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# Adaptive Finite Element Mesh Generation Schemes for Dynamic Structural Analyses

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## Abstract

Reliable dynamic analysis is essential in order to properly maintain structures so that structural hazards may be minimized. The finite element method (FEM) is proven to be an affective approximate method of structural analysis if proper element types and meshes are chosen. When the method is applied to dynamics analyzed in time domain, the meshes may need to be modified at each time step. As many meshes need to be generated, adaptive mesh generation schemes have become an important part in complex time domain dynamic finite element analyses of structures. In this paper, an adaptive mesh generation scheme for dynamic finite element analyses of structures is described. The concept of representative strain value is used for error estimates and the refinements of meshes use combinations of the h-method (node movement) and the r-method (element division). The validity of the scheme is shown through a cantilever beam example under a concentrated load with varying values. The example shows reasonable accuracy and efficient computing time. Furthermore, the study shows the potential for the scheme's effective use in complex structural dynamic problems such as those under seismic or erratic wind loads.

***Key words*** : structures, dynamic behavior, adaptive schemes, finite element method

## 요 지

구조물의 방재를 위해서 구조물의 효율적인 유지관리는 필수적이며, 여기서 신뢰 있는 구조물의 동적해석은 중요한 역할을 한다. 유한요소법은 구조해석법으로 가장 많이 사용되는 방법으로 자리 잡고 있으며, 요소와 요소망이 제대로 선택되면 신뢰 있는 해석 결과를 출력한다. 시간 영역 동적해석에 유한요소법을 사용하려면 각 시간 단계에서 요소망을 재형성할 필요가 생길 수 있는데, 여기에 연산 시간 측면에서 효율적인 적응적 요소망 전략을 사용하면 편리하다. 본 연구는 시간영역 동적해석에서 전 단계 해석 결과를 사용하여 계산된 대표 변형률 값을 오차 평가하는데 사용하고, 요소 세분화는 절점 이동인 r-법과 요소 분할인 h-법의 조합으로 효율적으로 계산하는 적응적 요소망 형성 전략을 제시한다. 적용한 캔틸레버보의 예제를 통하여 정확성과 연산 효율성을 검증하였고 나아가 방법의 간단함이 지진 하중, 풍하중 등에 의한 복잡한 구조 동적 해석에도 효율적으로 사용될 수 있는 것을 보여 준다.

**핵심용어** : 구조, 동적 거동, 적응적 전략, 유한요소법

## 1. Introduction

Reliable dynamic analysis is essential in order to properly maintain structures so that structural hazards may be minimized. The finite element method(FEM) is proven to be an affective approximate method of structural analysis (Bathe and Wilson, 1976; Belytschko et al., 1996; Cook et al., 1989; Reddy, 1993; Zienkiewcz et al., 2005). When the method is applied to dynamics analyzed in time domain, the meshes may need to be modified at each time step as the finite element results of an analysis largely depend of the mesh and the element types used (Heesom and Mahdjoubi, 2001). As many meshes need to be generated, adaptive mesh generation schemes have become an important part in complex time domain dynamic finite element analyses of structures.

In this paper, an adaptive mesh generation scheme for dynamic finite element analysis of structures is described. The adaptive mesh generation process involves estimation of error given a mesh and generation of a new improved mesh based on the error (Choi and Jung, 1998; Choi and Yu, 1998). A new mesh may be generated by moving an existing node (the r-method), by dividing an element into smaller elements (the h-method), by increasing the degree of the polynomial in the shape function of element used (the p-method), and combinations of these (de Las Casa, 1988;

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Zienkiewcz and Zhu, 1987). Structural loads in general are dynamic, that is, except the dead loads, the loads are by nature dynamic where the value and the position are functions of time. The responses for these problems are complex and generating an efficient mesh for these dynamic problems poses a difficult problem as the results must be accurate and computation times be minimal (Newmark, 1959; Stampgle et al., 2001). This paper presents an adaptive scheme for dynamic problems that uses error estimation that is computationally efficient but yielding reasonable results. The mesh generating scheme combines the r-method and the h-method. The scheme includes a check for limiting distortion. Time domain analysis is based on direct integration. As an example, undamped cantilever beam modeled with four node quadrilateral isoparametric elements for plane stress is considered. The initial mesh is defined by the user. The results of the analysis of the example shows that the proposed r-h method and the error estimation are efficient and have reasonable confidence level for complex dynamic problems in structural engineering.

## 2. Adaptive mesh scheme for dynamic analysis

### 2.1 Error estimation with representative strains

The results of a finite element analysis largely depend of the type of element and the mesh used. An adaptive mesh generation scheme attempts to automated the mesh generation and in the scheme, accuracy of error estimation of a given mesh is essential (Hwang, 1988; Jeong et al., 2003; McFee and Giannacopoulos, 2001; Ohnimus et al., 2001; Yoon, 2005; Zhu et al., 1991). In general engineering problems, accurate solutions are not known and thus accurate error estimation is inherently a difficult problem. In addition, error estimation for adaptive mesh needs to be computationally efficient, especially for dynamic problems, since there are extensive additional computations needed for many aspects of adaptive mesh generation and analyses that follow. Norm of a matrix is generally used to represent error of a mesh where the matrix includes values of stress, strain and displacements. The error in the domain O may be represented as follows:

$$\|E\| = \left[\int_{\Omega}(\varepsilon-\hat{\varepsilon})^T(\sigma-\hat{\sigma})d\Omega\right] \tag{1}$$

Here $\sigma$, $\varepsilon$ are the exact solutions for stress and strain and $\hat{\sigma}$, $\hat{\varepsilon}$ are the finite element solutions. The error defined in Eqs. (2-5) are the representative strain of element based on the standard deviations of the strains at the Gauss points in the element computed during the previous finite element analysis (Jeong and Yoon, 2003; Yoon, 2009). The z axis is not considered in x-y planar problems, and the representative strain values of element $i$ are represented by the following equations:

$$\|e\|_{ix} = \sqrt{\frac{\sum_{j=1}^{n_{gx}}(\varepsilon_{jx}-\varepsilon_x^*)^2}{n_{gx}-1}} \tag{2}$$

$$\|e\|_{iy} = \sqrt{\frac{\sum_{j=1}^{n_{gy}}(\varepsilon_{jy}-\varepsilon_y^*)^2}{n_{gy}-1}} \tag{3}$$

$$\|e\|_{ixy} = \sqrt{\frac{\sum_{j=1}^{n_{gxy}}(\gamma_{jxy}-\gamma_{xy}^*)^2}{n_{gxy}-1}} \tag{4}$$

$$\|e\|_i = \{\|e\|_{ix}+\|e\|_{iy}+\|e\|_{ixy}\} \times \frac{A_i}{A_{total}} \tag{5}$$

Here, $\|e\|_{ix}$ is the x directional standard deviation of the strain, $n_{gx}$ is the number of Gauss points in x direction, $\varepsilon_{jx}$ is the x directional strain of Gauss point $j$, and $\varepsilon_x^*$ is the y directional strain; $\|e\|_{iy}$, $n_{gy}$, $\varepsilon_{jy}$, $\varepsilon_y^*$ are the similar y directional values and $\|e\|_{ixy}$, $n_{gxy}$, $\gamma_{jxy}$, $\gamma_{xy}^*$ are the similar x-y shear strain values. In addition, is the element area and is the total area of the mesh.

### 2.2 Dynamic analysis

The direct numerical integration in the time domain is used for the dynamic analysis. Specifically, the Newmark-$\beta$ method, a reliable explicit method widely used in dynamic structural analysis, is used (Bathe and Wilson, 1976; Belytschko, 1974; Newmark, 1959). The equations in the Newmark-$\beta$ method may be summarized as follows:

$$\dot{u}_{i+1} = \dot{u}_i+(\Delta t)[(1-\gamma)\ddot{u}_i+\gamma\ddot{u}_{i+1}] \tag{6}$$

$$u_{i+1} = u_i+(\Delta t)\dot{u}_i+(\Delta t)^2\left[\left(\frac{1}{2}-\beta\right)\ddot{u}_i+\beta\ddot{u}_{i+1}\right] \tag{7}$$

Here, $u_i$, $\dot{u}_i$, $\ddot{u}_i$, are the displacement, velocity, and acceleration vectors in the $i$th step and $u_{i+1}$, $\dot{u}_{i+1}$, $\ddot{u}_{i+1}$ are the similar quantities in the $i+1$th step; $\Delta t$ is the time step length and 1/10 is used; $\beta$ and $\gamma$ are parameters selected by the user and the recommended values of $\beta$=1/4 and $\gamma$=1/2 are used.

The matrix equilibrium equations for the $i$th step may be represented as follows:

$$K'u_{i+1} = \dot{F}_{i+1} \tag{8}$$

The matrices $K'$ and $F'_{i+1}$ represent the following:

$$K' = K+\frac{1}{\beta(\Delta t)^2}M \tag{9}$$

$$F'_{i+1} = F_{i+1}+\frac{M}{\beta(\Delta t)^2}\left[u_i+(\Delta t)^2\dot{u}_i+\left(\frac{1}{2}-\beta\right)(\Delta t)^2\ddot{u}_i\right] \tag{10}$$

Here, $K$ is the stiffness matrix, $M$ is the mass matrix, and $F_{i+1}$ is the force vector for the $i+1$th time step.

### 2.3 Adaptive mesh generation scheme

The adaptive mesh generation based on the error estimation with representative strains is formulated combining the r-method and the h-method. The r-method moves existing nodes based on the following equations for the new coordinates $x_b$, $y_b$ :

$$x_b = \frac{\sum_{i=1}^{n_m} x_{ci}(\|e\|_i)}{\sum_{i=1}^{n_m} (\|e\|_i)} \tag{11}$$

$$y_b = \frac{\sum_{i=1}^{n_m} y_{ci}(\|e\|_i)}{\sum_{i=1}^{n_m} (\|e\|_i)} \tag{12}$$

Here, $x_{ci}$, $y_{ci}$ are the coordinates of the centroid of the element, $n$ is the number of elements and $\|e\|_i$ is the representative strain value of the element i. Eq. (11) and Eq. (12) for nodes on the boundary are modified according to the following equations:

$$x_b = \frac{\sum_{i=1}^{2} x_{lci}(\|e\|_i)}{\sum_{i=1}^{2} (\|e\|_i)} \tag{13}$$

$$y_b = \frac{\sum_{i=1}^{2} y_{lci}(\|e\|_i)}{\sum_{i=1}^{2} (\|e\|_i)} \tag{14}$$

Here, $x_{lci}$, $y_{lci}$ are the x, y coordinates of the central point on the boundary. A further attempt to keep the elements rectangular, and width to depth ratio to be close to 1 are maintained. A concept of shape factor defined in Eq. (15) and described in detail in reference Yoon(2009) is used to limit distortion of elements.

$$Shape\ Factor = \frac{\sqrt{A_i}}{0.25 L_{ti}} \tag{15}$$

Here, $L_{ti}$ is the length of the boundary of the element. The shape factor is 1 for square and an attempt to keep shape factor to be close to 1 is maintained; shapes with a shape factor less than 0.95 are not used.

The h-method divides an element into smaller elements of the same type. The elements to be subdivided are based on the following discretization parameter $d$ expressed in 1/force units:

$$d = \alpha \times mean[\|e\|_{initial}] \div \max P \tag{16}$$

Here, is a constant, $mean[\|e\|_{initial}]$ is the average representative strain value of the initial mesh and max$P$ is the maximum value of the applied load. After a parametric study, a value of 15.0 is used for $\alpha$.

The r-h method combines r-method and h-method. Various means of combining have been studied(de Las Casas, 1988; Hwang, 1988; Yoon and Jeong, 2005). To obtain an optimal combination of r-method and h-method, first the representative strain values are normalized at each step using the following equations.

$$\min[\|e\|_i] \times a + b = 0 \tag{17}$$

$$\max[\|e\|_i] \times a + b = 100 \tag{18}$$

Here, $\min[\|e\|_i]$, $\max[\|e\|_i]$ are the minimum and maximum values of the representative strains, and a, b are constants in each step determined from Eq. (17) and Eq. (18). These two equations are used to normalize the representative strains to range from 0 to 100.

A dispersion parameter $Cr$ is defined as follows:

$$Cr = |M_e - M_o| \tag{19}$$

Here, $M_e$ is the average of the normalized representative strains and $M_o$ is the mode of distribution of the representative strain values. A value for $Cr$ need to be set to combine the r-method and the h-method and a reasonable value, empirically obtained, is around 20 where if $Cr$ is larger than this value, r-method is used, and in other cases, the h-method is used.

In the h-method, the refinement of the mesh is terminated when the change in the representative strain values is less than 1% or when the element's discretization parameter is less than $d$ in Eq. (16). In the r-method, the refinement of the mesh is terminated when the element's discretization parameter is less than $d$ in Eq. (16).

## 3. Case Study

The example considered is a deep cantilever beam shown in Fig. 1 where the dimensions are shown. The beam is modeled with four node isoparametric quadrilateral elements. The dynamic loading is a lateral concentrated load P where the value is a function of time. The variation of value considered is given by the following equation:

$$P = -500 \sin(2\pi t) \tag{20}$$

The duration of the loading is 1 second as shown in Fig. 2 and the unit for loading is Newton. Since the free vibration response continues after 1 second, the response considered is for 5 seconds. A crude initial mesh formed by four

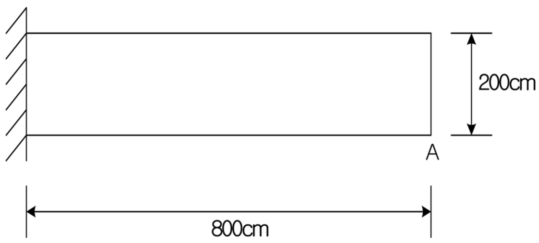


Fig. 1. Cantilever beam example

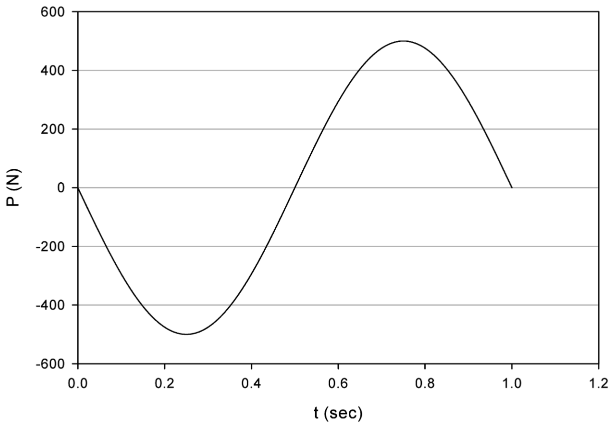


Fig. 2. Time variation of load P

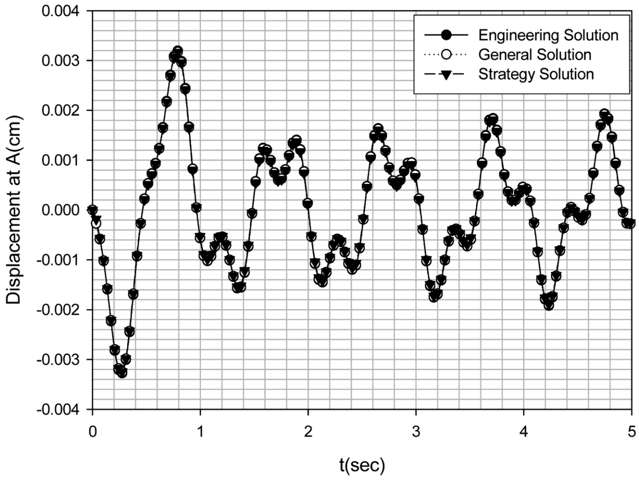


Fig. 3. Vertical displacement of the free end

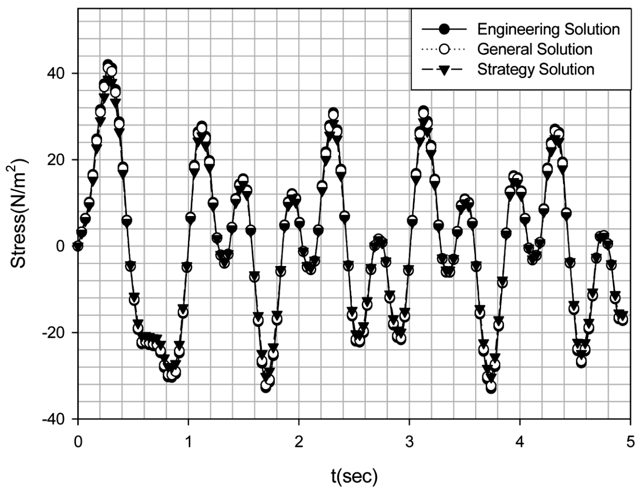


Fig. 4. Mid-horizontal normal stress at the fixed end

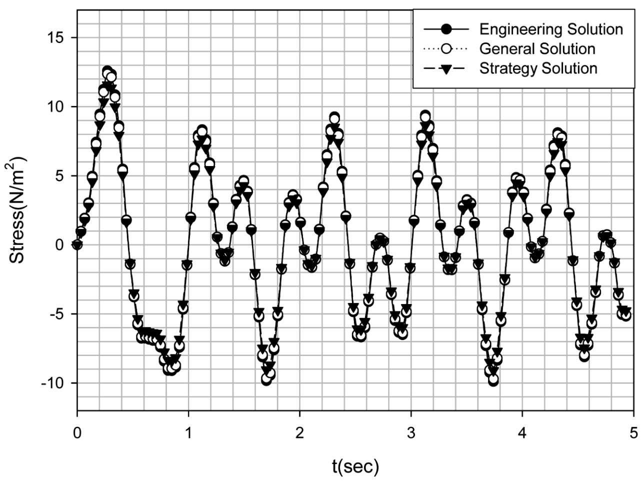


Fig. 5. Mid-vertical normal stress at the fixed end

equal elements arranged along the beam axis is used for the investigation. More appropriate initial mesh increases the efficiency of the algorithm.

The beam is 5 cm thick and the material obeys Hooke's law for elasticity with the modulus of elasticity of 210Þ105 N/cm$^2$ and Poisson's ratio of 0.3. The per unit mass is $7.85 \times 10^{-3}$ kg/cm$^3$. The time step selected for analysis is 0.034 seconds yielding 145 steps. For comparison purposes, solutions from a regularly discretized mesh of 1024 elements is termed the *engineering* solution, the ones from 256 elements is terms the *common* solution, and the solution from the converged adaptive mesh is termed the *strategy* solution. Fig. 3, Fig. 4, Fig. 5 and Fig. 6 show respectively, the comparisons of the vertical displacement of the free end, the mid-horizontal (x directional) normal stress at the fixed end, the mid-vertical (y directional) normal stress at the fixed end, the mid-shear stress at the free end of the engineering, the common, and the strategy solutions. The figures show close agreement among the three solutions.

Table 1 shows the representative strain values in the initial steps of the dynamic adaptive mesh where the transitions of the r-method and the h-method are shown. Fig. 7 shows some samples of the generated mesh during the analysis and Fig. 8 shows the mesh during the maximum deflection at the free end which is at 0.275 seconds, where the number of elements is 193 and the number of nodes is 232.

Although the computing efficiency is continuously increasing in general, the enormous amount of computing needed in the time domain finite element dynamic analysis of structures, computing efficiency in every aspect of the algorithm is important in order for the method to be practical (Ladeveze and Oden, 1998; MacLeod, 2002; Rafiz and Easterbrook, 2005). Table 2 shows the comparative computation times and error where it shows reasonable accuracy and extreme computational efficiency of the strategy solu-

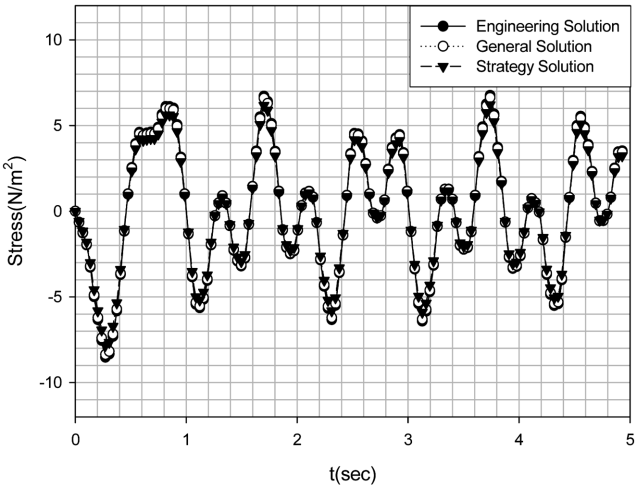



Fig. 6. Mid-shear stress at the free end

Table 1. Representative strain values for the adaptive scheme

| Step | Method | Maximum. Representative Strain(%) | Sum of Representative Strain(%) | Variation of Representative Strain(%) |
|---|---|---|---|---|
| 0 | | 0.0000297 | 0.0000659 | - |
| 1 | h | 0.0000057 | 0.0000456 | 30.81 |
| 2 | h | 0.0000011 | 0.0000253 | 44.53 |
| 3 | r | 0.0000009 | 0.0000251 | 0.72 |
| 4 | r | 0.0000008 | 0.0000251 | 0.14 |
| 5 | r | 0.0000008 | 0.0000250 | 0.16 |
| 6 | h | 0.0000002 | 0.0000147 | 41.22 |

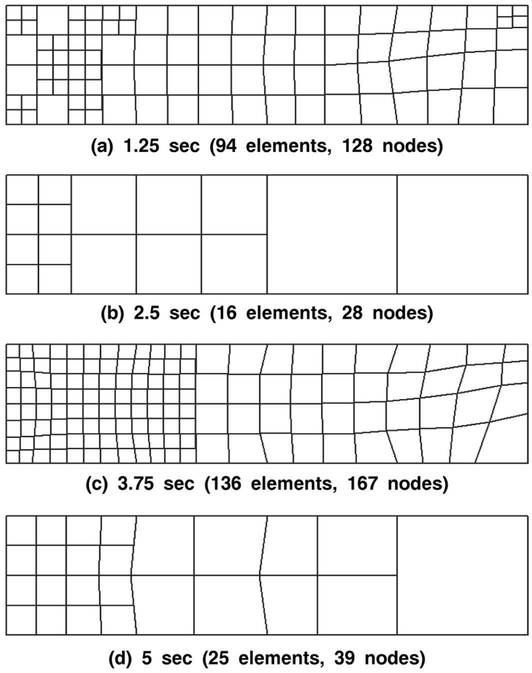
tion when compared to the engineering solution; only 8.03% error increase at 0.553% computing time. If more accurate results at the expense of increased computing time, the $\alpha$ in Eq. (16) for the discretization parameter $d$ may be reduced.

## 4. Conclusions

An adaptive mesh generation scheme for finite element analyses of complex dynamic problems in time domain is presented. The scheme uses representative strain value from each element computed from the previous time step for estimation of error and an efficient combination of the r-method and the h-method for mesh refinement. Applying the proposed scheme to a cantilever beam example, the following are the conclusions.

1) The representative strain value for estimation of error gives reasonable relative error. Since the computational requirement is small, the scheme is computationally efficient but successfully achieves the objective of identifying relative errors among the previous elements.
2) An efficient combination scheme combining the traditional r-method and the h-method is shown to be practical as checks for limiting distortions of elements are incorporated.

The proposed adaptive mesh algorithm is computationally efficient and thus the scheme is appropriate for real time computations of large complex structures under erratic time dependent loads such as earthquakes and turbulent winds, problems important in maintenance of structures and structural hazard mitigation. Some aspects of the scheme still need to be improved, one of which is the appropriate selection of the initial mesh that starts the algorithm.



**(a) 1.25 sec (94 elements, 128 nodes)**

**(b) 2.5 sec (16 elements, 28 nodes)**

**(c) 3.75 sec (136 elements, 167 nodes)**

**(d) 5 sec (25 elements, 39 nodes)**

Fig. 7. Generated meshes during analysi

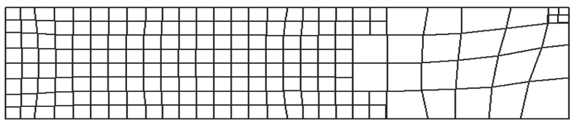


Fig. 8. Mesh during the maximum deflection at the free end (0.275 sec, 193 elements, 232 nodes)

Table 2. Comparative computation times and error

| Solution Method | Displacement Error vs Engineering(%) | Stress Error vs Engineering(%) | Analysis Time (Reduction vs Engineering in %) |
|---|---|---|---|
| Engineering | 0.00 | 0.00 | 6 hrs 4 min 10 sec (0.00) |
| Common | 0.72 | 1.84 | 8 min 14 sec (2.260) |
| Strategy | 3.11 | 8.03 | 2 min 1 sec (0.553) |

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