

# 레이리 페이딩 채널에서 차등 변조기법을 이용한 선택적 복호 후 재전송 중계 네트워크의 성능 분석

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## Performance Analysis for Selection Decode-and-Forward Relay Networks with Differential Modulation over Rayleigh Fading Channels

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### 요 약

본 논문을 통해 i.n.d 레이리 페이딩 채널에서 서로 다른 변조/복조 기법을 사용하는 중계기들로 구성된 선택적 복호 후 재전송 네트워크의 성능평가를 보여준다. 본 논문은 i.i.d와 i.n.d 레이리 페이딩 채널 모두에서 선택적 복호 후 전송 프로토콜이 최대 다이버시티를 얻을 수 있다는 것을 보여준다. 또한 SC(selection combining)기법을 사용하는 시스템과 MRC(maximal ratio combining)기법을 사용하는 시스템의 성능을 비교하여 결합기술의 효과를 연구하였다. 높은 SNR에서 시뮬레이션 결과와 수식적 분석 결과가 정확하게 일치하는 것을 보여준다.

**Key Words** : Cooperative Communication, Differential Modulation, Decode-and-forward, Maximal Ratio Combining(MRC), Selection Combining(SC)

### ABSTRACT

This paper offers performance analysis of selection decode and forward (DF) networks with differential modulation/demodulation for an arbitrary number of relays in independent but not identically distributed Rayleigh fading channels. We have shown that the selection DF protocol with differential modulation can achieve full diversity in both independent identically distributed (i.i.d.) and independent but not identically distributed (i.n.d.) Rayleigh fading channels, and the performance loss due to using non-coherent detection is not substantial. Furthermore, we study the impact of combining techniques on the performance of the system by comparing a system that uses selection combining (SC) to one that uses maximum ratio combining (MRC). Simulations are performed and show that they match exactly with analytic ones in high SNR regime.

### I. Introduction

The spatial diversity owing to the feasibility of deploying multiple antennas at both transmitter and receiver is an efficient solution to mitigate

the fading in wireless communications. However, when wireless mobiles may not be able to support multiple antennas due to size and power limitations or other constraints in wireless networks (especially, ad-hoc or sensor networks), this diver-

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sity technique is not exploited. To overcome such problem and still to obtain diversity gain, a technique is introduced that called cooperative communications. The main idea is that in a multi-user network, two or more users share their information and transmit jointly as a virtual antenna array. This enables them to obtain higher diversity than they could have individually <sup>[1]</sup>. Various protocols have been proposed to achieve the benefit offered by cooperative communication such as: amplify-and-forward (AF), decode-and-forward, coded cooperation.

In order to reduce hardware complexity at each node in the network and communication overhead among nodes, a combination of cooperative diversity schemes relaying and differential modulations has recently been introduced to obviate the need for channel state information (CSI) <sup>[2]-[7]</sup>. Specifically, in <sup>[2],[6]</sup>, new AF and DF schemes amenable to differential modulation for cooperative systems with one relay were proposed. In <sup>[7]</sup>, Laneman et al. considered maximum likelihood (ML) detection for differential modulation and derived the associated average BER and diversity order. In <sup>[3],[5]</sup>, the optimum resource allocation schemes, which minimize the system error, are studied for AF as well as DF relay networks with differential modulation.

In cooperative communication, beside amplify-and-forward, an important relaying scheme which also has attracted research interest is decode-and-forward. An adaptive version of DF with coherent modulation, in which the relays only assist the source-destination communications if the relay can correctly decode the source's messages, has been investigated in <sup>[8],[9]</sup>. With this adaptive DF strategy, the assumption of the perfect capability of decoding the cyclic redundancy check (CRC) at the relay has been applied. In contrast, for the fixed DF protocol, by relaxing this assumption the relay always decodes, re-encodes, and transmits the message, i.e., the relay does not need to check whether it's received data are right or wrong before forwarding to the destination. The performance of fixed DF relaying systems

equipped with relay selection and differential modulation in terms of diversity order over Rayleigh fading channels has been investigated in <sup>[4]</sup>. In that paper, the authors showed that if there are  $N$  relay nodes in the network to help the source to convey information to the destination, the selection DF relaying can achieve a diversity order of  $N+1$ . Also in <sup>[4]</sup>, a closed-form expression for the bit error probability of the selection DF relaying networks when the statistics of the channels between the source, relays and destination are assumed to be independent and identically distributed was provided. However, in certain environments, it may be more appropriate to consider independent but not identically distributed channels. In this paper, we present expressions for BER of the selection DF relaying networks with an arbitrary number of relays over dissimilar Rayleigh fading channels. We further investigate the performance loss of the system due to differential modulation/demodulation compared to that of coherent modulation/demodulation. Moreover, we also study the impact of combining techniques on the performance of the system by comparing a system that uses SC to one that uses MRC.

The rest of this paper is organized as follows. In Section II, we introduce the model under study. Section III shows the formulas allowing for evaluation of the average BER. Section IV, we contrast the simulations and the results yielded by theory. Finally, the paper is closed in Section V.

## II. System model

We consider a wireless relay network consisting of one source (S),  $N$  relays ( $R_i$ ) with  $i = 1, 2, \dots, N$  and one destination (D) as illustrated in Fig. 1. Each node is equipped with a single antenna and operates in half-duplex mode. It is assumed that every channel between the nodes experiences slow, flat, Rayleigh fading. For differential detection, the fading channel coefficients are assumed static over two-symbol intervals. Due to

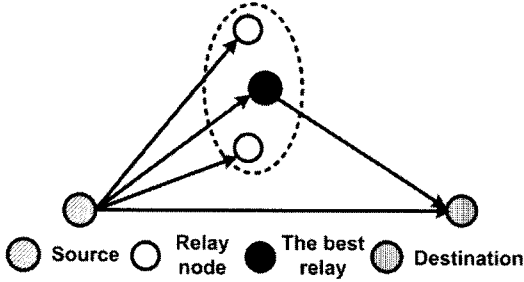


그림 1. 3개의 중계기 노드를 사용한 선택적 복호 후 전송  
Fig. 1. A Selection Decode-and-Forward Relay system with 3 relay nodes

Rayleigh fading, the channel powers denoted by  $\alpha_0 = |h_{SD}|^2$ ,  $\alpha_{1,i} = |h_{SR_i}|^2$  and  $\alpha_{2,i} = |h_{RD}|^2$  are independent and exponential random variables whose means are  $\lambda_0$ ,  $\lambda_{1,i}$  and  $\lambda_{2,i}$ , respectively where  $i = 1, 2, \dots, N$ . The average transmit powers for the source and the relays are denoted by  $\rho_1$  and  $\rho_2$ , respectively. Let us define the instantaneous signal-to-noise (SNR) for  $S \rightarrow R_i$  and  $R_i \rightarrow D$  links as  $\gamma_{1,i} = \rho_1 \alpha_{1,i}$  and  $\gamma_{2,i} = \rho_2 \alpha_{2,i}$ , respectively.

To eliminate mutual interference, the system uses orthogonal channels for transmission, either by time-division multiplexing (TDM) or by frequency-division multiplexing (FDM). To facilitate the explanation, we assume a time-division protocol with two time slots. In the first time slot, the source broadcasts the differential modulated symbols  $s(n)$  to both  $N$  relays and the destination where  $s(n)$  is defined as follows:

$$s(n) = s(n-1)d(n), \quad n = 1, \dots, L \quad (1)$$

where  $L$  denotes number of bits within one frame,  $d(n) \in \{1, -1\}$  are the information bits and the initially modulated symbol is 1, i.e.,  $s(0) = 1$ .

The received signals at the relays and the destination are written, respectively, as follows:

$$r_{SD}(n) = \sqrt{\rho_1} h_{SD} s(n) + n_{SD}(n) \quad (2a)$$

$$r_{SR_i}(n) = \sqrt{\rho_1} h_{SR_i} s(n) + n_{SR_i}(n) \quad (2b)$$

In the second time slot, relay selection is per-

formed, i.e. only the relay whose instantaneous SNR composed of the SNR across its two hops is highest is chosen for forwarding the information to the destination. To that effect, the clear-to-send (CTS) packet can be used for relay selection as suggested in [10]. Before forwarding to the destination, the received signals at the best relay are differentially decoded as

$$\hat{d}(n) = \text{sign}\{R\{r_{SR_b}^*(n-1)r_{SR_b}(n)\}\} \quad (3)$$

and re-encoded via a differential encoder as

$$\hat{s}(n) = \hat{s}(n-1)\hat{d}(n) \quad (4)$$

where  $R_b$  denotes the best relay,  $R(\cdot)$  denotes the real part of the argument,  $(\cdot)^*$  denotes complex conjugation and  $\hat{s}(0) = 1$ . Note that, owing to the imperfect detection at the best relay, it may forward incorrectly decoded signals to the destination. Hence, similarly as in [11], the relaying channel across two hops is dominated by the weaker link, and it can be modeled as an equivalent single hop whose output SNR can be tightly approximated in the high SNR regime as follows:

$$\gamma_i = \min(\gamma_{1,i}, \gamma_{2,i}) \quad (5)$$

Then, the dual hop instantaneous SNR of the best relay at the destination can be given by

$$\beta = \max_{i=1, \dots, N} \gamma_i \quad (6)$$

Under the assumption that all links are subject to independent fading, order statistics gives the cumulative distribution function (CDF) of  $\beta$  as

$$F_\beta(\gamma) = \Pr(\gamma_1 < \gamma, \dots, \gamma_N < \gamma) = \prod_{i=1}^N F_{\gamma_i}(\gamma) \quad (7)$$

where  $F_{\gamma_i}(\gamma) = P_r(\gamma_i < \gamma)$  is the corresponding CDF of  $\gamma_i$ . Hence, the joint pdf of is given by

differentiating (7) with respect to  $\gamma$ .

$$f_{\beta}(\gamma) = \frac{\partial F_{\beta}(\gamma)}{\partial \gamma} = \sum_{i=1}^N \left[ f_{\gamma_i}(\gamma) \prod_{\substack{j=1 \\ j \neq i}}^N F_{\gamma_j}(\gamma) \right] \quad (8)$$

Since  $\gamma_{1,i}$  and  $\gamma_{2,i}$  are exponentially distributed random variables with hazard rates  $\mu_{1,i} = 1/\overline{\gamma_{1,i}} = 1/(\rho_1 \lambda_{1,i})$  and  $\mu_{2,i} = 1/\overline{\gamma_{2,i}} = 1/(\rho_2 \lambda_{2,i})$ , respectively. From (5), it follows from the fact that the minimum of two independent exponential random variables is again an exponential random variable with a hazard rate equals to the sum of the two hazard rates <sup>[12]</sup>, i.e.,  $\mu_i = \overline{\gamma_i}^{-1} = \overline{\gamma_{1,i}}^{-1} + \overline{\gamma_{2,i}}^{-1}$ . Hence, we have

$$f_{\gamma_i}(\gamma) = \frac{1}{\gamma_i} e^{-\gamma/\gamma_i}, F_{\gamma_j}(\gamma) = 1 - e^{-\gamma/\gamma_j} \quad (9)$$

Substituting (9) into (8) gives us

$$f_{\beta}(\gamma) = \sum_{i=1}^N \left[ \frac{1}{\gamma_i} e^{-\frac{\gamma}{\gamma_i}} \prod_{\substack{j=1 \\ j \neq i}}^N (1 - e^{-\frac{\gamma}{\gamma_j}}) \right] \quad (10)$$

$$= \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i = 1 \\ n_1 < \dots < n_i}} \omega_i e^{-\gamma \omega_i}$$

where  $\omega_i = \sum_{l=1}^i \overline{\gamma_{l,1}}^{-1}$ .

The signal received at D at the second time slot is given by

$$\begin{aligned} r_{R_b,D}(n) &= \sqrt{\rho_2} h_{R_b,D} \hat{s}(n) + n_{R_b,D}(n) \\ &= \sqrt{\rho_2} h_{R_b,D} \hat{s}(n-1) \hat{d}(n) + n_{R_b,D}(n) \quad (11) \\ &= r_{R_b,D}(n-1) \hat{d}(n) + \widetilde{n}_{R_b,D}(n) \end{aligned}$$

where  $\widetilde{n}_{R_b,D}(n) = n_{R_b,D}(n) - n_{R_b,D}(n-1) \hat{d}(n)$ .

Finally, the destination combines the received signals from both the best relay and the source by using maximal ratio combining (MRC) technique to recover the signals. In fact, MRC performs a maximum likelihood detection operation on the received signals. Here, we take the imperfect de-

coding effect at the relays into consideration; hence, the output of combiners is modified by considering the SNR of the equivalent link as follows <sup>[21]</sup>:

$$\widetilde{d} = \text{sign} \left( R \left( r_{SD}^*(n-1) r_{SD}(n) + \left\{ \frac{\beta}{\gamma_{R_b,D}} r_{R_b,D}^*(n-1) r_{R_b,D}(n) \right\} \right) \right) \quad (12)$$

### III. Performance analysis

For differential BPSK modulation, the conditional BEP is given by [13, Eq. (12.1-13)]

$$P_b(\gamma_{eq}) = \frac{1}{8} (4 + \gamma_{eq}) e^{-\gamma_{eq}} \quad (13)$$

The bit error probability of D-BPSK modulation scheme in slow and flat Rayleigh fading channels can be derived by averaging the error probability for the AWGN channel over the pdf of the SNR in Rayleigh fading.

$$\overline{P}_b = \int_0^{\infty} P_b(\gamma) f_{\gamma_{eq}}(\gamma) d\gamma \quad (14)$$

Since maximal ratio combining technique is employed at the destination, the combined instantaneous SNR at the output of the maximal ratio combiner is given by

$$\gamma_{eq} = \gamma_0 + \beta \quad (15)$$

Assuming the independence of  $\gamma_0$  and  $\beta$ , the moment generating function (MGF) of  $\gamma_{eq}$  can be written as

$$M_{\gamma_{eq}}(s) = M_{\gamma_0}(s) M_{\beta}(s) \quad (16)$$

where  $M_{\gamma_0}(s)$  and  $M_{\beta}(s)$  are the MGFs of  $\gamma_0$  and  $\beta$ , respectively. Using the definition of the MGF as  $M_X(s) = E(e^{sX})$  [14, Eq. (2.4)] where

$E(\cdot)$  is statistical average operator, we have

$$M_{\gamma_0}(s) = (1 - s\gamma_0)^{-1} \quad (17a)$$

$$M_{\gamma_j}(s) = \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N (1 - sw_i^{-1})^{-1} \quad (17b)$$

Substituting (17) into (16) and using the partial fraction expansion of the MGF [15, chapter 9], (16) can be rewritten as Eq. (18) shown at the bottom of this page where

$$u(a, b) = \begin{cases} 1 & a \neq b \\ 0 & a = b \end{cases}, v(a, b) = \begin{cases} 0 & a \neq b \\ 1 & a = b \end{cases} \quad (19)$$

Finally, the joint pdf of  $\gamma_{eq}$  is determined by the inverse Laplace transform of  $M_{\gamma_{eq}}$  as Eq. (20) also shown at the bottom of this page.

With Rayleigh fading channels, using (13), (14) and (20), we have the average bit error rate of the system as.

$$\bar{P}_b = \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N \left\{ \begin{array}{l} u(\bar{\gamma}_0, \bar{\omega}_i^{-1}) * \\ \left[ \left( \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \bar{\omega}_i^{-1}} \right) I(\bar{\gamma}_0) \right. \\ + \\ \left. \left( \frac{\bar{\omega}_i^{-1}}{\bar{\omega}_i^{-1} - \bar{\gamma}_0} \right) I\left(\frac{1}{\bar{\omega}_i}\right) \right] \\ + v(\bar{\gamma}_0, \bar{\omega}_i^{-1}) J(\bar{\gamma}_0) \end{array} \right\} \quad (21)$$

where  $I(\alpha)$  and  $J(\alpha)$  are defined as follows:

$$\begin{aligned} M_{\gamma_{eq}}(s) &= (1 - s\gamma_0)^{-1} \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N [(1 - sw_i^{-1})^{-1}] \\ &= \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N [(1 - sw_i^{-1})^{-1} (1 - s\gamma_0)^{-1}] \\ &= \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N \left\{ u(\bar{\gamma}_0, \bar{\omega}_i^{-1}) \left[ \left( \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \bar{\omega}_i^{-1}} \right) (1 - s\gamma_0)^{-1} + \left( \frac{\bar{\omega}_i^{-1}}{\bar{\omega}_i^{-1} - \bar{\gamma}_0} \right) (1 - sw_i^{-1})^{-1} \right] + v(\bar{\gamma}_0, \bar{\omega}_i^{-1}) (1 - s\gamma_0) \right\} \end{aligned} \quad (18)$$

$$f_{\gamma_{eq}}(\gamma) = \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N \left\{ u(\bar{\gamma}_0, \bar{\omega}_i^{-1}) \left[ \left( \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \bar{\omega}_i^{-1}} \right) \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} + \left( \frac{\bar{\omega}_i^{-1}}{\bar{\omega}_i^{-1} - \bar{\gamma}_0} \right) \bar{\omega}_i e^{-\gamma \bar{\omega}_i} \right] + v(\bar{\gamma}_0, \bar{\omega}_i^{-1}) \frac{\gamma}{\gamma_0^2} e^{-\frac{\gamma}{\gamma_0}} \right\} \quad (20)$$

$$\begin{aligned} I(\alpha) &= \int_0^\infty \frac{1}{8} (4 + \gamma) e^{-\gamma} \frac{1}{a} e^{-\frac{\gamma}{a}} d\gamma \\ &= \frac{4 + 5a}{8(1 + a)^2} \end{aligned} \quad (22a)$$

$$\begin{aligned} J(a) &= \int_0^\infty \frac{1}{8} (4 + \gamma) e^{-\gamma} \frac{\gamma}{a^2} e^{-\frac{\gamma}{a}} d\gamma \\ &= \frac{(2 + 3a)}{4(1 + a)^3} \end{aligned} \quad (22b)$$

#### IV. Numerical results and discussion

In this section, computer simulations are carried out in order to verify the proposed analytical expressions. For a fair comparison to direct transmission, the uniform power allocation is employed in order to keep the total power constraint, i.e.,  $\rho_1 = \rho_2 = \rho/2$  where  $\rho$  is the transmit power of the source in case of direct transmission. This is due to the fact that for the selection DF relaying case, only the best relay is involved in the cooperative transmission. Fig. 2 shows the average bit error rate of the selection decode-and-forward relay network for D-BPSK with different number of cooperative nodes. The channel setup in this case is investigated as follows:  $\lambda_0 = 2$ ,  $\{\lambda_{1,i}\}_{i=1}^N = 1$  and  $\{\lambda_{1,i}\}_{i=1}^N = 3$  for all  $N$ . As shown in Fig. 2, the improvement of the BER is proportional to the number of relays. In addition, it can be seen that the agreement between analytic and simulation results is remarkable close at high SNR regime. However, with low SNR regime, there is a small gap between simulation results and ana-

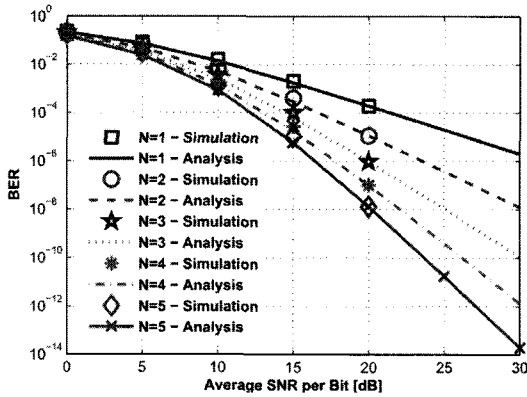


그림 2. 중계기의 개수를 다르게 하여 얻은 D-BPSK를 이용한 선택적 복호 후 전송 중계기 네트워크의 성능 그래프  
Fig. 2. Performance of Selection Decode-and-Forward Relay Networks with D-BPSK for different number of relays

lytical results due to the imperfect detection of fixed decode-and-forward protocol at the relays.

In Fig. 3, the performance of the system in both i.i.d. and i.n.d. channels are examined. The results are based on the assumption as follows: for i.i.d. case: all  $\lambda_0, \lambda_{1,i}$  and  $\lambda_{2,i}$  are

set to be 4 and for i.n.d. case:  $\lambda_0 = 1, \{\lambda_{1,i}\}_{i=1}^N = 2$  and  $\{\lambda_{2,i}\}_{i=1}^N = 3$ . The performance improvement by using more relays is impressive for both cases of i.i.d. and i.n.d. channels. For example, under same channel conditions and at

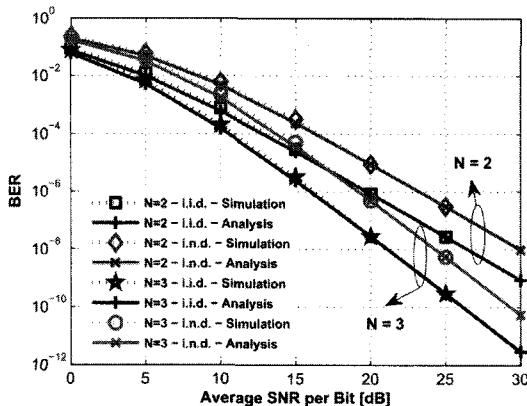


그림 3. I.i.d 와 I.n.d 레일리 페이딩 채널에서의 D-BPSK를 사용한 선택적 복호 후 전송 BER 그래프  
Fig. 3. BER of the DF relaying systems with D-BPSK under i.i.d. and i.n.d. Rayleigh fading channels

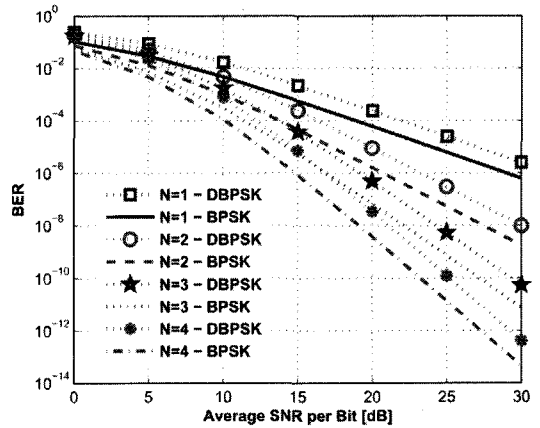


그림 4. 각기 다른 변조/복조 기법을 사용한 복호 후 전송 중계기 시스템의 BER 성능 그래프  
( $\lambda_0 = 1, \{\lambda_{1,i}\}_{i=1}^N = 2$  and  $\{\lambda_{2,i}\}_{i=1}^N = 3$ )  
Fig. 4. BER of the DF relaying systems with difference modulation/demodulation schemes  
( $\lambda_0 = 1, \{\lambda_{1,i}\}_{i=1}^N = 2$  and  $\{\lambda_{2,i}\}_{i=1}^N = 3$ )

the target BER of  $10^{-6}$ , we can see that the system with 3 relays in both cases outperform that with 2 relays around 3 dB.

In Fig. 4, we inspect how the modulation schemes affect the system performance. For comparison purposes, the closed form expression for the system with BPSK modulation is derived in [the Appendix A]. We can see that the modulation scheme does not change the diversity order of the system. Furthermore, it can be concluded that the BER of differential BPSK is around 3 dB worse in average fading SNR than that of BPSK. It is interesting to know that this is the same conclusion reached when comparing these two modulation/demodulation schemes over AWGN channels.

In Fig. 5, the performances of the selection DF relaying systems with difference diversity combining techniques at the destination are illustrated. Between them, the system with SC [see Appendix B] gives the worst performance. In addition, BER curves confirm that, under same channel conditions, the performance of a system employing MRC receiver is always better as compared to an equivalent system using SC by around 2 dB.

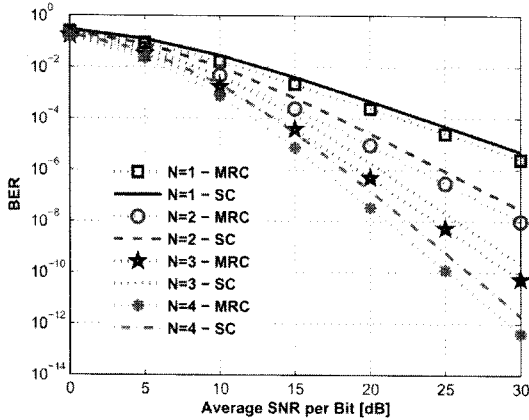


그림 5. 서로 다른 다이버시티 결합 기술을 사용한 D-BPSK 복호 후 전송 시스템의 수신단에서의 BER 그래프 ( $\lambda_0 = 1$ ,  $\{\lambda_{1,i}\}_{i=1}^N = 2$  and  $\{\lambda_{2,i}\}_{i=1}^N = 3$ )

Fig. 5. BER of the DF relaying systems for D-BPSK with difference diversity combining techniques at the destination

## V. Conclusion

The performance of selection DF relaying networks with differential modulation using SC or MRC technique at the destination under both i.i.d. and i.n.d. Rayleigh channels was examined in this paper. Simulation results are in excellent agreement with the derived results at high SNR regime.

Furthermore, a closed-form BER expression of the system for BPSK modulation was also established to facilitate in comparison. It can be seen that the performance loss due to non-coherent detection is not much. Hence, the combination of non-coherent detection and selection DF relaying, which utilizes both advantages of hardware complexity reducing at each node offered by differential modulation/demodulation and full diversity order offered by relay selection, could be gained actuality in cooperative wireless communication systems.

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### Appnedix

#### A. BEP of the system for BPSK scheme

The purpose of the appendix A is to derive the error probability which corresponds to coherent BPSK modulation to facilitate the comparison, but generalization to other modulation scheme is straightforward. More specifically, the bit error probability of the system for coherent BPSK scheme is

$$\begin{aligned} \bar{P}_b &= \int_0^\infty Q(\sqrt{2\gamma}) f_{\gamma_{eq}}(\gamma) d\gamma \\ &= \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N \left\{ u(\bar{\gamma}_0, w_i^{-1}) \times \right. \\ &\quad \left. \left[ \begin{aligned} &\left( \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - w_i^{-1}} \right) I_1(\bar{\gamma}_0) \\ &+ \left( \frac{w_i^{-1}}{w_i^{-1} - \bar{\gamma}_0} \right) I_1\left(\frac{1}{w_i}\right) \\ &+ v(\bar{\gamma}_0, w_i^{-1}) J_1(\bar{\gamma}_0) \end{aligned} \right] \right\} \quad (23) \end{aligned}$$

where  $Q(x)$  is the Gaussian Q-function defined ad  $Q(x) = 1/\sqrt{2\pi} \int_x^\infty e^{-t^2/2} dt$ . We further define  $I_1(\bar{\gamma}_0)$  and  $J_1(\bar{\gamma}_0)$  [13, Eq.(14.4-14)] as follows:

$$\begin{aligned} I_1(a) &= \int_0^\infty Q(\sqrt{2\gamma}) \frac{1}{a} e^{-\frac{\gamma}{a}} d\gamma \\ &= \frac{1}{2} \left( 1 - \sqrt{\frac{a}{1+a}} \right) \quad (24a) \end{aligned}$$

$$\begin{aligned} J_1(a) &= \int_0^\infty Q(\sqrt{2\gamma}) \frac{\gamma}{a^2} e^{-\frac{\gamma}{a}} d\gamma \\ &= \frac{1}{4} \left( 1 - \sqrt{\frac{a}{1+a}} \right)^2 \left( 2 + \sqrt{\frac{a}{1+a}} \right) \quad (24b) \end{aligned}$$

#### B. BEP of the system for Selection Combining at the destination

Since SC technique is used, the combined instantaneous SNR at the output of the selection combiner is written by

$$\gamma_{eq} = \max\{\gamma_0, \beta\} \quad (25)$$

From the elementary theory of order statistics, we can obtain the joint pdf for  $\gamma_{eq}$  [12, Eq.(6-79)] as follows:

$$\begin{aligned} f_{\gamma_{eq}}(\gamma) &= F_{\gamma_0}(\gamma) f_\beta(\gamma) + f_{\gamma_0}(\gamma) F_\beta(\gamma) \\ &= \sum_{i=1}^N (-1)^{i-1} \sum_{\substack{n_1, \dots, n_i=1 \\ n_1 < \dots < n_i}}^N \left[ \begin{aligned} &\frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} + \\ &w_i e^{-\gamma w_i} - \\ &\left( \left( w_i + \frac{1}{\gamma_0} \right)^* \right) \\ &e^{-\gamma \left( w_i + \frac{1}{\gamma_0} \right)} \end{aligned} \right] \quad (26) \end{aligned}$$

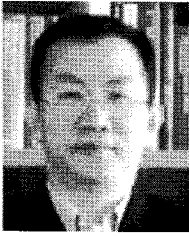
where  $F_{\gamma_0}(\gamma)$  and  $F_\beta(\gamma)$  are the CDFs of  $\gamma_0$  and  $\beta$ , respectively. Then, for this case, from (13), (14) and (26), the average bit error rate of the system for the case of SC can be written as

$$\bar{P}_b = \sum_{i=1}^N (-1)^{i-1} \sum_{n_1, \dots, n_i=1}^N \left[ \begin{aligned} &I(\bar{\gamma}_0) + I(w_i^{-1}) \\ &- I\left(\frac{\bar{\gamma}_0}{1 + \bar{\gamma}_0 w_i}\right) \end{aligned} \right] \quad (27)$$



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