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The Impacts of the Number of Suppliers on Inventory Management in a Make-to-order Manufacturer

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공급업체 수가 주문 생산 제조 기업의 재고 관리에 미치는 영향 분석

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We consider a supply chain consisting of a make-to-order manufacturer and N component suppliers and study the impacts of the number of suppliers on component inventory management. The manufacturer has full information and continuously observes the state of both component inventory level and customer backorders. Based on this information, the manufacturer determines whether or not to place a component purchasing order to a supplier among N suppliers even though some orders are in process by other suppliers. The goal of this paper is to numerically identify the manufacturer's purchasing policy which minimizes the total supply chain cost and the best choice of N. Our model contributes to the current literature in that the problem of simultaneously considering multiple outstanding orders and incorporating order setup cost into the model has not been covered yet. From numerical experiment, we investigate how much the policy with N suppliers can contribute to reducing the supply cost compared to the policy with a single supplier.

Keyword: multiple suppliers, multiple outstanding orders, make-to-order, supply chain management, setup cost, purchasing

1. Introduction

In this paper, we consider a supply chain consisting of a customized end item manufacturer and N component suppliers. The manufacturer faces customer orders from the market with the option to accept or reject and processes accepted customer orders using a single type of component. The manufacturer has full information and continuously observes the state of both component inventory level and customer backorder level, and the number of purchasing orders which are in process by suppliers. Based on this information, the manufacturer determines whether or not to place a purchasing order to a supplier that is not processing the manufacturer's purchasing order. In the context of inventory control, there can be multiple outstanding orders, since there are N suppliers. The goal of this paper is to identify the manufacturer's purchasing policy which minimizes the supply chain cost (consisting of customer order rejection, customer backorder, order setup, and component inventory holding), to find the best choice of N, and to examine the beneficial effects of having N

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suppliers rather than a single supplier.

The model presented in this paper has the operating feature similar to a make-to-order inventory-production (MTOPI) system in the literature. MTOPI system is concerned with a supply chain that is made up of a production facility and a raw material supplier. The production facility manufactures products on a make-to-order basis to meet customer demands using raw materials purchased from a supplier. Since the production facility carries no finished goods inventory, thus, inventory control is necessary only for raw materials replenishment. The unique aspect of MTOPI system compared to other inventory systems is that there is a delay between the arrival of a demand and the time raw materials are actually needed to produce a product for that demand. This is a result of explicitly modeling the production process as a queue. Consequently, inventory control in the production facility will depend on both the inventory level and the number of customer backorders, instead of just the inventory level (or inventory position) as in classical inventory control models (He *et al*., 2002).

He and Jewkes (2000) studied order-up-to purchasing policies when inventory is empty, and showed that the order size is the economic order quantity. In He and Jewkes (2000), lead time is not considered. He *et al*. (2002) considered a system with Poisson arrivals and exponential lead times, and studied a policy with varying order quantity according to the backorder level when the inventory becomes empty. The papers of jointly considering admission control and purchasing policy are very limited in the MTOPI literature. Berman and Kim (1999) characterized an optimal purchasing policy when admission control on customer order is based on a threshold and lead time is zero. Kim (2005) extended to the case with exponential lead time and identified an optimal purchasing policy as a dynamic reorder point which is a function of the size of customer backorders. However, all MTOPI references in the above assume that at most one purchasing order is outstanding.

Since customer order can be rejected and inventory is reviewed continuously, our model has some features similar to classical continuous review inventory models with lost sales. However, references in this stream has the constraint that at most one replenishment order may be outstanding at any time is almost universally applied. Archibald (1981) presented optimal and approximate solution procedures for an (s, S) model with compound Poisson demand and a fixed lead time. Buchanan and Love (1985) derived a solution procedure for the (r, Q) model with Poisson demand and an Erlang-distributed lead time. Beckmann and Srinivasan (1987) analysed the Poisson demand (s, S) model with exponential lead time as a queueing model. Mohebbi and Posner (1998) considered the behaviour of (r, Q) policies for compound Poisson demand models with Erlang or hyper-exponential lead time distributions. Hill and Johansen (2006) explored the behaviour of optimal inventory control policies under both continuous and periodic review.

In classical periodic review inventory models with lost sales, some references analyzed the case that more than one order may be outstanding at anytime. Morton (1971), Nahmias (1979) and, Van Donselaar *et al*. (1996) developed myopic heuristic solution procedures under a fixed lead time which is an integral number of review periods and a negligible set-up cost. When demand is discrete, Johansen (2001) explores optimal and near optimal base stock policies with negligible set-up costs and constant lead times.

In the context of already-published work in this area, the contribution of this paper can be established in the following two essential aspects. First, compared to MTOPI models and classical continuous review inventory models with lost sales, we study the issue of inventory management with multiple outstanding orders rather than at most single outstanding order. Under the mechanism of multiple outstanding orders, there exist multiple reorder points which depend on both the inventory level and the customer backorder level. In contrast, classical continuous review inventory models with lost sales have a single reorder point depending on just the inventory level. Second, we explicitly incorporate purchasing setup cost into the model and examine its impact on the optimal purchasing policy. Even though some classical periodic review inventory models with lost sales consider multiple outstanding orders, the setup cost is not included into the models.

The rest of the paper is organized as follows. We present a problem description in the next section. In Section 3, we provide a formulation of our model. Section 4 presents a numerical procedure that finds the optimal cost and characterizes the optimal purchasing policy based on numerical investigation. Section 5 discusses the results of numerical experiment and the last section states conclusions.

2. The Problem

The manufacturer faces stochastic customer orders from the market and rejects them if the number of customer orders in service delay (i.e. backordered) reaches a threshold, B. Whenever each customer order is rejected, a penalty cost of c_R is incurred. A backorder cost is assessed at rate c_1 for each customer order in service delay. The manufacturer manufactures accepted customer orders one by one using component. Each customized manufacturing requires one unit of component and takes random amounts of time with mean μ^{-1}.

Components are purchased from the group of N suppliers whose lead times are randomly distributed with mean d^{-1}. At each instant a purchasing order with Q units of component is placed to a supplier, a setup cost of c_K is incurred. Even when purchasing orders are in process by suppliers, the manufacturer can place a new purchasing order, depending on the inventory status. The maximum number of outstanding purchasing orders is limited to N (since there are N suppliers). For each unit of component in inventory, the manufacturer pays a holding cost incurred at rate c_2. <Figure 1> illustrates a schematic model of the problem described above.

3. The Model and Formulation

To provide insights into the effective inventory control, we model the problem using an Markov Decision Process (MDP). We assume that customer order process follows a Poisson process, and both customized manufacturing and component lead time processes have an exponential distribution.

Denote x_1 and x_2 the size of customer backorders and component inventory level, respectively. The system state is described by the vector (x_1, x_2, n). The indicator n describes the status of component lead time process. More specifically, $n>0$ implies that n purchasing orders are in process while $n=0$ means no purchasing orders are in process. The following state transitions can be considered in state (x_1, x_2, n) :

- Customer order arrival : When $x_1 < B$, the size of customer backorders increases by one, otherwise, it remains the same.
- Customized manufacturing completion : The size of customer backorders and the inventory level decrease by one, respectively.
- Arrival of purchasing order with size Q : The inventory level increases by Q and the indicator n decreases by one.
- Component purchasing order placement : The indicator n increases by one.

Let $\gamma \equiv \lambda + \mu + Nd$. We note that the expected state transition time is $1/\gamma$, since all transition processes are exponential. In the following, we define the value iteration (VI) operator T corresponding to the above state transitions :

$$Tv(x_1, x_2, n) = \begin{cases} \min\left\{ \begin{array}{l} T_u v(x_1, x_2, n), \\ T_p v(x_1, x_2, n) \end{array} \right\}, & \text{if } 0 \le n \le N-1 \\ T_u v(x_1, x_2, n), & \text{if } n = N \end{cases} \quad (1)$$

In (1), $T_u v(x_1, x_2, n)$ is the value function when purchasing order is not placed in state (x_1, x_2, n) and it is given by

$$\begin{aligned} T_u v(x_1, x_2, n) = {} & (c_1 x_1 + c_2 x_2)/\gamma \\ & + \mu/\gamma \{ v(x_1 - 1, x_2 - 1, n) 1(x_1 x_2 > 0) \\ & \quad + v(x_1, x_2, n) 1(x_1 x_2 = 0) \} \\ & + \lambda/\gamma \{ v(x_1 + 1, x_2, n) 1(x_1 < B) \\ & \quad + (v(x_1, x_2, n) + c_R) 1(x_1 = B) \} \end{aligned}$$

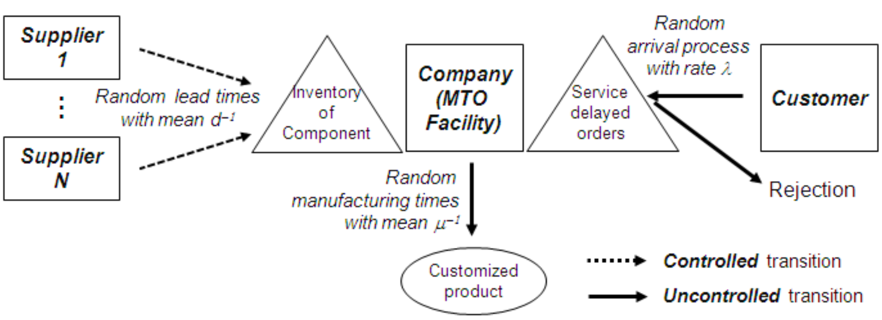

Figure 1. A supply chain model consisting of a MTO manufacturer and N suppliers

$$+\begin{cases} Nd/\gamma\, v(x_1,\, x_2,\, n) & \text{if } n=0 \\ nd/\gamma\, v(x_1,\, x_2+Q,\, n-1) \\ \quad +(N-n)d/\gamma\, v(x_1,\, x_2,\, n) & \text{if } n>0 \end{cases} \quad (2)$$

$T_p v(x_{1,}\, x_{2,}\, n)$ is the value function when a purchasing order with size Q is placed in state (x_1, x_2, n) and it is given by

$$T_p v(x_1,\, x_2,\, n) = c_K + T_u v(x_1,\, x_{2,}\, n+1). \quad (3)$$

Since the maximum number of outstanding orders is N, a purchasing order cannot be placed in state (x_1, x_2, N). In (2), $(c_1x_1+c_2x_2)/\gamma$ is the expected backorder and holding costs occurred during $1/\gamma$, the terms multiplied by λ/γ represent penalties and transitions generated with the arrival of customer order, the term multiplied by μ/γ implies transition associated with customized manufacturing, and the terms multiplied by d/γ imply transitions associated with purchasing order process. From ch. 8 in Puterman (2005), the Bellman's equation for the average cost Markov decision problem is

$$g(N)+v(x_1,\, x_2,\, n) = Tv(x_1,\, x_2,\, n), \quad 0 \le n \le N \quad (4)$$

where $g(N)$ is the optimal average cost during $1/\gamma$, given N.

To find the best value of N, we solve the following equation:

$$N^* = \arg min_N\ g(N). \quad (5)$$

If N^* is found using (5) via (4), then the optimal purchasing policy in state $(x_1,\, x_2,\, n)$, $0 \le n \le N^*$, is determined through the optimal solution procedure presented in the next section.

4. The Optimal Policy and Its Structural Properties

In this section, we investigated the structural properties of the optimal purchasing policy based on numerical study. We first show that there exists an optimal stationary policy.

Theorem 1 : For the average cost MDP problem defined in (4) with infinite state space can be transformed into new one with finite state space.

Proof : To show this theorem, we apply the same arguments as Carr and Duenyas (2000) used in proving Theorem 1 of their paper. The value x_2c_2/γ is the cost of holding x_2 components in inventory during the expected transition time, $1/\gamma$. When $x_2c_2/\gamma > c_K$, the manufacturer already has a sufficiently large number of components in inventory. Hence, the manufacturer does not have any motivation to place a purchasing order with Q in state $(x_1,\, x_2,\, n)$. This argument limits the maximum size of component inventory to $x_2 \le c_K\gamma/c_2$. We also have $x_1 \le B$ by problem assumption and the result follows.

Theorem 2 : When the average cost MDP problem defined in (4) has a finite state space, there exists a stationary policy that is average cost optimal.

Proof : It can be easily seen that the finite MDP problem generates an irreducible aperiodic Markov chain. In other words, every state in the state space communicate each other and there are no transient and absorbing states. Furthermore, the action space is finite because there are at least two control actions applied to state $(x_1,\, x_2,\, n)$. Therefore, the result follows from Theorem 8.4.5. in Puterman (2005).

Now consider a VI algorithm (see ch. 8 in Puterman, 2005) to solve for (4) :

$$v^{k+1}(x_1,\, x_2,\, n) = Tv^k(x_1,\, x_2,\, n),\ n \ge 0 \quad (6)$$

where

$$Tv^k(x_{1,}\, x_{2,}\, n) = \begin{cases} \min\left\{ \begin{array}{l} T_u v^k(x_1,\, x_2,\, n), \\ T_p v^k(x_1,\, x_2,\, n) \end{array} \right\}, & \text{if } 0 \le n \le N-1 \\ T_u v^k(x_{1,}\, x_{2,}\, n), & \text{if } n = N \end{cases} \quad (7)$$

where

$$\begin{aligned} T_u v^k(x_1,\, x_2,\, n) &= (c_1x_1+c_2x_2)/\gamma \\ &+ \mu/\gamma\left\{ v^k(x_1-1,\, x_2-1,\, n)1(x_1x_2>0) \right. \\ &\quad \left. + v^k(x_1,\, x_2,\, n)1(x_1x_2=0) \right\} \\ &+ \lambda/\gamma\left\{ v^k(x_1+1, x_2, n)1(x_1<B) \right. \end{aligned}$$

$$+ (v^k(x_1, x_2, n) + c_R)1(x_1 = B)\}$$

$$+ \begin{cases} Nd/\gamma\, v^k(x_1, x_2, n) & \text{if } n = 0 \\ nd/\gamma\, v^k(x_1, x_2 + Q, n-1) \\ \quad + (N-n)d/\gamma\, v^k(x_1, x_2, n) & \text{if } n > 0 \end{cases} \quad (8)$$

$$T_p v^k(x_1, x_2, n) = c_K + T_u v^k(x_1, x_2, n+1), \quad (9)$$

and $v^0(x_1, x_2, n) = 0$ for every state (x_1, x_2, n). Here $v^k(x_1, x_2, n)$ is the optimal value function in state (x_1, x_2, n) when the problem is terminated after the k^{th} iteration of VI algorithm. VI algorithm for the average cost MDP approximately finds a constant to which component-wide deviations of Tv^k and v^k converge and this constant is known to be the optimal average cost. If VI algorithm stops at the $(l+1)^{th}$ iteration, the optimal value function in state (x_1, x_2, n) is given by $v(n_1, n_2, n) = v^l(n_1, n_2, n)$ and the optimal average cost $g(N)$ is given by the average of the maximum and the minimum of component-wide deviations of Tv^l and v^l.

In the following, we present a numerical procedure which given N and Q, jointly finds the optimal value function v and the optimal average cost $g(N)$. Let ϵ be the termination criterion of the numerical procedure.

<Optimal solution procedure>

① **Initialization :** Set $k = 0$, and for each state (x_1, x_2, n), pick the value function $v^0(x_1, x_2, n) = 0$.

② **Value iteration step** : Implement a VI on the current value function estimate v^k :

$$Tv^k(x_1, x_2, n) = \begin{cases} \min\left\{ \begin{array}{l} T_u v^k(x_1, x_2, n), \\ T_p v^k(x_1, x_2, n) \end{array} \right\}, & \text{if } 0 \le n \le N-1 \\ T_u v^k(x_1, x_2, n), & \text{if } n = N \end{cases}$$

where the operators $T_u v^k$ and $T_p v^k$ are defined in (7) and (8).

③ **Termination test** : Perform the following convergence test:

$$\underline{b_k} = \min_{(x_1, x_2, n)}\{Tv^k(x_1, x_2, n) - v^k(x_1, x_2, n)\} \quad (12)$$

$$\overline{b_k} = \max_{(x_1, x_2, n)}\{Tv^k(x_1, x_2, n) - v^k(x_1, x_2, n)\} \quad (13)$$

If $(\overline{b_k} - \underline{b_k}) \ge \epsilon$, for every state (x_1, x_2, n), let

$$v^{k+1}(x_1, x_2, n) = Tv^k(x_1, x_2, n) \quad (14)$$

Then, increase k by one and go to *Value iteration step*. If $(\overline{b_k} - \underline{b_k}) < \epsilon$, go to *Evaluation step*.

④ **Evaluation step :** Let the optimal value function be

$$v(x_1, x_2, n) = Tv^k(x_1, x_2, n)$$

and the optimal average profit be

$$g(N) = (\overline{b_k} + \underline{b_k})/2.$$

Stop the procedure.

Since we cannot prove the convexity of $g(N)$ with respect to N, we implement the one dimensional search to find N^* which minimizes $g(N)$ (see (5)). In other words, we iterate the above numerical procedure over the set of possible values of N and find a certain value of N which achieves the best performance.

In the following theorem we show that $v^k(x_1, x_2, n)$ converges to $v(x_1, x_2, n)$ and (4) has a well-defined solution.

Theorem 3 : There exist an integer J, a constant $g(N)$, and a function v such that

$$v^{kJ+j}(x_1, x_2, n) - (kJ + j)g(N) \to v(x_1, x_2, n)$$

for all $j = 0, \cdots, J-1$ as $k \to \infty$.

Proof : Since the original problem can be transformed into a finite state problem from Theorem 1, the results follow by Theorem 8.4.1. in Puterman (2005).

Numerical investigation suggests that the optimal purchasing policy has the following set of the structural properties even though we cannot prove it:

Conjecture. (i) Let

$$r(x_1, n) := \min_{n < N}\left\{ x_2 : \begin{array}{l} T_u v(x_1, x_2, n) \\ \le T_p v(x_1, x_2, n) \end{array} \right\}, \quad (10)$$

Then, given x_1 and x_2, it is optimal to purchase Q units of component if $x_2 < r(x_1, n)$. Otherwise, the purchasing order should not be placed.

(ii) $r(x_1, n) \le r(x_1 + 1, n)$, i.e., $r(x_1, n)$ is increasing in x_1 given n.

(iii) $r(x_1, n) \geq r(x_1, n+1)$, i.e., $r(x_1, n)$ is decreasing in n given x_1.

Property (i) states that the purchasing decision point is managed by N switching curves which are the function of the size of customer backorders. Optimal switching curve $r(x_{1,} n)$ in (10) can be derived through the optimal value function found by the evaluation step of the optimal solution procedure. Properties (ii) and (iii) say that the switching curve increases in the size of customer backorders but decreases in the number of outstanding orders. To illustrate the structure of the optimal purchasing policy, we compute a numerical example with $c_K = 400$, $c_R = 40$, $c_1 = 3$, $c_2 = 1$, $\lambda = 0.3$, $\mu = 1$, $d = 0.01$, $B = 15$, and $N = 4$. The purchasing order quantity is set to the economic order quantity (EOQ); $Q = 16$. <Figure 2> shows that the optimal switching curves depend on the system status. For example, when $x_1 = 10$ and $x_2 = 5$, it is optimal to place a purchasing order if $n \leq 2$. When $x_1 = 10$ and $x_2 = 17$, it is optimal to place a purchasing order if $n = 0$. <Figure 2> confirms that $r(x_1, n)$ is increasing in x_1 given n and decreasing in n given x_1.

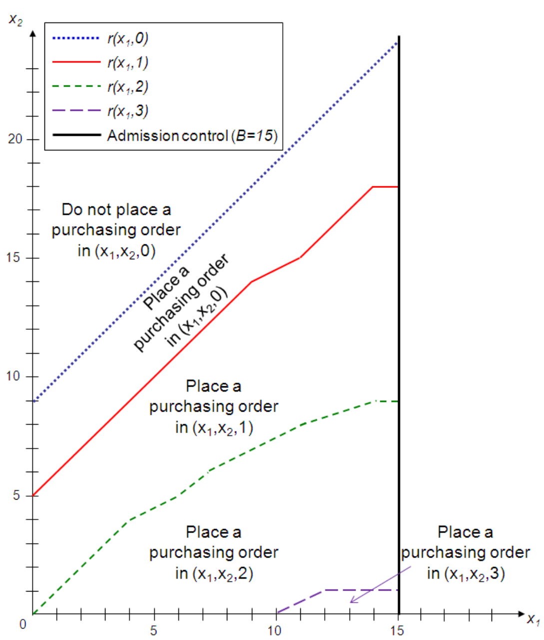

Figure 2. Illustration of the optimal switching curves

5. The Numerical Analysis

In this section, we investigate how much the purchasing policy with N suppliers ($O(N)$) contributes to lowering the total supply chain cost, and under what parameter conditions it is more effective over the purchasing policy with a single supplier ($O(1)$). We observe that the performance of $O(N)$ and $O(1)$ can be varied depending on the parameter values. For this reason, we present the results with the specific sets of examples rather than with randomly generated examples, which are reported in <Tables 1>.

In <Tables 1>, columns 2~9 show the problem parameters, column 10 column shows the purchasing order quantity given by EOQ : $\sqrt{2c_K\lambda/c_2}$, Column 11 displays the optimal average cost for $O(1)$ denoted by g, columns 12~13 display the optimal performance for $O(N)$, and % in column 14 denotes the percentage gap of $O(1)$ over $O(N)$. For example, 58.2% of example 1 in Table 1 is obtained by the following formular : $(g - g(N^*))/g(N^*) \times 100$.

We conduct eight scenarios of computational experiment and the examples are shown in <Table 1>. In the following, we present the details of numerical experiment and managerial insights :

- Scenario 1 (examples 1~4) : We analyze the impacts of the customer order arrival rate on the performance of $O(N)$ and $O(1)$ by varying λ from 0.3 to 0.9 in steps of 0.2, ceteris paribus. The result shows that $O(N)$ significantly outperforms $O(1)$ under a light traffic (λ/μ) than under a heavy traffic. The result also shows that N^* weakly decreases in λ, which implies that under a low traffic, utilizing more suppliers with smaller EOQ is essential for effective inventory management.
- Scenario 2 (examples 5~8) : We analyze the impacts of customized manufacturing rate on the performance of $O(N)$ and $O(1)$ by varying μ from 1 to 1.4 in steps of 0.1, ceteris paribus. We observe that $O(N)$ performs much better than $O(1)$ but % is not nearly affected by the change in μ. We also observe that the increase in μ does not affect N^* even though a faster customized manufacturing seems to increase N^*. The reasoning behind this observation is that there is no motivation to keep more component inventory by increasing N^* despite of the increase in μ, since the threshold for customer order admission control remains the same.
- Scenario 3 (examples 9~12) : We analyze the impacts of lead time rate on the performance of $O(N)$ and $O(1)$ by varying d from 0.02 to 0.16 in steps of twice, ceteris paribus. The result indicates that $O(N)$ can be more effective when

Table 1 Effects of multiple order quantities on the supply chain cost

Ex.	Parameters									Optimal performance			
	λ	μ	d	c_K	c_R	c_1	c_2	B	EOQ	g	N^*	$g(N^*)$	%
1	0.3	1	0.01	400	40	3	1	15	16	42.641	4	26.950	58.2
2	0.4								18	48.863	3	34.412	42.0
3	0.5								20	54.252	2	45.186	20.1
4	0.6								22	59.309	2	51.746	14.6
5	0.7	1	0.10	200	30	4	1	15	17	36.972	2	33.735	9.6
6		1.1							17	34.034	2	30.893	10.2
7		1.2							17	32.039	2	29.000	10.5
8		1.3							17	30.591	2	27.650	10.6
9	0.55	1	0.01	300	10	2.5	1	15	19	40.207	2	36.862	9.1
10			0.02						19	36.570	2	32.043	14.1
11			0.04						19	31.234	2	28.299	10.4
12			0.08						19	26.443	2	25.813	2.4
13	0.65	1	0.025	200	50	3	1	15	17	53.233	3	35.821	48.6
14				300					20	52.131	2	41.395	25.9
15				400					23	51.787	2	43.358	19.4
16				500					26	51.962	2	49.529	4.9
17	0.75	1	0.20	150	20	3.5	1	15	15	31.337	2	30.424	3.0
18					30				15	31.439	2	30.469	3.2
19					40				15	31.538	2	30.515	3.4
20					50				15	31.635	2	30.559	3.5
21	0.45	1	0.01	250	20	3	1	15	15	47.917	4	32.405	47.9
22						4			15	60.467	4	37.231	62.4
23						5			15	73.016	4	41.81	74.6
24						6			15	85.565	4	46.189	85.2
25	0.6	1	0.05	300	15	3	1	15	17	27.07	2	24.286	11.5
26							1.5		14	32.55	2	28.816	13.0
27							2		12	37.141	2	32.562	14.1
28							2.5		11	40.549	2	35.678	13.7
29	0.8	1	0.10	200	30	4	1	5	18	30.75	1	30.75	0.0
30								10	18	38.935	2	37.69	3.3
31								15	18	45.353	2	41.725	8.7
32								20	18	49.645	2	43.986	12.9

lead time process is slow than when it is fast, which is consistent with other research works in the inventory control literature. Interestingly, N^* appears to be not affected by the change in d.

- Scenario 4 (examples 13～16) : We analyze the impacts of setup cost on the performance of $O(N)$ and $O(1)$ by varying c_K from 200 to 500 in steps of 100, ceteris paribus. The result shows that $O(N)$ outperforms $O(1)$ but % significantly decreases in c_K. We explain this phenomenon as follows. Since purchasing order placement is more restricted under a high setup cost than under a low setup cost, the beneficial effect of having multiple suppliers appears to be reduced as c_K increases.

- Scenario 5 (examples 17～20) : We analyze the impacts of customer order rejection cost on the performance of $O(N)$ and $O(1)$ by varying c_R from 5 to 20 in steps of 5, ceteris paribus. We observe that $O(N)$ performs better than $O(1)$ but the scenario has the least average performance gap compared to other scenarios. Furthermore, the performance gap is nearly affected by the change in c_R. The reasoning behind this is that the increase in c_R does not directly impact the number of customer backorders because we have a static customer order admission control based on a threshold.
- Scenario 6 (examples 21～24) : We analyze the impacts of backorder cost on the performance of $O(N)$ and $O(1)$ by varying c_1 from 2 to 5 in steps of 1, ceteris paribus. It is observed that % increases in c_1. In particular, the performance gap between $O(N)$ and $O(1)$ is significantly large and N^* is almost twice, compared to other scenarios. This observation implies that when service delay penalty is expensive, giving more flexibility to the order placement with more suppliers appears to be important in order to efficiently manage inventory.
- Scenario 7 (examples 25～28) : We analyze the impacts of inventory holding cost on the performance of $O(N)$ and $O(1)$ by varying c_2 from 1 to 4 in steps of 1, ceteris paribus. The result shows that % tends to increase in c_2. In other words, $O(N)$ is usually more effective than $O(1)$ when holding components in inventory is expensive. It implies that since EOQ becomes smaller as holding components becomes expensive, having multiple suppliers depending on the system status is essential in order to efficiently manage inventory. Interestingly, it is observed that N^* is not affected by the change in c_2.
- Scenario 8 (examples 29～32) : We analyze the impacts of the maximum size of backorders on the performance of $O(N)$ and $O(1)$ by varying B from 5 to 20 in steps of 5, ceteris paribus. The result shows that % increases in B, that is, $O(N)$ contributes to saving more costs when the maximum size of backorders increases. The result also indicates that N^* weakly increases in B. Increased backorders due to the increase in B appears to push N^* to be higher than before.

Next, we discuss the cross impacts of two parameters on the performance of $O(N)$ and $O(1)$. Even though there exist a variety of combinations among parameters, we present the following cases which exhibit the meaningful and explicable pattern :

Impacts of λ and c_K : <Figure 3> shows that given λ, the percentage gap between $O(N)$ and $O(1)$ increases as c_K decreases and the marginal increment of this gap is larger under low λ than under high λ. This result implies that the purchasing order placement with multiple suppliers, rather than with a single supplier, can greatly reduce the cost when the supply chain has a light traffic at customer order arrival and a low setup cost.

Impacts of λ and c_1 : <Figure 4> shows that given λ, the percentage gap between $O(N)$ and $O(1)$ increases in c_1. The pattern of the percentage gap between $O(N)$ and $O(1)$, presented in <Figure 4>, is similar to that of <Figure 3>. It is intuitive to see that the motivation to use multiple suppliers in purchasing order placement will become tangible when backorder cost is expensive under heavy traffic intensity.

Impacts of λ and B : <Figure 5> shows that given λ, the percentage gap between $O(N)$ and $O(1)$ in-

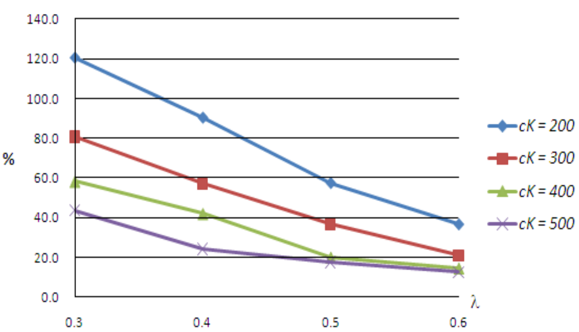

Figure 3. Impacts of λ and c_K on the performance of $O(N)$ and $O(1)$

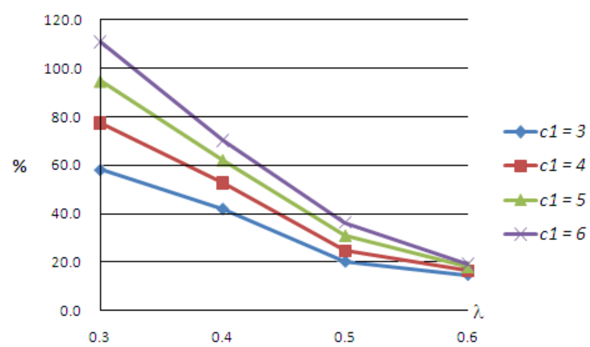

Figure 4. Impacts of λ and c_1 on the performance of $O(N)$ and $O(1)$

creases in B and this phenomenon is more apparent under low λ than under high λ. The increase in both λ and B implies that more customer orders are accepted and thus the chance of having increased backorder level is higher. Therefore, it becomes more cost effective to use the option of multiple suppliers in purchasing components.

Impacts of c_1 *and* d : <Figure 6> shows that given d, the percentage gap between $O(N)$ and $O(1)$ decreases in c_1 and its marginal increment decreases in d. In particular, the percentage gap between $O(N)$ and $O(1)$ is almost independent of c_1. Increase in c_1 and decrease in d imply that the manufacturer usually faces the risk of the extra payment of customer backorder costs unless inventory of components is timely acquired. In such case, the purchasing policy with multiple suppliers has an advantage over that of a single supplier.

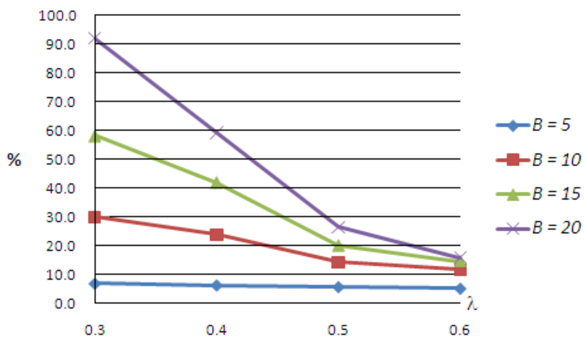

Figure 5. Impacts of λ and B on the performance of $O(N)$ and $O(1)$

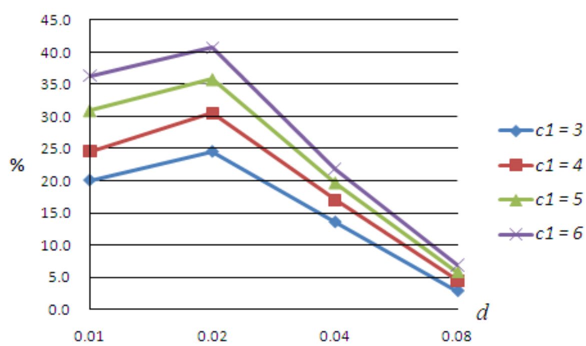

Figure 6. Impacts of d and c_1 on the performance of $O(N)$ and $O(1)$

6. Conclusions

We investigated the optimal purchasing policy for a supply chain consisting of a MTO manufacturer and N component suppliers. We analyzed the impact of the number of suppliers on the structure of the optimal policy through a numerical experiment. Specifically, we showed the existence of the optimal policy and the optimal average cost, and present the optimal solution procedure that computes these optimal performances. Based on numerical investigation, we conjectured that the optimal purchasing policy should be characterized by N switching curves which are the function of the customer backorder level and the size of multiple outstanding order.

To provide a better understanding of the structure of the optimal purchasing policy with multiple outstanding orders, we numerically compared its performance with the policy with a single supplier. From this comparison, we examined how much the option of using multiple suppliers can contribute to reducing the supply chain cost and under what conditions it is much more favorable over the policy with a single supplier. Numerical experiment showed that the performances comparison can be varied depending on the setting of the parameter values. It also suggests that developing more efficient and computationally inexpensive heuristic policy is demanding. We leave it for future research.

The methodology developed here provides a simple basis for estimating the impacts of utilizing multiple suppliers rather than a single supplier in purchasing components, based on test examples. However, this approach limits the applicability of the numerical results obtained in this paper. To provide managerial insights for component suppliers and end item manufacturer, a more elaborate computational study is needed through real manufacturer data, which will be a viable direction of further research.

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