 1, \dots, M$ which are all less than 1.

Theorem 2: Let U be a given unitary space-time code achieving full spatial diversity and maximizing the coding advantage under noncoherent decoding. Suppose that $\Phi_l^H \Phi_{l'} = Re\{\Phi_l^H \Phi_{l'}\}$ for $\Phi_l, \Phi_{l'} \in \{\Phi_1, \dots, \Phi_L\}$, $\forall l \neq l'$. Then, U also maximizes the coding advantage under coherent

decoding.

Proof: The coding advantage of the code with coherent decoding is given by

$$\begin{aligned} & |(\Phi_l - \Phi_{l'})^H (\Phi_l - \Phi_{l'})|^{1/M} \\ &= |2I_M - 2Re\{\Phi_l^H \Phi_{l'}\}|^{1/M} \\ &= \left\{ \prod_{m=1}^M (2 - 2\lambda_{l'm}) \right\}^{1/M} \end{aligned} \quad (20)$$

where $\lambda_{l'M} \leq \dots \leq \lambda_{l'1}$ are the singular values of $Re\{\Phi_l^H \Phi_{l'}\}$. Since $\Phi_l^H \Phi_{l'} = Re\{\Phi_l^H \Phi_{l'}\}$, $\lambda_{l'm} = d_{l'm}$, $m = 1, \dots, M$. Then, the coding advantage of U under coherent decoding can be written as

$$\begin{aligned} & |(\Phi_l - \Phi_{l'})^H (\Phi_l - \Phi_{l'})|^{1/M} \\ &= \left\{ \prod_{m=1}^M (2 - 2d_{l'm}) \right\}^{1/M}. \end{aligned} \quad (21)$$

By *Lemma 2*, if $\left\{ \prod_{m=1}^M (1 - d_{l'm}^2) \right\}^{1/M}$ is maximized,

then $\left\{ \prod_{m=1}^M (2 - 2d_{l'm}) \right\}^{1/M}$ is also maximized.

Therefore, the coding advantage of U under coherent decoding is maximized. ■

In addition, since $0 \leq d_{l'M} \leq \dots \leq d_{l'1} \leq 1^{[2]}$, we have

$$\left\{ \prod_{m=1}^M \frac{(2 - 2d_{l'm})}{(1 - d_{l'm}^2)} \right\}^{1/M} = \left\{ \prod_{m=1}^M \frac{2}{(1 + d_{l'm})} \right\}^{1/M} \geq 1$$

from (19) and (21). Thus, we have

$$\left\{ \prod_{m=1}^M (2 - 2d_{l'm}) \right\}^{1/M} \geq \left\{ \prod_{m=1}^M (1 - d_{l'm}^2) \right\}^{1/M}. \quad (22)$$

Therefore, the coding advantage under coherent decoding is greater than or equal to the coding advantage under noncoherent decoding.

The coding advantage of the code with coherent decoding, on the other hand, is given by

$$|(\Phi_l - \Phi_r)^H(\Phi_l - \Phi_r)|^{1/M} = \left\{ \prod_{m=1}^M \sigma_{l'm}^2 \right\}^{1/M} \quad (23)$$

where $0 \leq \sigma_{l'M} \leq \dots \leq \sigma_{l'1} \leq 2$ are the singular values of $\Phi_l - \Phi_r$ ^[2]. Hence, for coherent decoding, we wish to maximizing the singular values $\sigma_{l'm}$, $m = 1, \dots, M$. In general, there is no direct relationship between the singular values $d_{l'm}$ and $\sigma_{l'm}$. However, for $M=1$, $d_{l'1} = |\Phi_l^H \Phi_r|$ and $\sigma_{l'1} = |\Phi_l - \Phi_r| = \sqrt{2 - 2\text{Re}(\Phi_l^H \Phi_r)}$ which implies that $\sqrt{2(1 - d_{l'1})} \leq \sigma_{l'1} \leq \sqrt{2(1 + d_{l'1})}$ ^[2]. For the special case $d_{l'1} = \dots = d_{l'M} = 0$, then $\sigma_{l'1} = \dots = \sigma_{l'M} = \sqrt{2}$.

V. Conclusions

In this paper, we studied the relation between the performance criteria on the diversity gain with coherent and noncoherent decoding of unitary space-time codes. From our study, we knew the fact that a unitary space-time code with full spatial transmit diversity under noncoherent decoding also guarantees full spatial transmit diversity under coherent decoding.

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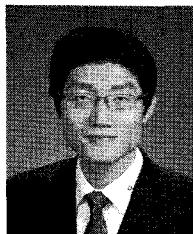


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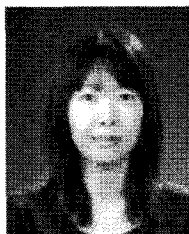


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