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A new approach to moment inequalities for NBRU class of life distributions with hypothesis testing applications

M. A. W. Mahmoud*
Mathematics Department, Al-Azhar University, Cairo, Egypt

M. S. Albassam
Department of Statistics, King Abdulaziz University, Jeddah, Saudi Arabia

E. H. Abdulfattah
Department of Statistics, King Abdulaziz University, Jeddah, Saudi Arabia

Abstract. The main objective of this study is to present a new approach to obtain moment inequalities for the new better than renewal used (NBRU) class of life distributions. In order to achieve our main objective, the moment inequalities for NBRU class of life distribution using the new approach has been derived and then a new test for testing exponentiality against NBRU class based on these inequalities has been constructed. Then we calculate the Pitman asymptotic efficiency for the proposed test using some alternative distributions and comparing it with the other tests. Moreover, we make a comparison between Pittman asymptotic efficiencies (PAE's) and PAE's of some other tests. A simulation study is conducted to calculate the upper critical values and the power estimate of the proposed test for some common alternatives. Finally, we apply the suggested test to some real data.

Key Words: *NBRU Class of Life Distributions, Pitman asymptotic efficiency*

1. INTRODUCTION

The mathematical theory of reliability has developed in two main branches. The first branch is mostly concentrated on the mathematical aspects of practical problems that face reliability and design engineers and reliability analysis specialists. The second branch concentrates on the mathematical and statistical aspects of reliability such as life testing and estimation. Note that the exponential distribution forms the back bone of statistical

* Corresponding Author.
E-mail address: *mawmahmoud11@hotmail.com*

reliability theory and maintenance modeling, see for example Barlow and Proschan (1981) and Zachks (1992).

For over three decades, studies have led to declaring several families of life distributions to characterize aging. Many classes of life distributions have been introduced in reliability, see for example Barlow et. al. (1963), Bryson and Siddiqui (1969),Barlow and Proschan (1981), Abouammoh and Ahmad (1992), Abouammoh et. al. (1994, 2000), Elbatal (2007) and Kayid (2007).

Testing exponentiality against various classes of life distributions has got a good deal of attention. For testing against IFR class, see Proschan and Pyke (1967), Barlow (1968) and Ahmad (1975, 1976) among others. For testing IFRA, see Deshpandeet.al. (1986), inmk (1989), Aly (1989) and Ahmad (1994). For testing against NBU, see Hollander and Proschan (1972), Koul (1977), Kumazawa (1983) and Ahmad (1994).

For testing against NBUE, NBUFR, NBAFR, NBURFR, NBRU, DVRL and NBUL classes, we refer to Klefsjo (1981, 1982), Deshpande et. al. (1986), Abouammo hand Ahmad (1988), Loh (1984), Hendiet. al. (2000), Mahmoud and AbdulAlim (2003,2008) and Abuammaoh et. al. (1994), Al-Zahrani and Stoyanov (2008), Mahmoud and Diab (2008), and Diab et. al. (2009) respectively. Finally, for testing against RNBU and HNRBUE, see Mahmoud et. al. (2005) and Mahmoud and Diab (2007).

The rest of the article is structured as follows. In section 2, the moment inequalities based on the new approach have been derived. Section 3 is devoted to construct the test statistic for testing exponentiality against NBRU class of life distributions based on the moment inequality presented in section 2. The asymptotic properties of the mentioned test has been stated and proved. The Pitman asymptotic efficiencies of the proposed test is calculated in Section 4 for three common alternative distributions (linear failure rate, Weibull and Make ham distributions). Section 5 presents calculations, based on simulation data, of the power estimates of the proposed test for the same common alternatives mentioned in section 4. To study the effectiveness of the proposed test, three real examples have been presented in section 6 and then apply our suggested approach to these real data.

2. MOMENTS INEQUALITIES FOR NBRU CLASS

Definition 2.1. We say the distribution $F \in$ NBRU if

$$\overline{W}(x+t) \leq \overline{F}(x)\overline{W}(t), \quad t \geq 0\,, x \geq 0$$

where

$$\overline{W}(z) = \frac{1}{\mu}\int_z^\infty \overline{F}(u)du\,, \text{ and } \mu = \int_0^\infty \overline{F}(u)du$$

We need the following lemma:

Lemma 2.1. Suppose that *F* is NBRU life distribution and its (r+3)rd moment is finite for some integer $r \geq 0$, then

$$\frac{\mu_{(r+3)}}{(r+2)(r+3)} \le \sum_{i=0}^{r} C_i^r \frac{\mu_{(i+1)}\mu_{(r-i+2)}}{(i+1)(r-i+1)(r-i+2)} \tag{2.1}$$

Proof.

We have

$$\bar{W}(x+t) \le \bar{F}(x)\bar{W}(t), \quad t \ge 0, x \ge 0$$

Then

$$\int_0^\infty \int_0^\infty (x+t)^r \bar{W}(x+t)\, dx\, dt \le \int_0^\infty \int_0^\infty (x+t)^r \bar{F}(x)\bar{W}(t)\, dx\, dt \tag{2.2}$$

Consider

$$I_1 = \int_0^\infty \int_0^\infty (x+t)^r \bar{W}(x+t)\, dx\, dt \tag{2.3}$$

and

$$I_2 = \int_0^\infty \int_0^\infty (x+t)^r \bar{F}(x)\bar{W}(t)\, dx\, dt \tag{2.4}$$

Let $(x + t) = y$ in the integral (2.3), then I_1 can be in the form

$$\begin{aligned} I_1 &= \int_0^\infty \int_t^\infty y^r \bar{W}(y) dy\, dt = \int_0^\infty t^r \bar{W}(t) \int_0^t dy\, dt \\ &= \int_0^\infty t^{r+1} \bar{W}(t) dt = \frac{1}{\mu}\int_0^\infty t^{r+1} \int_t^\infty \bar{F}(u) du\, dt \\ &= \frac{1}{\mu}\int_0^\infty \bar{F}(t) \int_0^\infty u^{r+1}\, du\, dt = \frac{1}{\mu}\int_0^\infty \frac{t^{r+2}}{r+2} \bar{F}(t) dt \\ &= \frac{1}{\mu}\frac{\mu_{(r+3)}}{(r+2)(r+3)} \end{aligned} \tag{2.5}$$

Now

$$\begin{aligned} I_2 &= \int_0^\infty \int_0^\infty (x+t)^r \bar{F}(x)\bar{W}(t)\, dx\, dt \\ &= \sum_{i=0}^{r} C_i^r \int_0^\infty \int_0^\infty x^i t^{r-i} \bar{F}(t)\bar{W}(t)\, dx\, dt \\ &= \sum_{i=0}^{r} C_i^r \left[\int_0^\infty x^i \bar{F}(t) dx\right]\left[\int_0^\infty t^{r-i}\bar{W}(t) dt\right] \\ &= \sum_{i=0}^{r} C_i^r \left[\frac{\mu_{(i+1)}}{i+1}\right]\left[\frac{1}{\mu}\int_0^\infty t^{r-i} \int_t^\infty \bar{F}(u) du\, dt\right] \\ &= \sum_{i=0}^{r} C_i^r \frac{\mu_{(i+1)}}{i+1}\frac{1}{\mu}\int_0^\infty \bar{F}(t)\int_0^t u^{r-i}\, du\, dt \\ &= \sum_{i=0}^{r} C_i^r \frac{\mu_{(i+1)}}{i+1}\frac{1}{\mu}\int_0^\infty \frac{t^{r-i+1}}{r-i+1}\bar{F}(t) dt \\ &= \sum_{i=0}^{r} C_i^r \frac{\mu_{(i+1)}}{i+1}\frac{\mu_{r-i+2}}{(r-i+1)(r-i+2)} \end{aligned} \tag{2.6}$$

Making use (2.5), (2.6) and (2.2), we get (2.1).

3. TESTING HYPOTHESIS AGAINST NBRU

In this section, we present the procedures of testing hypothesis for exponentiality against NBRU class of life distribution based on the moment inequality (2.1), i.e. the following hypotheses will be tested:

$$H_0 = \text{F is exponential}$$

and

$$H_1 = \text{F belongs to NBRU class and not exponential}$$

Let X_1, X_2, …, X_n be a random sample from a population with distribution F. Using Lemma 2.1, we use δ_r as a measure of departure from exponentiality where

$$\delta_r = \sum_{i=0}^{r} C_i^r \frac{\mu_{(i+1)}\mu_{(r-i+2)}}{(i+1)(r-i+1)(r-i+2)} - \frac{\mu_{(r+3)}}{(r+2)(r+3)} \quad (3.1)$$

It is easy to show that $\delta_r = 0$ under H_0 and $\delta_r > 0$ under H_1. Now consider δ_1,

$$\delta_1 = \sum_{i=0}^{1} \frac{\mu_{(i+1)}\mu_{(3-i)}}{(i+1)(2-i)(3-i)} - \frac{\mu_{(4)}}{12}$$

This implies that

$$\delta_1 = \frac{\mu\mu_3}{6} + \frac{\mu_2^2}{4} - \frac{\mu_{(4)}}{12}$$

The empirical form of δ_1 is given by

$$\hat{\delta} = \frac{1}{n^2}\sum_{j=1}^{n}\sum_{k=1}^{n}\left[\frac{X_jX_k^3}{6} + \frac{X_j^2X_k^2}{4} - \frac{X_j^4}{12}\right]$$

By taking

$$\emptyset(X_1, X_2) = \frac{X_1X_2^3}{6} + \frac{X_1^2X_2^2}{4} - \frac{X_1^4}{12}$$

and defining the symmetric kernel

$$\varphi(X_1, X_2) = \frac{1}{2!}\sum_{R} \emptyset(X_{j1}, X_{j2})$$

where the summation is over all arguments of X_{j1} and X_{j2}, then $\hat{\delta}$ is equivalent to U-statistic, such that

$$U_n = \frac{1}{C_n^2}\sum_{R} \varphi(X_j, X_k)$$

To make the test statistic invariant, let

$$\Delta_1 = \frac{\delta_1}{\mu^4} \text{ which is estimated by } \hat{\Delta} = \frac{\hat{\delta}}{\bar{X}^4}$$

The following theorem is summarizing the asymptotic properties of δ_1.

Theorem 3.1. As $n \to \infty$

$$\sqrt{n}(\hat{\Delta} - \Delta_1) \sim N(0, \sigma^2)$$

where $\sigma^2 = \text{Var}(\eta(X))$ and

$$\eta(X) = \frac{X_{\mu(3)}}{6} + \frac{X_{\mu(2)}^2}{2} + \frac{X_{\mu^3}}{6} + \frac{X^4}{12} - \frac{\mu_{(4)}}{12}$$

Proof.

Consider

$$\eta_1(X_1) = \emptyset(X_1, X_2|X_1)$$

And

$$\eta_2(X_2) = \emptyset(X_1, X_2|X_2)$$

Therefore

$$\eta_1(X_1) = \frac{X_{1\mu_{(3)}}}{6} + \frac{X^2_{1\mu_{(2)}}}{4} - \frac{X_1^4}{12}$$

And

$$\eta_2(X_2) = \frac{\mu X_2^3}{6} + \frac{X_2^2 \mu_{(2)}}{4} - \frac{X_1^4}{12}$$

Suppose that $\eta(X) = \eta_1(X) + \eta_2(X)$, this implies that

$$\eta(X) = \frac{X_{\mu(3)}}{6} + \frac{X^2_{\mu(2)}}{2} + \frac{X_{\mu^3}}{6} + \frac{X^4}{12} - \frac{\mu_{(4)}}{12}$$

It is easy to show that $\bar{X}^4 \to \mu^4$ as $n \to \infty$ and $E(\widehat{\Delta_1}) = \Delta_1$. Using Serfling (1980) results, we conclude that: As $n \to \infty$,

$$\sqrt{n}(\hat{\Delta} - \Delta_1) \sim N(0, \sigma^2)$$

where $\sigma^2 = Var(\eta(X))$

It is easy to show that under H_0:

$$E[\eta(X)] = 0$$

and

$$\begin{aligned} \sigma^2 &= E[\eta^2(X)] \\ &= E\left[\frac{X^3}{6} - \frac{X^4}{12} + X^2 + X - 2\right]^2 \\ &= E\left[\frac{1}{144}X^8 - \frac{1}{36}X^7 - \frac{5}{36}X^6 + \frac{1}{6}X^5 + \frac{5}{3}X^4 + \frac{4}{3}X^3 - 3X^2 - 4X + 4\right] \end{aligned}$$

This implies that

$$\sigma_0^2 = \frac{1}{144}8! - \frac{1}{36}7! - \frac{5}{36}6! + \frac{1}{6}5! + \frac{5}{3}4! + \frac{4}{3}3! - 3(2!) - 4 + 4 = 102$$

and this completes the proof.

4. THE PITMAN ASYMPTOTIC EFFICIENCY(PAE)

In this section, we compute the PAE of our test δ_1, where

$$PAE(\delta_1(\theta)) = \frac{\left|\frac{d}{d\theta}\delta_1(\theta)\right|_{\theta \to \theta_0}}{\sigma_0} \tag{4.1}$$

and

$$\delta_1(\theta) = \frac{\mu_\theta \mu_{(3)\theta}}{6} + \frac{\mu^2_{(2)\theta}}{4} - \frac{\mu_{(4)\theta}}{12}$$

This implies that

$$\delta_1' = \frac{\mu_\theta \mu'_{(3)\theta}}{6} + \frac{\mu'_\theta \mu_{(3)\theta}}{6} + \frac{\mu_{(2)\theta}\mu'_{(2)\theta}}{2} - \frac{\mu'_{(4)\theta}}{12}$$

Here, we use three alternatives, Weibull, linear failure rate and Makeham families, $\bar{F}_1(x), \bar{F}_2(x)$ and $\bar{F}_3(x)$, therefore

$(i)\ \bar{F}_1(\mathrm{x}) = \exp(-x^\theta)\ , x > 0\ , \theta\ \geq 1$
$(ii)\ \bar{F}_2(\mathrm{x}) = \exp\left(-x - \frac{\theta}{2}\mathrm{x}^2\right)\ , x > 0\ , \theta\ \geq$
$(iii)\ \bar{F}_3(x) = \exp[-x + \theta(x + e^{-x} - 1)]\ , x > 0\ , \theta\ \geq 0$

Note that H_0 is attained at θ= 1 in (i) and θ= 0 in (ii) and (iii).

4.1. PAE ($\boldsymbol{\delta_1}$) for Weibull Family

Using Maple Package, we get the following:

$$\mu = 1,\ \mu_{(2)} = 2,\ \mu_{(3)} = 6 \qquad ,\ \acute{\mu} = -0.4227$$
$$\acute{\mu}_{(2)} = -3.691,\ \acute{\mu}_{(3)} = -22.61,\ \acute{\mu}_{(4)} = -144.58$$

where

as $\theta \to 1\ \ \mu = \mu_\theta$
as $\theta \to 1\ \ \mu_{(2)} = \mu_{(2)\theta}$
as $\theta \to 1\ \ \mu_{(3)} = \mu_{(3)\theta}$
as $\theta \to 1\ \ \acute{\mu}_{(2)} = \acute{\mu}_{(2)\theta} = 2\ \int_0^\infty x^2 \ln x\, e^{-x}\, dx$
as $\theta \to 1\ \ \acute{\mu}_{(3)} = \acute{\mu}_{(3)\theta} = 3\ \int_0^\infty x^3 \ln x\, e^{-x}\, dx$
as $\theta \to 1\ \ \acute{\mu}_{(4)} = \acute{\mu}_{(4)\theta} = 4\ \int_0^\infty x^4 \ln x\, e^{-x}\, dx$

Now, by using Equation (4.1), we calculate PAE of ±1, then we get that

$$PAE(\delta_1) = 0.4125$$

4.2. PAE (δ_1) for Makeham Family

In this case, we have

$$\mu = 1,\ \mu_{(2)} = 2,\ \mu_{(3)} = 6 \qquad ,\ \acute{\mu} = -0.5$$
$$\acute{\mu}_{(2)} = -2.5,\ \acute{\mu}_{(3)} = -12.75,\ \acute{\mu}_{(4)} = -73.5$$

Therefore, by applying Equation (4.1), we get

$$\mathrm{E}(\delta_1) = 1$$

4.3. PAE ($\boldsymbol{\delta_1}$) for Linear Failure Family

In this case, we have

$$\mu = 1,\ \mu_{(2)} = 2,\ \mu_{(3)} = 6 \qquad ,\ \acute{\mu} = -1$$

$$\acute{\mu}_{(2)} = -6, \acute{\mu}_{(3)} = -36, \acute{\mu}_{(4)} = -240$$

Therefore, by applying Equation (4.1), we get

$$PAE(\delta_1) = 0.693$$

Table 4.1 gives the efficiencies of our proposed test δ_1 comparing with the tests given by Kango (1993) (U_n), Mugdadi and Ahmad (2005) (δ_3), Abdel-Aziz (2007) (Δ_{RN}) and Mahmoud and Abdul Alim (2008) (($\delta_{F_n}^{(1)}, \delta_{F_n}^{(2)}$)).

It is clear from Table 4.1 that our test is more efficient than all other tests for all alternatives.

Table 4.1. The PAE's of δ_1, U_n, δ_3, Δ_{RN}, $\delta_{F_n}^{(1)}, \delta_{F_n}^{(2)}$

Alternatives ↓ Tests →	δ_1	U_n	δ_3	Δ_{RN}	$\delta_{F_n}^{(1)}$	$\delta_{F_n}^{(2)}$
Linear failure rate	0.693	0.433	0.408	0.535	0.433	0.217
Weibull	0.412	0.132	0.170	0.223	0.405	0.05
Makeham	1	0.144	0.039	0.184	0.289	0.144

5. THE POWER OF THE PROPOSED TEST

The power of the proposed test at a significant level αw.r.t. the linear failure rate, Weibull and Makeham alternatives are calculated based on simulation data. In such simulation, 10,000 samples are generated with sizes 10, 20 and 30 from the alternatives for different values of θ at $\alpha = 0.05$. Algorithms for computing the percentiles and thepower of the test are explained as follows:

Algorithm for computing the percentiles

1. Determine the required sample size and the simulated number of samples.
2. Generate an exponential random sample.
3. Use $\hat{\delta}_1$to compute the values of our test statistic.
4. Save the values of $\hat{\delta}_1$
5. Repeat steps (2) to (4) for the determined number of simulated samples and sort the values of the test statistic.
6. For the specified simulated number of samples, compute the upper percentiles of the computed values of $\hat{\delta}_1$
7. Repeat steps (1) to (6) for all different simulated number of sample and sample sizes.

Table 5.1. Power of the proposed test

n	θ	Linear failure rate	Weibull	Makeham
10	1	0.9958	0.9486	0.9830
10	2	0.9976	1.0000	0.9924
10	3	0.9990	1.0000	0.9950
10	1	0.9984	0.9416	0.9890
20	2	1.0000	1.0000	0.9968
20	3	0.9998	1.0000	0.9974
30	1	0.9990	0.9416	0.9904
30	2	1.0000	1.0000	0.9982
30	3	1.0000	1.0000	0.9986

Table 5.2. Critical values of the proposed test

n	90%	95%	98%	99%	n	90%	95%	98%	99%
5	0.7501	0.8071	0.8839	0.9457	28	0.6273	0.6562	0.6846	0.7058
6	0.7413	0.8055	0.8779	0.9314	29	0.6258	0.6546	0.6867	0.7101
7	0.7377	0.7956	0.8649	0.9216	30	0.6232	0.6520	0.6823	0.7014
8	0.7218	0.7785	0.8444	0.8883	31	0.6213	0.6472	0.6763	0.6952
9	0.7179	0.7633	0.8242	0.8648	32	0.6198	0.6460	0.6785	0.6977
10	0.7019	0.7474	0.8015	0.8308	33	0.6153	0.6390	0.6723	0.6918
11	0.6904	0.7273	0.7782	0.8186	34	0.6142	0.6410	0.6686	0.6918
12	0.6837	0.7241	0.7731	0.8073	35	0.6123	0.6385	0.6676	0.6884
13	0.6794	0.7190	0.7656	0.7981	36	0.6124	0.6384	0.6685	0.6866
14	0.6725	0.7112	0.7560	0.7933	37	0.6079	0.6336	0.6626	0.6820
15	0.6704	0.7078	0.7530	0.7876	38	0.6055	0.6336	0.6602	0.6774
16	0.6633	0.7007	0.7400	0.7684	39	0.6051	0.6306	0.6598	0.6764
17	0.6591	0.6951	0.7339	0.7620	40	0.6045	0.6305	0.6562	0.6727
18	0.6535	0.6843	0.7250	0.7510	41	0.6031	0.6303	0.6593	0.6748
19	0.6521	0.6842	0.7216	0.7493	42	0.6006	0.6262	0.6533	0.6705
20	0.6466	0.6793	0.7116	0.7318	43	0.5997	0.6250	0.6527	0.6701
21	0.6441	0.6780	0.7146	0.7355	44	0.5959	0.6231	0.6519	0.6699
22	0.6427	0.6757	0.7090	0.7323	45	0.5952	0.6198	0.6466	0.6638
23	0.6368	0.6681	0.7038	0.7276	46	0.5960	0.6210	0.6471	0.6641
24	0.6346	0.6639	0.6959	0.7160	47	0.5938	0.6188	0.6444	0.6655
25	0.6321	0.6619	0.6973	0.7182	48	0.5905	0.6166	0.6434	0.6591
26	0.6317	0.6628	0.6940	0.7147	49	0.5925	0.6188	0.6433	0.6599
27	0.6295	0.6568	0.6883	0.7115	50	0.5908	0.6163	0.6394	0.6560

Algorithm for computing the power

1. Determine the values percentiles that we shall compared with sample sizes and the umber of simulated samples.
2. Put the first value of θ.
3. Generate LFR, Makehaman Weibull random samples and save them.
4. Use $\hat{\delta}_1$ to compute the values of our test statistic for all generated samples one by one.
5. Compare the values of $\hat{\delta}_1$ for the specified sample with the corresponding percentile which leads the sample to reject or not.
6. Repeat steps (3) to (5) for the determined number of simulated samples.
7. For specified simulated number of samples and the parameter θ, compute and save the power in this case.
8. Repeat steps (2)-(5) for all different values of θ and sample sizes.

From Table 5.1, it is noticed that the power of the proposed test is high and increases by increasing the value of the parameter θ and the sample sizes.

Table 5.2 shows the critical values of the proposed test in different sample sizes and it can be noted that the critical values decrease when the sample sizes increase.

6. SOME APPLICATIONS

One of the main objectives of this research is to study the effectiveness of the proposed test. In this section, the proposed test is employed, with three sets of data, to identify the distribution where the data is drawn from via comparison between the tabulated value and calculated value.

Example 6.1. The following data representing failure times in hours for a specific type of electrical insulation in an experiment in which insulation was subjected to a continuously increasing voltage stress (see Lawless 1982, p.138):

0.205	0.363	0.407	0.770	0.720	0.782
1.178	1.255	1.592	1.635	2.310	-

In the above case, we found $\hat{\delta}_1 = 0.499$, which is less than the tabulated value, consequently the given set of data is drawn from the exponential distribution.

Example 6.2. Consider the well-known Darwin data that represent the differences in heights between cross and self-fertilized plants of the same pair grown together in one pot. The data is as follows (see Fisher 1966) :

4.9	-6.7	0.8	1.6	0.6	2.3	2.8	4.1
1.4	2.9	5.6	2.4	7.5	6.0	-4.8	-

In the above data, $\hat{\delta}_1 = 2.26$, which exceeds the tabulated value, consequently the given set of data is drawn from NBRU class and not exponential.

Example 6.3. The following data set consists of 16 intervals in operating days between successive failures of air conditioning equipment in a Boeing 720 aircraft (see Edgeman et. al. 1988) :

4.25	8.708	0.583	2.375	2.25	1.333	2.792	2.458
5.583	6.333	1.125	0.583	9.583	2.75	2.542	1.417

One can show that $\hat{\delta}_1$= 0.549, which is less than the tabulated value, and then the given set of data is drawn from exponential distribution.

7. SUMMARY AND CONCLUSIONS

The main objective of this research is to derive moment inequalities for NBRU class of life distribution using a new approach. Based on these inequalities, new testing hypothesis for exponentiality against the NBRU class are introduced. The Pitman asymptotic efficiency of this test is calculated for some alternative distributions and compared with other tests for exponentiality. The critical values and the power of the proposed test are calculated. Finally, the proposed test is applied to some real data.

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