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Stochastic analysis of a non-identical two-unit parallel system with common-cause failure, critical human error, non-critical human error, preventive maintenance and two type of repair

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Abstract. This paper investigates a mathematical model of a system composed of two non-identical unit parallel system with common-cause failure, critical human error, non-critical human error, preventive maintenance and two type of repair, i.e. cheaper and costlier. This system goes for preventive maintenance at random epochs. We assume that the failure, repair and maintenance times are independent random variables. The failure rates, repair rates and preventive maintenance rate are constant for each unit. The system is analyzed by using the graphical evaluation and review technique (GERT) to obtain various related measures and we study the effect of the preventive maintenance preventive maintenance on the system performance. Certain important results have been derived as special cases. The plots for the mean time to system failure and the steady-state availability $A(\infty)$ of the system are drawn for different parametric values.

Key Words: Preventive maintenance (PM), Common-cause failure (CCF), W-function, graphical evaluation and review technique (GERT), busy-time, critical human error, non-critical human error

1. INTRODUCTION

Many authors (Goel and Gupta, 1984; Goel et al., 1985) have studied a two similar or dissimilar unit cold standby redundant system with preventive maintenance, inspection and two types of repairs. Mokkaddis et.al (2008) studied a two-unit cold standby system with four different modes [normal, human errors, partial hardware failure and total hardware failures]. The cost-benefit analysis of a two-unit cold standby system with two types of repair - minor (regular) and major (expert) are considered by (El-Sherbeny et al., 2009). Mokkaddis et al. (2005) studied some characteristics of a single-man-machine system unit operating under different physical conditions. The failure, repair and change

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of physical conditions (good - poor) are stochastically independent random variables each having an arbitrary distribution. The stochastic analysis of a non-identical two-unit parallel system with common-cause failure by graphical evaluation and review technique (GERT) are investigated by Sridharan and Kalyani (2002). Mokkaddis et al. (2005) studied a single-unit repairable system with helping unit. The single unit is main and the other is helping. Initially both units are operative, the functioning of the main unit is assumed dependent on the helping unit if it is operative, and otherwise it is independent with an increasing failure rate. A stochastic analysis of a dissimilar two-unit parallel system with preventive maintenance and common-cause failure are investigated by Mokkaddis et al. (2008). Wang et al. (2006) studies the cost benefit analysis of series systems with cold standby components and a repairable service station, when the service times and the failure times of the primary components are assumed exponentially distributed. The breakdown times and the repair times of the service station are also assumed exponentially distributed. Mokkaddis et al. (2009) considered the reliability and mean time to system failure (*MTTF*) analysis of a two-state complex with repairable system, consisting of two sub-systems A and B arranged in series, incorporating the concept of hardware and human failures. Wang et al. (2006) considered four different system configurations with warm standby components and standby switching failures are compared based on their reliability and availability, when the time-to-repair and the time-to-failure for each of the primary and warm standby components are assumed to follow the negative exponential distribution. A two-unit warm standby system with constant failure rate and two types of repairmen and patience time are investigated by Mokkaddis et al. (2009). The reliability analysis of two mathematical models representing electric power systems operating in fluctuating outdoor weather (i.e., normal and stormy weather) and compared between two models are investigated by Mokaddis et al., (2010). Mahmoud and Moshref (2010) studies the stochastic analysis of a two-unit cold standby system considering hardware failure, human error failure and preventive maintenance (PM). Human error is defined as a failure to perform the performance of a prohibited action, which could result in damage to equipment and property or disruption of scheduled operations. It can be classified as either critical or non-critical. A critical human error is one which causes the failure of the entire system, for example, fire due to a human error in a room where a redundant system is located will cause total system failure. On the other hand, a non-critical human error does not lead to a catastrophic result. The occurrence of human errors can due to incorrect actions, maintenance errors, misinterpretation of instruments and so on.

A CCF is defined as the failure of single unit or multiple units due to a single common-cause. Some of the CCF may occur due to the following reasons.

- abnormal environmental conditions, e.g. temperature, pressure;
- defective design;
- improper maintenance of machines by workers; and
- natural catastrophe like fire, . . . etc.

The present paper is devoted to deal two-unit (non-identical) parallel system with additional preventive maintenance on regenerative state S_0 at random epochs. The main-time to system failure, steady-state availability, busy-time and idle-time for the service facility are obtained using the graphical evaluation and review technique (GERT). The effect of preventive maintenance on the system performance is shown graphically.

The results obtained by Sridharan and Kalyani (2002) and Goel and Shrivastava (1992) can be derived from present paper as a special case.

2. ASSUMPTIONS

1. The system consists of a single unit having two non-identical parallel components, say A and B.
2. The system remains operative even if a single life component operates.
3. The failure of a component changes the lift time parameter of the other.
4. The common cause failure and other failures are independent of each other.
5. The system is said to be down when both units are failed are inoperative.
6. The online unit suffers four types of failures, namely, hardware error, non-critical human error, critical human error and common-cause failure.
7. Common cause failure, hardware error, non-critical human error and critical human error can occur when one unit has failed and the other is operating.
8. There are two types of repairs i.e. 1) repair low-cost with probability π and 2) costly repairs with probability $\bar{\pi} = 1 - \pi$.
9. Both the components (when failed) can be replaced simultaneously.
10. Preventive maintenance (e.g. overhaul, inspection, minor, repairs, etc.) is provided to this system at random epochs when the system is in the state S_0 defined below.
11. The repaired or (replaced) unit works like new.

3. NOTATION

α constant hardware failure rate of component *A*,

β constant hardware failure rate of component *B*,

α_1 constant non-critical human error failure rate of component *A*,

β_1 constant non-critical human error failure rate of component *B*,

α_2 constant hardware failure rate of component *A* when *B* has already failed,

β_2 constant hardware failure rate of component *B* when *A* has already failed,

α_3 constant common cause failure rate of component *A* when *B* has already failed,

β_3 constant common cause failure rate of component *B* when *A* has already failed,

α_4 constant critical human error failure rate of component *A* when *B* has already failed,

β_4 constant critical human error failure rate of component *B* when *A* has already failed,

η constant common cause failure rate of the system when both the units are operating,

η_1 constant critical human error failure rate of the system when both the units are operating,

θ rate of simultaneous replacement of components *A* and *B* (due to hardware error failure),

θ_1 rate of simultaneous replacement of components *A* and *B* (due to common cause failure),

θ_2 rate of simultaneous replacement of components *A* and *B* (due to critical human error failure),

$\delta \mid \varphi$ costlier repair rate of a component *A* or component *B* receptivity,

$\delta_1 \mid \varphi_1$ cheaper repair rate of a component *A* or component *B* receptivity,

λ constant rate of PM time,

κ constant rate of time for taking a unit into PM,

μ_i mean sojourn in state S_i , $\forall i = 0,...,8$.

$p_{i,j}$ The transition probability from state *i* to state *j*.

From the above assumptions of our problem, it must be assumed that $(\alpha_2 > \alpha)$, $(\beta_2 > \beta)$, $(\delta_1 > \delta)$, $(\varphi_1 > \varphi)$, $(\pi > \bar{\pi})$ and $\{\theta, \theta_1, \theta_2\} < \{\delta, \delta_1, \varphi, \varphi_1\}$.

Symbols for the states of the system

A_N Component *A* in normal mode and operative.

B_N Component *B* in normal mode and operative.

A_{FS1} Component *A* in failure (hardware failure or non-critical human error) mode and under repair of type 1.

B_{FS1} Component *B* in failure (hardware failure or non-critical human error) mode and under repair of type 1.

A_{FS2} Component *A* in failure (hardware failure or non-critical human error) mode and under repair of type 2.

B_{FS2} Component *B* in failure (hardware failure or non-critical human error) mode and under repair of type 2.

A_F Component *A* in failure (hardware failure or non-critical human error) mode and needs replacement.

B_F Component *B* in failure (hardware failure or non-critical human error) mode and needs replacement.

A_{NP} Component *A* in normal mode and under preventive maintenance.

B_{NP} Component B in normal mode and under preventive maintenance.
A_{FC} Component A in failure (common cause) mode and needs replacement.
B_{FC} Component B in failure (common cause) mode and needs replacement.
A_{FCH} Component A in failure (critical human error) mode and needs replacement.
B_{FCH} Component B in failure (critical human error) mode and needs replacement.

Considering these symbols, the system can be in any one of the following states.

$S_0=(A_N,B_N)$ $\quad S_1=(A_{FS1},B_N)$ $\quad S_2=(A_{FS2},B_N)$

$S_3=(A_N,B_{FS1})$ $\quad S_4=(A_N,B_{FS2})$ $\quad S_5=(A_{NP},B_{NP})$

$S_6=(A_F,B_F)$ $\quad S_7=(A_{FC},B_{FC})$ $\quad S_8=(A_{FCH},B_{FCH})$

The system can have any one of the following states:

S_0, S_1, S_2, S_3, S_4 and S_5 are up states and S_6, S_7 and S_8 are down states (see Table 3.1).

Table 3.1. Transition rates

From \ To	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
S_0	---	$\pi(\alpha+\alpha_1)$	$\bar{\pi}(\alpha+\alpha_1)$	$\pi(\beta+\beta_1)$	$\bar{\pi}(\beta+\beta_1)$	κ	---------	η	η_1
S_1	δ_1	-----------	-----------	-----------	-----------	---	$\beta_2+\beta_1$	β_3	β_4
S_2	δ	-----------	-----------	-----------	-----------	---	$\beta_2+\beta_1$	β_3	β_4
S_3	φ_1	-----------	-----------	-----------	-----------	---	$\alpha_2+\alpha_1$	α_3	α_4
S_4	φ	-----------	-----------	-----------	-----------	---	$\alpha_2+\alpha_1$	α_3	α_4
S_5	λ	-----------	-----------	-----------	-----------	---	---------	---	---
S_6	θ	-----------	-----------	-----------	-----------	---	---------	---	---
S_7	θ_1	-----------	-----------	-----------	-----------	---	---------	---	---
S_8	θ_2	-----------	-----------	-----------	-----------	---	---------	---	---

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The probability of transition from state *i* to state *j* is given by

$$p_{0,1}=\frac{\pi(\alpha+\alpha_1)}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},\quad p_{0,2}=\frac{\bar{\pi}(\alpha+\alpha_1)}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},$$

$$p_{0,3}=\frac{\pi(\beta+\beta_1)}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},\quad p_{0,4}=\frac{\bar{\pi}(\beta+\beta_1)}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},$$

$$p_{0,5}=\frac{\kappa}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},\quad p_{0,7}=\frac{\eta}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},$$

$$p_{0,8}=\frac{\eta_1}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},\quad p_{1,0}=\frac{\delta_1}{\delta_1+\beta_1+\beta_2+\beta_3+\beta_4},$$

$$p_{1,6}=\frac{\beta_1+\beta_2}{\delta_1+\beta_1+\beta_2+\beta_3+\beta_4},\quad p_{1,7}=\frac{\beta_3}{\delta_1+\beta_1+\beta_2+\beta_3+\beta_4},$$

$$p_{1,8}=\frac{\beta_4}{\delta_1+\beta_1+\beta_2+\beta_3+\beta_4},\quad p_{2,0}=\frac{\delta}{\delta+\beta_1+\beta_2+\beta_3+\beta_4},$$

$$p_{2,6}=\frac{\beta_1+\beta_2}{\delta+\beta_1+\beta_2+\beta_3+\beta_4},\quad p_{2,7}=\frac{\beta_3}{\delta+\beta_1+\beta_2+\beta_3+\beta_4},$$

$$p_{2,8}=\frac{\beta_4}{\delta+\beta_1+\beta_2+\beta_3+\beta_4},\quad p_{3,0}=\frac{\varphi_1}{\varphi_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4},$$

$$p_{3,6}=\frac{\alpha_1+\alpha_2}{\varphi_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4},\quad p_{3,7}=\frac{\alpha_3}{\varphi_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4},$$

$$p_{3,8}=\frac{\alpha_4}{\varphi_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4},\quad p_{4,0}=\frac{\varphi}{\varphi+\alpha_1+\alpha_2+\alpha_3+\alpha_4},$$

$$p_{4,6}=\frac{\alpha_1+\alpha_2}{\varphi+\alpha_1+\alpha_2+\alpha_3+\alpha_4},\quad p_{4,7}=\frac{\alpha_3}{\varphi+\alpha_1+\alpha_2+\alpha_3+\alpha_4},$$

$$p_{4,8}=\frac{\alpha_4}{\varphi+\alpha_1+\alpha_2+\alpha_3+\alpha_4},\quad p_{5,0}=p_{6,0}=p_{7,0}=p_{8,0}=1. \tag{4.1}$$

The mean sojourn time in state S_i is defined as the time of stay in state S_i before transiting to any other state is given by

$$\mu_0=\frac{1}{\alpha+\beta+\alpha_1+\beta_1+\kappa+\eta+\eta_1},\quad \mu_1=\frac{1}{\delta_1+\beta_1+\beta_2+\beta_3+\beta_4},$$

$$\mu_2=\frac{1}{\delta+\beta_1+\beta_2+\beta_3+\beta_4},\ \mu_3=\frac{1}{\varphi_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4},\ \mu_4=\frac{1}{\varphi+\alpha_1+\alpha_2+\alpha_3+\alpha_4},$$

$$\mu_5 = \frac{1}{\lambda},\ \mu_6 = \frac{1}{\theta},\ \mu_7 = \frac{1}{\theta_1},\ \mu_8 = \frac{1}{\theta_2}. \tag{4.2}$$

Hence from the above states the W-function from state *i* to state *j* is given by

$$W_{ij}(s) = p_{ij}\left(1-\mu_i s\right)^{-1}, \tag{4.3}$$

where p_{ij} is the probability of transition from state *i* to state *j* and μ_i is the mean sojourn time.

5. MEAN TIME TO SYSTEM FAILURE (MTSF)

The mean-time to system failure is defined as the time until the system is completely inoperative. This is accomplished by finding the *W*-function from the initial state S_0 to terminal state $\{S_1,....,S_8\}$. Making use of Mason's rule we get.

$$W(s) = \frac{N_0(s)}{D_0(s)}. \tag{5.1}$$

where

$$N_0(s) = W_{0,1}(s)\{W_{1,6}(s) + W_{1,7}(s) + W_{1,8}(s)\} + W_{0,2}(s)\{W_{2,6}(s) + W_{2,7}(s) + W_{2,8}(s)\} + W_{0,7}(s)$$

$$+W_{0,8}(s) + W_{0,3}(s)\{W_{3,6}(s) + W_{3,7}(s) + W_{3,8}(s)\} + W_{0,4}(s)\{W_{4,6}(s) + W_{4,7}(s) + W_{4,8}(s)\}$$

and,

$$D_0(s) = 1 - \{W_{0,1}(s)W_{1,0}(s) + W_{0,2}(s)W_{2,0}(s) + W_{0,3}(s)W_{3,0}(s) + W_{0,4}(s)W_{4,0}(s)$$
$$+ W_{0,5}(s)W_{5,0}(s)\}.$$

where

$$W_{i,j}(s) = p_{i,j}\left(1-\mu_i s\right)^{-1},\ i,j = 0,1,...,6,\ i \neq j$$

The transition probabilities are $W_{i,j}(0) = p_{i,j}$.

The mean sojourn time can be written as follows:

$$\mu_0 = \frac{d}{ds}\{W_{0,1}(s) + W_{0,2}(s) + W_{0,3}(s) + W_{0,4}(s) + W_{0,5}(s) + W_{0,7}(s) + W_{0,8}(s)\}_{s=0},$$

$$\mu_1 = \frac{d}{ds}\{W_{1,0}(s) + W_{1,6}(s) + W_{1,7}(s) + W_{1,8}(s)\}_{s=0},$$

$$\mu_2 = \frac{d}{ds}\{W_{2,0}(s) + W_{2,6}(s) + W_{2,7}(s) + W_{2,8}(s)\}_{s=0},$$

$$\mu_3 = \frac{d}{ds}\{W_{3,0}(s) + W_{3,6}(s) + W_{3,7}(s) + W_{3,8}(s)\}_{s=0},$$

$$\mu_4 = \frac{d}{ds}\{W_{4,0}(s) + W_{4,6}(s) + W_{4,7}(s) + W_{4,8}(s)\}_{s=0},$$

$$\mu_5 = \frac{d}{ds}\{W_{5,0}(s)\}_{s=0},\ \mu_6 = \frac{d}{ds}\{W_{6,0}(s)\}_{s=0},\ \mu_7 = \frac{d}{ds}\{W_{7,0}(s)\}_{s=0},$$

$$\mu_8 = \frac{d}{ds}\{W_{8,0}(s)\}_{s=0}. \tag{5.2}$$

Hence, the mean-time to system failure is given by $MTSF = E(t) = \frac{1}{p}\left[\frac{d}{ds}W(s)\right]_{s=0}$.

where

$$p = [W(s)]_{s=0} = 1.$$

The mean-time to system failure (*MTSF*) when the system starts from S_0 is.

$$MTSF = E(t) = \frac{N_0}{D_0}. \tag{5.3}$$

where

$$N_0 = \mu_0 + p_{0,1}\,\mu_1 + p_{0,2}\,\mu_2 + p_{0,3}\,\mu_3 + p_{0,4}\,\mu_4 + p_{0,5}\,\mu_5,$$

and,

$$D_0 = 1 - \{p_{0,1}\,p_{1,0} + p_{0,2}\,p_{2,0} + p_{0,3}\,p_{3,0} + p_{0,4}\,p_{4,0} + p_{0,5}\,p_{5,0}\}.$$

6. MEAN TIME TO REPAIR / REPLACEMENT (MTTR)

The repair facility of the system is available in states $\{S_6, S_7, S_8\}$.
Hence, the mean-time to repair is given by:

$$MTTR(s) = \frac{N_1(s)}{D_1(s)}. \tag{6.1}$$

where

$$N_1(s) = \frac{d}{d\,s}\{(p_{0,1}\,p_{1,6} + p_{0,2}\,p_{2,6} + p_{0,3}\,p_{3,6} + p_{0,4}\,p_{4,6})W_{6,0}(s) + (p_{0,1}\,p_{1,7} + p_{0,2}\,p_{2,7}$$

$$+ p_{0,3}\,p_{3,7} + p_{0,4}\,p_{4,7})\,W_{7,0}(s) + (p_{0,1}\,p_{1,8} + p_{0,2}\,p_{2,8} + p_{0,3}\,p_{3,8} + p_{0,4}\,p_{4,8})W_{8,0}(s)\}_{s=0}$$

and,

$$D_1(s) = 1 - \{p_{0,1}\,p_{1,0} + p_{0,2}\,p_{2,0} + p_{0,3}\,p_{3,0} + p_{0,4}\,p_{4,0} + p_{0,5}\,p_{5,0}\}.$$

The mean-time to repair (*MTTR*) is given by:

$$MTTR = \frac{N_1}{D_1}. \tag{6.2}$$

where

$$N_1 = \left(p_{0,1}\,p_{1,6} + p_{0,2}\,p_{2,6} + p_{0,3}\,p_{3,6} + p_{0,4}\,p_{4,6}\right)\mu_6 + \left(p_{0,1}\,p_{1,7} + p_{0,2}\,p_{2,7} + p_{0,3}\,p_{3,7} + p_{0,4}\,p_{4,7} + p_{0,7}\right)\mu_7 + \left(p_{0,1}\,p_{1,8} + p_{0,2}\,p_{2,8} + p_{0,3}\,p_{3,8} + p_{0,4}\,p_{4,8} + p_{0,8}\right)\mu_8 ,$$

and,

$$D_1 = 1 - \left\{p_{0,1}\,p_{1,0} + p_{0,2}\,p_{2,0} + p_{0,3}\,p_{3,0} + p_{0,4}\,p_{4,0} + p_{0,5}\,p_{5,0}\right\} .$$

6.1. Availability analysis

The system availability $A(t)$, is defined as Pr (that the system is in an operating state at time t). The steady-state availability is defined as:

$$A(\infty) = \lim_{t\to\infty} A(t) = \frac{MTSF}{MTSF + MTTR} = \left[1 + \frac{N_1}{N_0}\right]^{-1} . \tag{6.3}$$

7. BUSY-TIME AND IDLE-TIME FOR THE SERVICE FACILITY

When the process is available in states S_0, the repair facility is idle. This path is represented by.

$$W(s) = W_{0,1}(s) + W_{0,2}(s) + W_{0,3}(s) + W_{0,4}(s) + W_{0,5}(s) + W_{0,7}(s) + W_{0,8}(s) . \tag{7.1}$$

Thus, the mean idle-time is $= \mu_0 = \dfrac{1}{\alpha + \alpha_1 + \beta + \beta_1 + \kappa + \eta + \eta_1} = \dfrac{1}{x}$. (7.2)

The repair facility is available in states $\{S_1, S_2, S_3, S_4, S_6, S_7, S_8\}$ with repair rates δ_1, δ, φ_1, φ, θ, θ_1 and θ_2 respectively. Hence, the equivalent *W*-function $W(s)$ is obtained as.

$$W(s) = p_{0,1}\,p_{1,0}\left(1 - \frac{s}{\delta_1}\right)^{-1} + p_{0,2}\,p_{2,0}\left(1 - \frac{s}{\delta}\right)^{-1} + p_{0,3}\,p_{3,0}\left(1 - \frac{s}{\varphi_1}\right)^{-1} + p_{0,4}\,p_{4,0}\left(1 - \frac{s}{\varphi}\right)^{-1}$$
$$+ p_{0,5}\,p_{5,0}\left(1 - \frac{s}{\lambda}\right)^{-1} + \left(p_{0,1}\,p_{1,6} + p_{0,2}\,p_{2,6} + p_{0,3}\,p_{3,6} + p_{0,4}\,p_{4,6}\right)\left(1 - \frac{s}{\theta}\right)^{-1} + p_{0,7}\,p_{7,0}\left(1 - \frac{s}{\theta_1}\right)^{-1}$$
$$+ p_{0,8}\,p_{8,0}\left(1 - \frac{s}{\theta_2}\right)^{-1} . \tag{7.3}$$

Therefore, the mean busy time of service facility given as

$$p_{0,1}\,\mu_1 + p_{0,2}\,\mu_2 + p_{0,3}\,\mu_3 + p_{0,4}\,\mu_4 + p_{0,5}\,\mu_5 + \left(p_{0,1}\,p_{1,6} + p_{0,2}\,p_{2,6} + p_{0,3}\,p_{3,6} + p_{0,4}\,p_{4,6}\right)\mu_6 + p_{0,7}\,\mu_7 + p_{0,8}\,\mu_8 = B \text{ (say)}. \tag{7.4}$$

Therefore, the Busyness of the service facility may be defined as.

Busyness = Mean busy-time / (Mean busy-time + Mean idle-time) ,

$$\text{Busyness } = \frac{B}{\frac{1}{x}+B} = \frac{x\,B}{1+x\,B}. \tag{7.5}$$

8. SPECIAL CASES

8.1. Study the system without preventive maintenance.

The mean time to system failure ($MTSF_1$) *is given as.*

$$MTSF_1 = E_1(t) = \frac{\mu_0 + p_{0,1}\,\mu_1 + p_{0,2}\,\mu_2 + p_{0,3}\,\mu_3 + p_{0,4}\,\mu_4}{1-\left\{p_{0,1}\,p_{1,0} + p_{0,2}\,p_{2,0} + p_{0,3}\,p_{3,0} + p_{0,4}\,p_{4,0}\right\}}. \tag{8.1}$$

The mean time to repair/replacement ($MTTR_1$) *is given as.*

$$MTTR_1 = \frac{N_1}{D_2}. \tag{8.2}$$

where

$$D_2 = 1-\left\{p_{0,1}\,p_{1,0} + p_{0,2}\,p_{2,0} + p_{0,3}\,p_{3,0} + p_{0,4}\,p_{4,0}\right\}$$

The steady-state availability is given by

$$A_1(\infty) = \left[1+\frac{N_1}{\mu_0 + p_{0,1}\,\mu_1 + p_{0,2}\,\mu_2 + p_{0,3}\,\mu_3 + p_{0,4}\,\mu_4}\right]^{-1}. \tag{8.3}$$

Busy-Time and Idle-Time for the Service Facility

When the process is available in states S_0, the repair facility is idle. This path is represented as.

$$W(s) = W_{0,1}(s) + W_{0,2}(s) + W_{0,3}(s) + W_{0,4}(s) + W_{0,7}(s) + W_{0,8}(s). \tag{8.4}$$

$$\text{Thus, the mean idle-time is } = \mu_0 = \frac{1}{\alpha+\alpha_1+\beta+\beta_1+\eta+\eta_1} = \frac{1}{x_1}. \tag{8.5}$$

Therefore, the mean busy time of service facility given as

$$p_{0,1}\,\mu_1 + p_{0,2}\,\mu_2 + p_{0,3}\,\mu_3 + p_{0,4}\,\mu_4 + \left(p_{0,1}\,p_{1,6} + p_{0,2}\,p_{2,6} + p_{0,3}\,p_{3,6} + p_{0,4}\,p_{4,6}\right)\mu_6$$

$$+ p_{0,7}\,\mu_7 + p_{0,8}\,\mu_8 = B_1 \text{ (say)}. \tag{8.6}$$

$$\text{Busyness } = \frac{B_1}{\frac{1}{x_1}+B_1} = \frac{x_1 B_1}{1+x_1 B_1}. \tag{8.7}$$

8.2. Study the system without common-cause failure.

The mean time to system failure ($MTSF_2$) *is given by*

$$MTSF_2 = E_2(t) = \frac{N_0}{D_0}. \tag{8.8}$$

The mean time to repair ($MTTR_2$) *is given by*

$$MTTR_2=\frac{N_2}{D_1}. \quad (8.9)$$

where

$$N_2=\left(p_{0,1}\,p_{1,6}+p_{0,2}\,p_{2,6}+p_{0,3}\,p_{3,6}+p_{0,4}\,p_{4,6}\right)\mu_6$$
$$+\left(p_{0,1}\,p_{1,8}+p_{0,2}\,p_{2,8}+p_{0,3}\,p_{3,8}+p_{0,4}\,p_{4,8}+p_{0,8}\right)\mu_8.$$

The steady-state availability is given by

$$A_2(\infty)=\left[1+\frac{N_2}{N_0}\right]^{-1}. \quad (8.10)$$

Busy-Time and Idle-Time for the Service Facility

When the process is available in states S_0, the repair facility is idle. This path is represented by.

$$W(s)=W_{0,1}(s)+W_{0,2}(s)+W_{0,3}(s)+W_{0,4}(s)+W_{0,8}(s). \quad (8.11)$$

Thus, the mean idle-time is $=\mu_0=\dfrac{1}{\alpha+\alpha_1+\beta+\beta_1+\kappa+\eta_1}=\dfrac{1}{x_2}$. (8.12)

Therefore, the mean busy time of service facility given by

$$p_{0,1}\,\mu_1+p_{0,2}\,\mu_2+p_{0,3}\,\mu_3+p_{0,4}\,\mu_4+\left(p_{0,1}\,p_{1,6}+p_{0,2}\,p_{2,6}+p_{0,3}\,p_{3,6}+p_{0,4}\,p_{4,6}\right)\mu_6$$
$$+p_{0,8}\,\mu_8=B_2\ \text{(say)}. \quad (8.13)$$

$$\text{Busyness}=\frac{B_2}{\frac{1}{x_2}+B_2}=\frac{x_2B_2}{1+x_2B_2}. \quad (8.14)$$

8.3. Study the system without critical human error.

The mean time to system failure ($MTSF_3$) *is given by*

$$MTSF_3=E_3(t)=\frac{N_0}{D_0}. \quad (8.15)$$

The mean time to repair/replacement ($MTTR_3$) *is given by*

$$MTTR_3=\frac{N_3}{D_1}. \quad (8.16)$$

where

$$N_3=\left(p_{0,1}\,p_{1,6}+p_{0,2}\,p_{2,6}+p_{0,3}\,p_{3,6}+p_{0,4}\,p_{4,6}\right)\mu_6+\left(p_{0,1}\,p_{1,7}+p_{0,2}\,p_{2,7}+p_{0,3}\,p_{3,7}\right.$$
$$\left.+p_{0,4}\,p_{4,7}+p_{0,7}\right)\mu_7\ .$$

The steady-state availability is given by

$$A_3(\infty) = \left[1 + \frac{N_3}{N_0}\right]^{-1}. \quad (8.17)$$

Busy-Time and Idle-Time for the Service Facility

When the process is available in states S_0, the repair facility is idle. This path is represented as.

$$W(s) = W_{0,1}(s) + W_{0,2}(s) + W_{0,3}(s) + W_{0,4}(s) + W_{0,5}(s) + W_{0,7}(s). \quad (8.18)$$

$$\text{Thus, the mean idle-time is} = \mu_0 = \frac{1}{\alpha + \alpha_1 + \beta + \beta_1 + \kappa + \eta} = \frac{1}{x_3}. \quad (8.19)$$

Therefore, the mean busy time of service facility given by.

$$p_{0,1}\mu_1 + p_{0,2}\mu_2 + p_{0,3}\mu_3 + p_{0,4}\mu_4 + p_{0,5}\mu_5 + \left(p_{0,1}p_{1,6} + p_{0,2}p_{2,6} + p_{0,3}p_{3,6} + p_{0,4}p_{4,6}\right)\mu_6$$
$$+ p_{0,7}\mu_7 = B_3 \text{ (say)}. \quad (8.20)$$

$$\text{Busyness} = \frac{B_3}{\frac{1}{x_3} + B_3} = \frac{x_3 B_3}{1 + x_3 B_3}. \quad (8.21)$$

8.4. Study the system without preventive maintenance, common-cause failure, critical human error, non- critical human error and one type of repar "i.e $\pi = 1$" . all results of Ref. (Goel and Shrivastava, 1992) are deduced.

8.5. Study the system without common-cause failure, critical human error, non-critical human error and one type of repar "i.e $\pi = 1$" . we have the results of Ref. (Goel and Shrivastava, 1992).

8.6. Study the system without preventive maintenance, critical human error, non-critical human error and one type of repar "i.e $\pi = 1$" . all results of Ref. (Sridharan and Kalyani, 2002) are deduced.

9. COMPARISON BETWEEN MTSF AND $A(\infty)$ UNDER EFFECT THE PREVENTIVE MAINTENANCE(PM)

The purpose of this section study the effect of PM on the system. The following numerical results are obtained by considering the following system parameters:

We fix $\alpha_1 = 0.1$, $\alpha_2 = 0.9$, $\alpha_3 = 0.01$, $\alpha_4 = 0.02$, $\beta_1 = 0.2$, $\beta_2 = 0.8$, $\beta_3 = 0.03$, $\beta_4 = 0.05$, $\eta = 0.01, \eta_1 = 0.05$, $\theta = 0.06$, $\theta_1 = 0.02$, $\theta_2 = 0.04$, $\delta = 0.2$, $\delta_1 = 0.4$, $\varphi = 0.3$, $\varphi_1 = 0.5$, $\pi = 0.7$, $\bar{\pi} = 0.3$ and vary the values of α and β from 0.1 to 0.8 (see Tables 9.1-9.4).

These curves are shown in Figures (10.5-10.8). From these figures we conclude that *MTSF* and the steady-state availability for system (*with PM*) are always above as compared to those for system (*without PM*) which implies that the preventive maintenance "***PM***" leads to an improvement in overall system performance.

In Figures (10.1-10.4) show that, the *MTSF* and $A(\infty)$ are increases when the time for taking a unit into *PM* (κ)greater then the *PM* time rate (λ) *i.e.* $\kappa > \lambda$.

Table 9.1. Comparison of the mean time to system failure (*MTSF*) by using four cases of PM.

	MTSF			
α	$\lambda=0.2 < \kappa=0.9$	$\lambda=\kappa=0.9$	$\lambda=0.9>\kappa=0.2$	*without PM*
0.1	7.97779	3.46542	2.46267	1.00733
0.2	5.68832	2.56662	1.87291	0.92899
0.3	5.31039	2.4918	1.86545	0.977804
0.4	6.3694	2.88621	2.11217	0.997539
0.5	5.98479	2.74771	2.02835	0.994778
0.6	5.65094	2.62748	1.9556	0.992221
0.7	4.72635	2.2529	1.70324	0.94509
0.8	5.10003	2.42909	1.83555	0.987639

Table 9.2. Comparison of the mean time to system failure (*MTSF*) by using four cases of PM.

	MTSF			
β	$\lambda=0.2 < \kappa=0.9$	$\lambda=\kappa=0.9$	$\lambda=0.9>\kappa=0.2$	*without PM*
0.1	6.77572	2.94403	2.09254	0.931478
0.2	5.40334	2.39584	1.7275	0.866463
0.3	4.77925	2.14849	1.56388	0.833203
0.4	5.55351	2.49252	1.8123	0.905347
0.5	5.2485	2.37985	1.74237	0.897896
0.6	4.97955	2.28049	1.6807	0.890965
0.7	4.47105	2.07521	1.5428	0.860533
0.8	4.52697	2.1133	1.57693	0.878461

Table 9.3. Comparison of the availability by using four cases of PM.

	$A(\infty)$			
α	$\lambda=0.2 < \kappa=0.9$	$\lambda=\kappa=0.9$	$\lambda=0.9>\kappa=0.2$	*without PM*
0.1	0.378198	0.198747	0.143839	0.126741
0.2	0.305672	0.157832	0.116004	0.103279
0.3	0.272173	0.140713	0.104781	0.0939566
0.4	0.331142	0.172924	0.127035	0.112974
0.5	0.310499	0.161902	0.119745	0.10691
0.6	0.292671	0.15268	0.1137	0.101895
0.7	0.248966	0.130327	0.0986868	0.0892196
0.8	0.263432	0.138121	0.138121	0.0940867

Table 9.4. Comparison of the availability by using four cases of PM.

	$A(\infty)$			
β	$\lambda=0.2<\kappa=0.9$	$\lambda=\kappa=0.9$	$\lambda=0.9>\kappa=0.2$	*without PM*
0.1	0.272173	0.140713	0.104781	0.0939566
0.2	0.232532	0.119034	0.0890986	0.0801682
0.3	0.211398	0.108443	0.0818043	0.0738984
0.4	0.236996	0.123125	0.0930462	0.0840695
0.5	0.22752	0.118546	0.0900187	0.0815258
0.6	0.218912	0.114442	0.0873151	0.0792566
0.7	0.20199	0.105675	0.0810269	0.0737332
0.8	0.203861	0.107392	0.0826917	0.0753813

10. CONTRIBUTIONS AND CONCLUDING REMARKS

In this paper, GERT is used to derive some reliability measures of a of a non-identical two-unit parallel system. Some previous results are deduced as a special case of our model. The effect of PM to the measures of the system is studied graphically. Finally, for achieving high reliability of the system, we recommend adding PM to the model and $\kappa > \lambda$ because of its positive effect on the model as shown in.

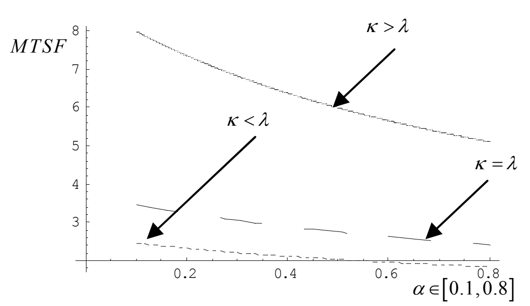

Figure 10.1.

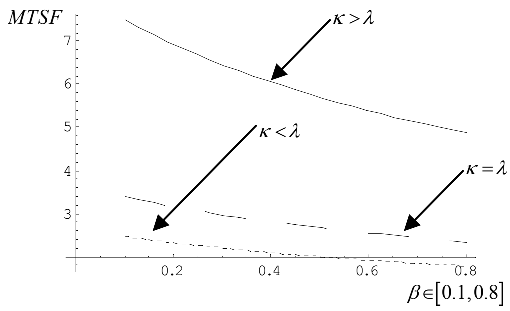

Figure 10. 2.

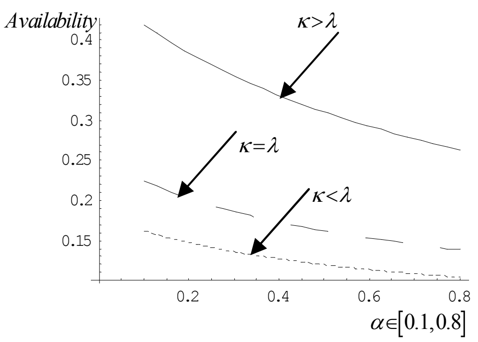

Figure 10. 3.

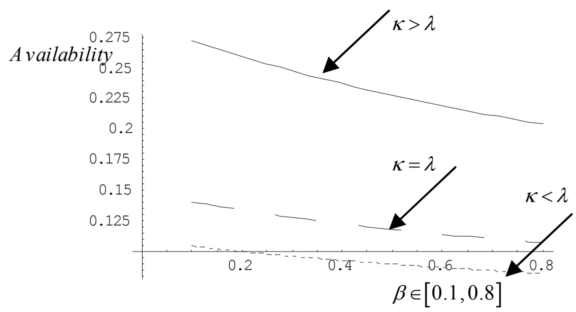

Figure 10. 4.

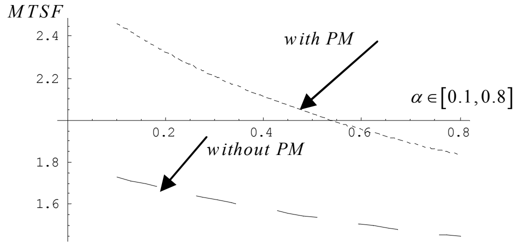

Figure 10. 5.

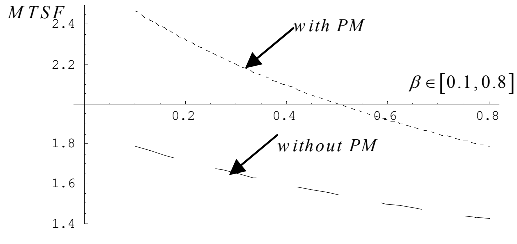

Figure 10. 6.

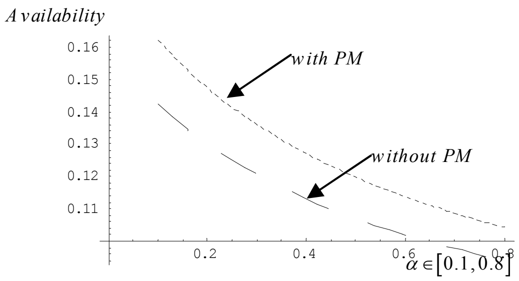

Figure 10. 7.

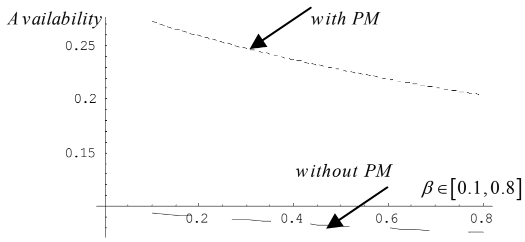

Figure 10. 8.

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