A Novel Resource Allocation Algorithm in Multi-media Heterogeneous Cognitive OFDM System

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Abstract

An important issue of supporting multi-users with diverse quality-of-service (QoS) requirements over wireless networks is how to optimize the systematic scheduling by intelligently utilizing the available network resource while, at the same time, to meet each communication service QoS requirement. In this work, we study the problem of a variety of communication services over multi-media heterogeneous cognitive OFDM system. We first divide the communication services into two parts. Multimedia applications such as broadband voice transmission and real-time video streaming are very delay-sensitive (DS) and need guaranteed throughput. On the other side, services like file transmission and email service are relatively delay tolerant (DT) so varying-rate transmission is acceptable. Then, we formulate the scheduling as a convex optimization problem, and propose low complexity distributed solutions by jointly considering channel assignment, bit allocation, and power allocation. Unlike prior works that do not care computational complexity. Furthermore, we propose the FAASA (Fairness Assured Adaptive Sub-carrier Allocation) algorithm for both DS and DT users, which is a dynamic sub-carrier allocation algorithm in order to maximize throughput while taking into account fairness. We provide extensive simulation results which demonstrate the effectiveness of our proposed schemes.

Keywords: Cognitive multi-user OFDM, rate adaptive, margin adaptive, geometric progression, linear water-filling, service outage probability

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1. Introduction

It is well known that we are running out of usable radio frequencies, so cognitive radio as a feasible solution for the short of frequency resource is one of the most popular wireless technologies recently [1][2][3]. Cognitive radio enhance the spectrum efficiency by allowing cognitive users to access to the spectrum holes [4][5][6][7] or transmit with primary users at the same time and frequency band, via power control.

Fig. 1 shows a system model for spectrum sharing in Cognitive Radio Network (CRN). In this system network, there are one Primary User Transmitter (PU-Tx), one Primary User Receiver (PU-Rx) and one Cognitive User Transmitter (CU-Tx), one Cognitive User Receiver (CU-Rx). The link between PU-Rx and CU-Tx is supposed to be a flat fading channel with instantaneous channel gain $g_{k,n}$, the channel between CU-Rx and CU-Tx is also a flat fading channel characterized by instantaneous channel gain $h_{k,n}$. We use OFDM method to divide the channel into many sub-channels. Every sub-channel has a bandwidth of Δf .

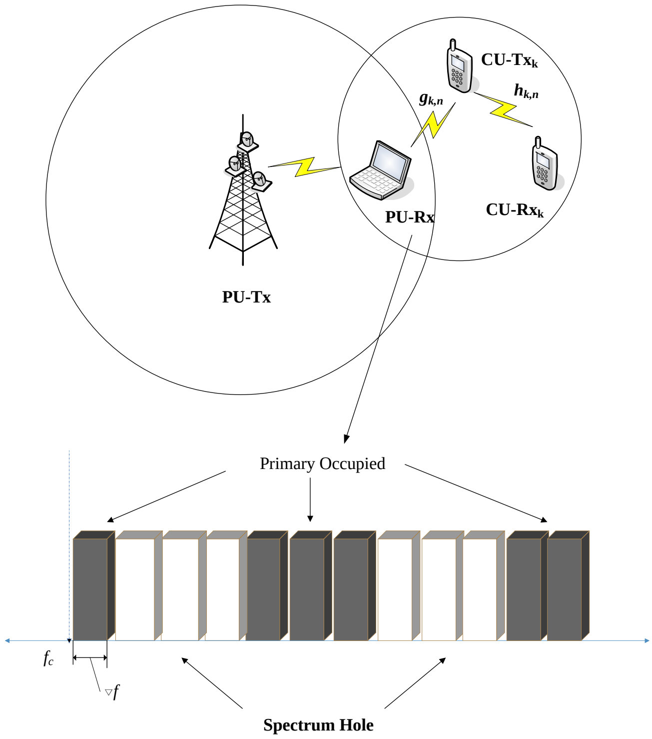

Fig. 1. A system model for sharing spectrum of PU and CU in Cognitive Radio Network (CRN)

The cognitive radio enables the usage of temporally unused sub-carrier, which is referred to as "Spectrum Hole" [5]. If this band is further used by a PU, the CU moves to another spectrum hole or stays in the same band, altering its transmission power level to avoid interference as shown in **Fig. 2**.

In the heterogeneous cognitive OFDM system, our research work focus on how to dynamically allocated wireless resources such as transmission power and frequency bandwidth so as to achieve the good performance of all the DS and DT users meanwhile satisfy the primary user interferene constrants. Transmission of DS service can be regarded as a data guaranteed service [8][9] in which a constant transmission rate should be

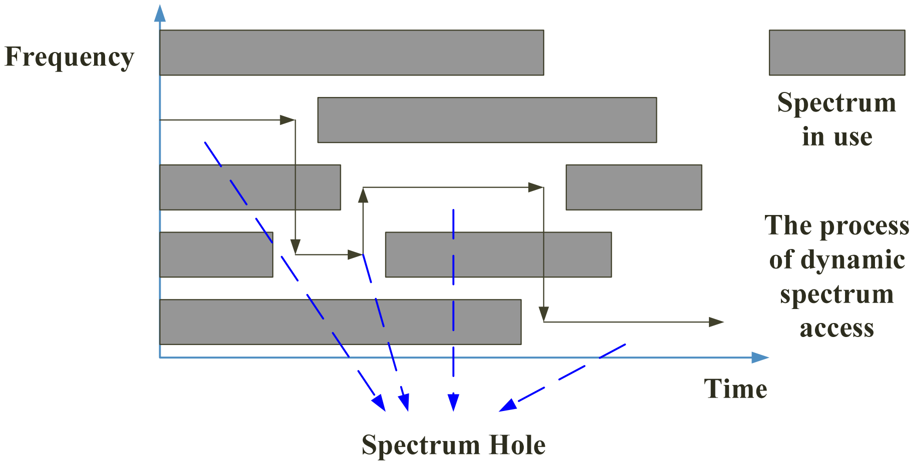

Fig. 2. Dynamic spectrum access process

maintained and transmission of DT service can be regarded as a best-effort service [10] in which the total transmission power is limited. The objective of DS service is to minimize the transmission power while maintaining the guaranteed throughput of DS users. And DT service is to maximize the transmission rate with the power constraint. The main contributions of this paper are summarized as follows:

- We discuss the multi-media heterogeneous cognitive OFDM with both delay-sensitive and delay-tolerant services. We conbine the rate adaptive (RA) [11][12][13][14] and margin adaptive (MA) [15][16][17][18] principle then establish a multi-user mathematical model of the heterogeneous cognitive OFDM system.
- Based on the heterogeneous cognitive OFDM system mathematical model, we propose the FAASA (Fairness Assured Adaptive Sub-carrier Allocation) algorithm for both DS and DT users, which is a adaptive sub-carrier allocation algorithm in order to maximize throughput while taking into account fairness.
- After sub-carrier allocation, we propose a low complexity (LC) bit allocation algorithm for DS users based on geometric progression and we propose a linear water-filling (LWF) algorithm for DT users. Compared with the tranditional bit and power allocation algorithm, the proposed algorithms are efficient and low complexity.
- We perform simulation experiments to evaluate joint FAASA and LC/LWF allocation algorithm by several performance criteria. The results show that proposed algorithms achieve higher efficiency performance than the static resource allocation algorithm proposed in [16].

The rest of this paper is organized as follows. Setion 2 presents some related works and section 3 introduces the heterogeneous system model and problem formulation. In section 4, we analyze multi-user resource allocation and propose the joint resource allocation FAASA, LC and LWF algorithms. In section 5, the performance evaluation of the proposed joint resource allocation algorithm is demonstrated. Finally, we summarize our work and conclude the paper in secion 6.

2. Related Work

Resource allocation has become a crucial technology in cognitive OFDM system, as many valuable research results have been obtained. Most resource allocation schemes are based on RA or MA principle.

Firstly, RA principle:

In [11], Kivanc et al. investigated the problem of finding an sub-optimal subcarrier and power allocation strategy for downlink communication to multiple users in an OFDM wireless system. The problem of minimizing total power consumption with constraints on bit-error rate and transmission rate for users requiring different classes of service is formulated and simple algorithms with good performance are derived.

In [12], Zhang et al. proposed an adaptive resource allocation methodology for cellular orthogonal frequency division multiplexing systems in a multiuser environment. The proposed method is featured as a low-complexity algorithm that involves not only adaptive modulation, but also adaptive multiple-access control and cell selection. A dynamic cell selection scheme is proposed to deal with the problem of overloading and nonuniform traffic density.

In [13], Rhee et al. derived a multi-user convex optimization problem to find the optimal allocation of subchannels, and propose a low-complexity adaptive subchannel allocation algorithm in the downlink of OFDM systems.

Secondly, MA principle:

In [16], Anas et al. propose a resource allocation scheme for an OFDMA based system to simultaneously provide two services, guaranteed performance (GP) and best effort (BE). Sub-carrier and power allocation are carried out sequentially to reduce the complexity, and an optimal power allocation procedure is derived. However resource allocation algorithm is static

In [17], Musavian et al. investigated the fundamental capacity limits of opportunistic spectrum-sharing channels in fading environments. The concept of opportunistic spectrum access is motivated by the frontier technology of cognitive radio which offers a tremendous potential to improve the utilization of the radio spectrum by implementing efficient sharing of the licensed spectrum. In [18], Kang et al. proposed an optimal power allocation strategies to achieve the ergodic capacity and the outage capacity of the SU fading channel under different types of power constraints and fading channel models. Besides the interference power constraint at PU, the transmit power constraint of SU is also considered in this paper.

The weakness of all the above research works is that most of them only consider wireless networks support the communication services with the same QoS requirements. In [16], Anas et al. presented two services based on diverse QoS requirements and the resource allocation algorithm proposed is static. However our research work is focus on the heterogeneous cognitive OFDM networks which support a variety of communication services with diverse (QoS) requirements. The joint allocation of sub-carriers and power/bit for both delay-sensitive (DS) and delay-tolerant (DT) users in this heterogeneous cognitive OFDM system is proposed.

3. Heterogeneous System Model and Problem Formulation

The down link heterogeneous C-OFDM system is show n in **Fig. 3**. We can see that the users are divided into two groups. One group is multimedia users, the other is file and email transmission users. We consider a heterogeneous system model of multi-users accessing to the unauthorized spectrum in CRN shown in **Fig. 4**. The C-OFDM system consists of a couple of primary users and K couples of cognitive mobile users. The first K_1 pairs of users have DS services and require a constant transmission rate of $R_k(k=1,......,K_1)$ for each. The services of the remaining $K-K_1$ couples of users have no delay constraint and can be delivered in a best-effort manner. In the heterogeneous system model, the cognitive network is divided into two parts. One is the multimedia network, the other is the network with file and email transmission services. In each part there is a Cognitive Base Station (CBS). The CBS is a central controller which is responsible for data fusion and the controlling of sub-carriers, power and bits allocation. Primary user transmitter (PU-Tx) utilized the licensed spectrum to contact with primary user receiver (PU-Rx), meanwhile the cognitive user transmitter (CU-Tx) transmit data to corresponding cognitive user receiver (CU-Rx). The total channel bandwidth is BHz and is divided into N orthogonal sub-carriers, which are shared among the K couples

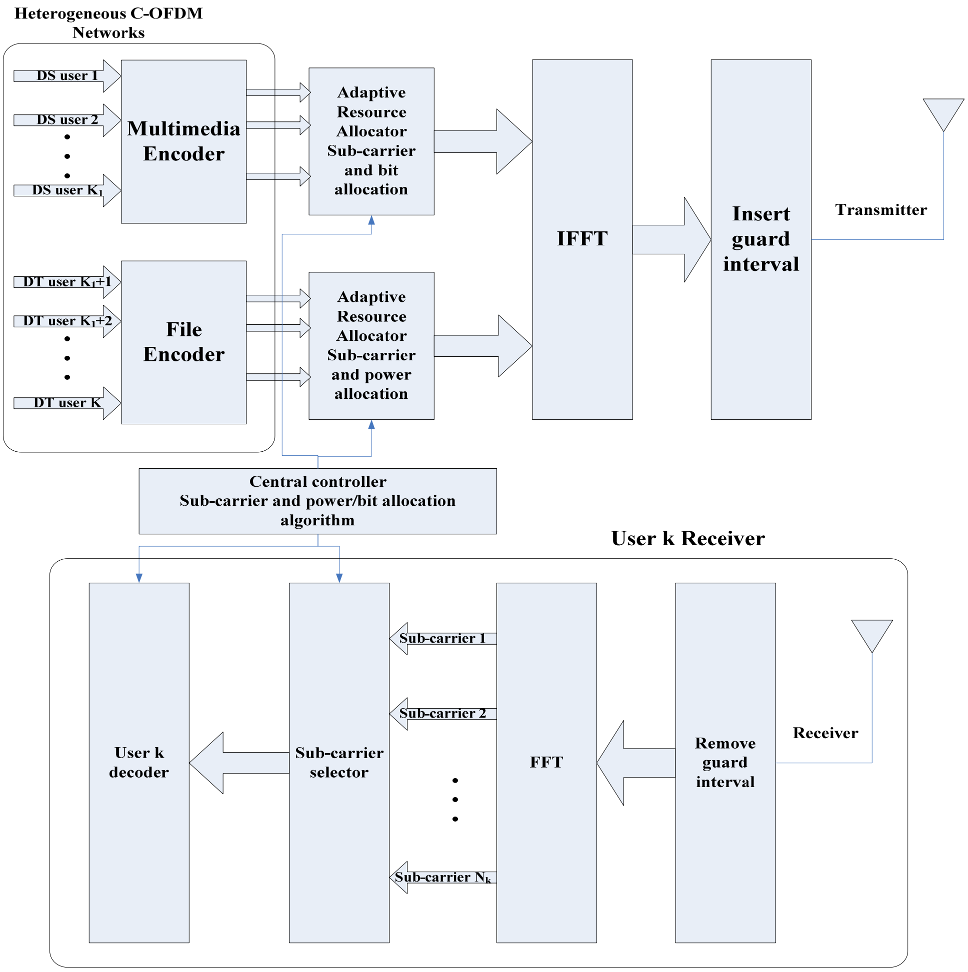

Fig. 3. The down link heterogeneous C-OFDM system

of users. In this paper, the channel between CU-Tx and PU-Rx is called interfering channel, while the one between CU-Tx and CU-Rx is called cognitive channel. Cognitive users employ OFDMA method for transmission. Here $|g_{k,n}|^2$ and $|h_{k,n}|^2$ denote the sub-carrier gain of the user k in sub-carrier n of interfering channel and cognitive channel respectively. These fading coefficients of all users are assumed to remain unchanged within each transmission frame but can vary from one frame to another. All the interfering and cognitive channel information is assumed perfectly known at the central controller (CBS). Typically, the channel information

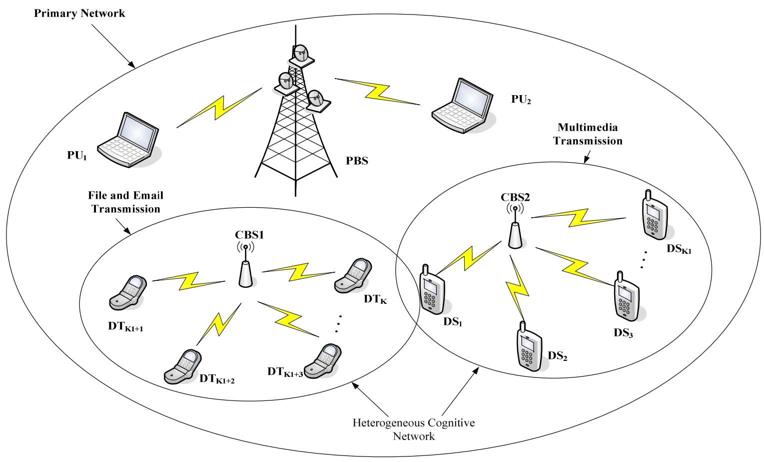

Fig. 4. Heterogeneous system model of multi-users accessing to the unauthorized spectrum in CRN

can be collected by estimating it at each user terminal and sending it to the CBS via a feedback channel. Based on the instantaneous channel inputs, the CBS allocates different sub-carriers to different users and determines the amount of power or bit to be transmitted on each sub-carrier through the sub-carrier and power or bit allocation algorithm. The total interference power from the base station is fixed and is given by P_{total} . For simplicity, the heterogeneous system model can be abstract in the **Fig. 5**.

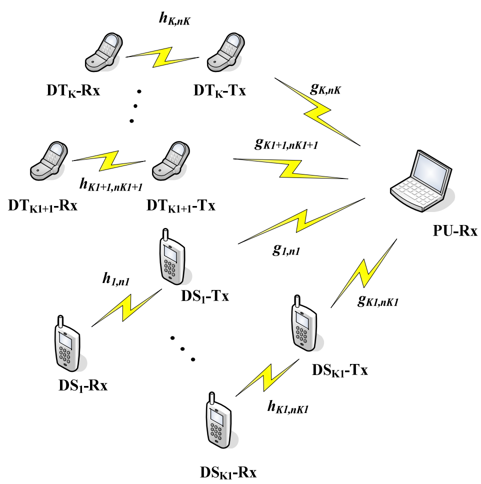

Fig. 5. Simplified system model of heterogeneous multi-user Cognitive OFDM

We assume that all the channels are *Rayleigh* fading channels. However, the channel in each sub-carrier is narrow enough to experience flat fading. We denote $r_{k,n}$ as the transmission rate of user k on sub-carrier n . In general, $r_{k,n}$ can be expressed as

$$r_{k,n} = log_2(1+\frac{p_{k,n}\cdot|h_{k,n}|^2}{\Gamma N_0 B/N}) \tag{1}$$

where N_0 is the power spectral density of additive white Gaussian noise and Γ is a constant, usually called the signal-to-noise ratio (SNR) gap. When instantaneous mutual information is used to characterize the achievable transmission rate, we have $\Gamma = 1(0dB)$. If practical signal constellations are used, Γ is a constant related to a given bit-error-rate (BER) requirement. For example, when MQAM modulation is used we have $\Gamma = -\frac{ln(5BER)}{1.5}$.

The goal of the resource allocation is to optimize the allocation of sub-carriers and power or bit under the total interference power constraint so as to maximize the sum-rate of all the $K-K_1$ DT users while satisfying the individual QoS requirement for each of the K_1 DS users. Mathematically, we can formulate the problem as

$$\max_{\{p_{k,n}\}} \sum_{k=K_1+1}^{K}\sum_{n=1}^{N}\rho_{k,n}\cdot r_{k,n}$$

$$s.t \begin{cases} \sum_{n=1}^{N}\rho_{k,n}\cdot r_{k,n} = R_k, k=1,\ldots,K_1 \\ R_1 : R_2 : \ldots : R_{K_1} = r_1 : r_2 : \ldots r_{K_1} \\ p_{k,n} \geq 0, \forall k,n \\ \sum_{k=1}^{K}\sum_{n=1}^{N}\rho_{k,n}\cdot|g_{k,n}|^2\cdot p_{k,n} = P_{total} \\ \rho_{k,n} = \{0,1\}, \sum_{k=1}^{K}\rho_{k,n} = 1, \forall k,n \end{cases} \tag{2}$$

where P_{total} is total interference power, and $\rho_{k,n}$ only be either 1 or 0, indicating whether sub-carrier n is used by user k or not. And each sub-carrier can only be used by one user. $\{r_i\}_{i=1}^{K}$ is a set of predetermined values that are used to ensure proportional fairness among DS users.

4. Proposed Resource Allocation Algorithm

In this paper, we consider the resource allocation problem in a heterogeneous multi-user C-OFDM system where DS and DT users are supported simultaneously. Users in the system are classified into DS users and DT users based on their service delay requirements. We assume that the total interference power for the primary user is fixed. Now we divided the problem into two parts. One is that for DS users, the objective is to minimize the total transmit power while satisfying a basic transmission rate for each user. This is often referred to as margin adaption [15][16][17][18]. The other is that for DT users, the problem is often formulated as maximizing the sum-rate of the DT users subject to a total interference power constraint. This formulation is also known as rate adaption [11][12][13][14]. We consider a low complexity bit allocation algorithm based on geometric progression for the resource allocation of the DS users, and use a linear water-filling algorithm for the power allocation of the DT users. The specific steps of the proposed algorithms are as follow.

4.1 FAASA and Bit Allocation Algorithm for DS Users

First we conduct sub-carrier allocation for DS users. This FAASA algorithm has a low complexity, and satisfying the proportional rate fairness between DS users. First we suppose that the bit is equally distributed among all sub-carriers. The user with the smallest percentage rate has the priority to choose sub-carrier. Specific allocation is as follow:

Here Ω_k is the set of sub-carriers assigned to user k , A is the set of all the sub-carriers.

- Initialization
  Set $\Omega_k = \Phi, R_k = 0 (k = 1,2,...,K_1)$, $A = \{1,2,...,N\}$
- $k = 1,2,...,K_1$
- find n satisfying $|h_{k,n}| \geq |h_{k,j}|$ for all $j \in A$
- let $\Omega_k = \Omega_k \cup \{n\}$, $A = A - \{n\}$ and update $R_k = R_k + R_{total} / N$,
- While $A \neq \Phi$

a) find k^* satisfying $R_{k^*} / r_{k^*} \leq R_i / r_i$ for all $1 \leq i \leq K$

b) for the find k^*, find n^* satisfying $|h_{k^*,n^*}| \geq |h_{k^*,j}|$, $j \in A$

c) for the find k^* *and* n^*, *let* $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$, $A = A - \{n^*\}$, $R_{k^*} = R_{k^*} + R_{total} / N$

d) until $A = \Phi$

After sub-carrier allocation, we must load every bit in the assigned sub-carriers. From [19], we know that the lower bound of the transmission power required by sub-carrier n of user k for $r_{k,n}$ bits to be transmitted at a given BER P_e is obtained as

$$f(r_{k,n}) = \frac{N_0}{3}[Q^{-1}(P_e / 4)]^2 (2^{r_{k,n}} - 1) \tag{3}$$

We further realize that $f(r_{k,n})$ is a convex and increasing function with $f(0) = 0$. This condition essentially means that no power is needed when no bit is transmitted and that the required additional power to transmit an additional bit increases with $r_{k,n}$.

This bit allocation algorithm is a two-dimension bit allocation algorithm, $|g_{k,n}|^2$ and $|h_{k,n}|^2$ denote the sub-carrier gain of the user k in sub-carrier n of interfering channel and cognitive channel respectively.

$$r_{k,n} = f(|g_{k,n}|^2, |h_{k,n}|^2) \tag{4}$$

The algorithm is based on the geometric progression of the additional transmission power required by the sub-carriers and the AM-GM means inequality. For K_1 DS users and the given sub-carriers, the transmission power allocated of user k to the nth sub-carrier $p_{k,n}$, can be expressed as the sum of the additional transmission power required to successively allocate up to $r_{k,n}$ bits, one bit at a time to the nth sub-carrier

$$p_{k,n}(r_{k,n}) = \sum_{m=1}^{r_{k,n}} \Delta p_{k,n,m} \tag{5}$$

where $\Delta P_{k,n,m} = [f(m) - f(m-1)] / |h_{k,n}|^2$. Here $\Delta P_{k,n,m}$ indicates the additional power needed to

transmit one additional bit through the sub-carrier n for user k when the number of bits loaded on the sub-carrier is $m-1$.
For the problem of minimizing the DS users' interference power, (2) can then be rewritten as

$$\begin{aligned} P_{DS} &= min\sum_{k=1}^{K_1}\sum_{n=1}^{N}\sum_{m=1}^{r_{k,n}}\rho_{k,n}\left|g_{k,n}\right|^2\Delta p_{k,n,m} \\ R_k &= \sum_{k=1}^{K_1}\sum_{n=1}^{N}\rho_{k,n}r_{k,n} \end{aligned} \tag{6}$$

From (3) and (5) it is understood that the sequence $\left|g_{k,n}\right|^2\Delta p_{k,n,1}, \left|g_{k,n}\right|^2\Delta p_{k,n,2}, ..., \left|g_{k,n}\right|^2\Delta p_{k,n,r_{k,n}}$ is the geometric progression in which the initial term is $f(1)\cdot\left|g_{k,n}\right|^2/\left|h_{k,n}\right|^2$ and the common ratio is 2. It has been proven in the Appendix that if and only if the sum of the last additional transmission power required by N sub-carriers, $\sum_{n\in\Omega_k}\left|g_{k,n}\right|^2\Delta p_{k,n,r_{k,n}}$ is minimum, then the total interference power is minimum. To find the minimum value of $\sum_{n=1}^{N_k}\left|g_{k,n}\right|^2\Delta p_{k,n,r_{k,n}}$, advantage is taken of the arithmetic-geometric means inequality: N_k is the number of sub-carriers assigned to user k, Ω_k is the set of sub-carriers assigned to user k.

$$\sum_{n=1}^{N_k}\left|g_{k,n}\right|^2\Delta p_{k,n,r_{k,n}} = \sum_{n=1}^{N_k}\frac{\left|g_{k,n}\right|^2 f(1)}{\left|h_{k,n}\right|^2}2^{r_{k,n}-1} \geq N_k\cdot\sqrt[N_k]{\frac{\left|g_{k,1}\right|^2\left|g_{k,2}\right|^2\cdots\left|g_{k,N_k}\right|^2[f(1)]^{N_k}}{\left|h_{k,1}\right|^2\left|h_{k,2}\right|^2\cdots\left|h_{k,N_k}\right|^2}2^{r_{k,1}+r_{k,2}+\cdots+r_{k,N_k}-N_k}} \tag{7}$$

When all terms on the left-hand side of (7) are equal to each other, the sum is the minimum. Therefore, if each term of the left-hand has the same value as

$$\frac{\left|g_{k,n}\right|^2 f(1)}{\left|h_{k,n}\right|^2}2^{r_{k,n}-1} = \sqrt[N_k]{\frac{\left|g_{k,1}\right|^2\left|g_{k,2}\right|^2\cdots\left|g_{k,N_k}\right|^2[f(1)]^{N_k}}{\left|h_{k,1}\right|^2\left|h_{k,2}\right|^2\cdots\left|h_{k,N}\right|^2}2^{r_{k,1}+r_{k,2}+\cdots+r_{k,N_k}-N_k}} \quad n=1,2,...,N_k, k=1,2,...,K_1 \tag{8}$$

Then the total interference power has the minimum value. From (8), the optimum number of bits for the *nth* sub-carrier is obtained as

$$r_{k,n} = 2\,log_2(\left|h_{k,n}\right|/\left|g_{k,n}\right|)+\frac{R_k}{N_k}-\frac{2}{N_k}log_2(\omega_{T_k}) \quad n=1,2,...N_k, k=1,2,...K_1 \tag{9}$$

where $\omega_{T_k} = \prod_{n=1}^{N_k}(\left|h_{k,n}\right|/\left|g_{k,n}\right|)$.

Here is a problem, the $r_{k,n}$ means the number of bits allocated in sub-carrier n, so it must be integer, and also $\sum_{n=1}^{N_k}r_{k,n}=R_k$. By using equation (9) we can calculate $r_{k,n}$. If $r_{k,n}$ is not an integer, we should round off it as an integer such that $\hat{r}_{k,n}=round(r_{k,n})$ $n=1,2,...,N_k$. When the sum of $\hat{r}_{k,n}$ coincides with R_k, this bit allocation procedure is finished. But if $R_k-\sum_{n=1}^{N_k}\hat{r}_{k,n}$ is not equal to zero, we must use a mathematical optimal algorithm to add or subtract the additional bits from the appropriate sub-carriers.

The exact steps of this proposed algorithm are as follow

- Compute the number of bits for each sub-carrier allocated to user k using equation (9)
- Round off each $r_{k,n}$ such as $\hat{r}_{k,n} = round(r_{k,n})$ and calculate $R_l = R_k - \sum_{n=1}^{N_k} \hat{r}_{k,n}$

  a) If $R_l = 0$, end this bit allocation procedure.

  b) If $R_l > 0$, then select R_l sub-carriers based on a descending order of the value of $r_{k,n} - \hat{r}_{k,n}$ and add one bit to each of these sub-carriers.

  c) If $R_l < 0$, then select $|R_l|$ sub-carriers based on an ascending order of the value of $r_{k,n} - \hat{r}_{k,n}$ and subtract one bit from each of these sub-carriers.

By now the bit allocation for K_1 DS users is finished. Next is the linear water-filling for $K - K_1$ DT users.

4.2 FAASA and Power Allocation Algorithm for DT Users

First is sub-carrier allocation. The sub-carrier allocation for DT users is different from the one for DS users. The specific steps are as follow:

Ω_k is the set of sub-carriers assigned to user k, A is the set of all the sub-carriers. The algorithm can be described as

- Initialization
- Set $\Omega_k = \Phi, R_k = 0 (k = K_1+1, K_1+2, ..., K)$, $A = \{1,2,...,N\}$
- $k = K_1+1, K_1+2, ..., K$

  a) Find n satisfying $|h_{k,n}|^2 \geq |h_{k,j}|^2$ for all $j \in A$

  b) Let $\Omega_k = \Omega_k \cup \{n\}$, $A = A - \{n\}$ and update $R_k = R_k + log_2(1 + \frac{|h_{k,n}|^2 \times (P_{total} - P_{DS})}{\Gamma N_0 B / N})$

- While $A \neq \Phi$

  a) Find k^* satisfying $R_{k^*} / r_{k^*} \leq R_i / r_i$ for all $K_1 + 1 \leq i \leq K$

  b) For the find k^*, find n^* satisfying $|h_{k^*,n^*}|^2 \geq |h_{k^*,j}|^2$, $j \in A$

  c) For the find k^* and n^*, let $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$, $A = A - \{n^*\}$,

  $$R_{k^*} = R_{k^*} + log_2(1 + \frac{|h_{k^*,n^*}|^2 \times (P_{total} - P_{DS})}{\Gamma N_0 B / N})$$

  d) Until $A = \Phi$

After sub-carrier allocation, we should inject power to every assigned sub-carrier. The objective of the remaining $K - K_1$ DT users is to maximize the sum-rate subject to a total interference power constraint. This is the best-effort method. Water-filling algorithm is fit in with this requirement. Water-level calculation is a critical part of water-filling algorithm. **Fig. 6** describes the optimal power allocation based on multi-level water-filling.

In [20] the power allocated in sub-carrier n for user k is

$$p_{k,n} = \left(\frac{1}{\lambda_k |g_{k,n}|^2} - \frac{\Gamma N_0 B/N}{|h_{k,n}|^2}\right)^+ \quad k = (K_1+1,......K) \tag{10}$$

where $(x)^+ = \begin{cases} x, x>0 \\ 0, x \le 0 \end{cases}$. The water-level is $\frac{1}{\lambda_k |g_{k,n}|^2}$. In [20] the adaptive water-filling

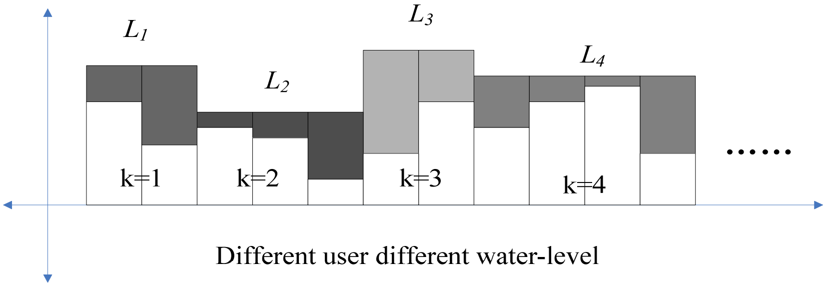

Fig. 6. Multi-level water-filling for 4 DT users and 11sub-carriers

algorithm needs iterate for many times, this makes the calculation complexity unacceptable especially in multi-user system. So we propose a linear water-filling algorithm. This algorithm could determine the sub-carriers that do not need power injection at the beginning and greatly reduce the calculate complexity. The detail of the algorithm is as follow

By analyzing the equation (10), we can get

$$\frac{\frac{|h_{k,n}|^2}{\Gamma N_0 B/N}}{1+\frac{|h_{k,n}|^2}{\Gamma N_0 B/N} p_{k,n}} - \lambda_k |g_{k,n}|^2 = 0, \text{ and } \frac{\frac{|h_{k,n}|^2}{\Gamma N_0 B/N |g_{k,n}|^2}}{1+\frac{|h_{k,n}|^2}{\Gamma N_0 B/N} p_{k,n}} = \lambda_k, \text{ so } \frac{\frac{|h_{k,n}|^2}{\Gamma N_0 B/N |g_{k,n}|^2}}{1+\frac{|h_{k,n}|^2}{\Gamma N_0 B/N} p_{k,n}} = \frac{\frac{|h_{k,m}|^2}{\Gamma N_0 B/N |g_{k,m}|^2}}{1+\frac{|h_{k,m}|^2}{\Gamma N_0 B/N} p_{k,m}}$$

$$(n,m = 1,2,......N_k),(k = K_1+1,...,K) \tag{11}$$

We set $\frac{|h_{k,n}|^2}{\Gamma N_0 B/N} = H_{k,n}$, $\frac{|h_{k,m}|^2}{\Gamma N_0 B/N} = H_{k,m}$, then the above equation simplifies to $\frac{H_{k,n}/|g_{k,n}|^2}{1+H_{k,n} p_{k,n}} = \frac{H_{k,m}/|g_{k,m}|^2}{1+H_{k,m} p_{k,m}}$, so we can get

$$|g_{k,n}|^2 p_{k,n} = |g_{k,m}|^2 p_{k,m} + \left(\frac{|g_{k,m}|^2}{H_{k,m}} - \frac{|g_{k,n}|^2}{H_{k,n}}\right) \tag{12}$$

As long as the power of one sub-carrier is determined, the rest can be calculated by equation (12). Thus, the sum power of $K-K_1$ DT users is $P_{total} - P_{DS}$

$$\sum_{n=1}^{N_k} |g_{k,n}|^2 p_{k,n} = N_k \left(|g_{k,n}|^2 p_{k,n} + \frac{|g_{k,n}|^2}{H_{k,n}}\right) - \sum_{m=1}^{N_k} \frac{|g_{k,m}|^2}{H_{k,m}} \le \frac{\sum_{n=1}^{N_k} |h_{k,n}|^2 \cdot (P_{total} - P_{DS})}{\sum_{k=K_1+1}^{K} \sum_{n=1}^{N_k} |h_{k,n}|^2} \tag{13}$$

Transfer the inequality, we can get

$$\left|g_{k,n}\right|^2 p_{k,n} \le \frac{1}{N_k}\left[\frac{\sum_{n=1}^{N_k}\left|h_{k,n}\right|^2 \cdot (P_{total}-P_{DS})}{\sum_{k=K_1+1}^{K}\sum_{n=1}^{N_k}\left|h_{k,n}\right|^2} - \frac{N_k\left|g_{k,n}\right|^2}{H_{k,n}} + \sum_{m=1}^{N_k}\frac{\left|g_{k,m}\right|^2}{H_{k,m}}\right] \tag{14}$$

We can calculate the power injected in every sub-carrier according to equation (14), if $\left|g_{k,n}\right|^2 p_{k,n} < 0$, because $\left|g_{k,n}\right|^2 > 0$, so $p_{k,n} < 0$, and set $p_{k,n} = 0$. Meanwhile exclude this channel and use $\sum_{m=1}^{N_k}\frac{\left|g_{k,m}\right|^2}{H_{k,m}} - \frac{\left|g_{k,n}\right|^2}{H_{k,n}}$ instead of $\sum_{m=1}^{N_k}\frac{\left|g_{k,m}\right|^2}{H_{k,m}}$, we suppose $H_{k,1} \le H_{k,2} \le \ldots\ldots \le H_{k,N_k}$, then $p_{k,1} \le p_{k,2} \le \ldots\ldots \le p_{k,N_k}$. According to equation (14) can be

$$\left|g_{k,1}\right|^2 p_{k,1} = \frac{1}{N_k}\left[\frac{\sum_{n=1}^{N_k}\left|h_{k,n}\right|^2 \cdot (P_{total}-P_{DS})}{\sum_{k=K_1+1}^{K}\sum_{n=1}^{N_k}\left|h_{k,n}\right|^2} - \frac{N_k\left|g_{k,1}\right|^2}{H_{k,1}} + \sum_{m=1}^{N_k}\frac{\left|g_{k,m}\right|^2}{H_{k,m}}\right] \tag{15}$$

If $\left|g_{k,1}\right|^2 p_{k,1} < 0$, then set $p_{k,1} = 0$, inject the power to another sub-carrier

$$\left|g_{k,2}\right|^2 p_{k,2} = \frac{1}{N_k}\left[\frac{\sum_{n=1}^{N_k}\left|h_{k,n}\right|^2 \cdot (P_{total}-P_{DS})}{\sum_{k=K_1+1}^{K}\sum_{n=1}^{N_k}\left|h_{k,n}\right|^2} - \frac{(N_k-1)\left|g_{k,2}\right|^2}{H_{k,2}} + \sum_{m=2}^{N_k}\frac{\left|g_{k,m}\right|^2}{H_{k,m}}\right] \tag{16}$$

Until find $p_{k,m} > 0$, we calculate the power allocation in following sub-carriers according to equation (12). Compared with the traditional water-filing method, this algorithm does not need calculate the power allocated in all sub-carriers and then revise the water-filling level, so it reduce the calculation complexity.

5. Experimental Classification Results and Analysis

In this section we present numerical performance results of proposed resource allocation algorithm. We consider a heterogeneous Cognitive OFDM system with $N = 64$ sub-carriers and $K = 8$ users. Here, $K_1 = 4$ users have DS services and rest have DT services. For simplicity, we set the rate requirements of all DS users be identical and equal to R_{DS} / K_1 bits/OFDM symbol, where R_{DS} denotes the sum of the DS users' rate. The channel coefficient $g_{k,n}$ and $h_{k,n}$ subject to *Rayleigh* complex Gaussian fading. The system total transmit SNR is defined as P_{total}, $BER = 10^{-5}$.

We denote P_{DS} the actual power consumption of all the K_1 DS users. If P_{DS} is larger than the total interference power constraint P_{total}, a service outage occurs. Otherwise, the residual transmit power $P_{total} - P_{DS}$ together with the residual sub-carrier set A are allocated among the $K - K_1$ DT users. Specifically, each sub-carrier in A is assigned to the DT user with the highest channel-noise-ratio (CNR), and the power is distributed over these sub-carriers in the form of linear water-filling (14), where the water level can be determined by $P_{total} - P_{DS}$.

To evaluate the performance of the two proposed adaptive resource allocation algorithm, we also present the results for two non-adaptive schemes in comparison.

In the first scheme, all the 8 users are treated equally and each is assigned 8 sub-carriers. We refer to this scheme as Fixed Sub-carrier Assignment with Water-filling (FSA-WF) Algorithm. In the second scheme, DS users are given higher priority than DT users and each is assigned 12 sub-carriers, whereas each DT user is allocated 4 sub-carriers only. This scheme is called Fixed Sub-carrier Assignment with Priority and with Water-filling (FSAP-WF) Algorithm.

Firstly, we conduct the comparison of the service outage behavior. **Fig. 7** illustrates the service outage probability versus total transmit SNR when the total target transmission rate of DS users is $R_{DS} = 96$ bits/OFDM symbol. Total transmit SNR $P_{total} = 4:14$. The proposed algorithm is with the FAASA, low complexity bit allocation and linear water-filling algorithms. There also are two scheme of proposed algorithm. First is that the number of sub-carriers that can be assigned to DS users is 32. The second is that the number is 48.

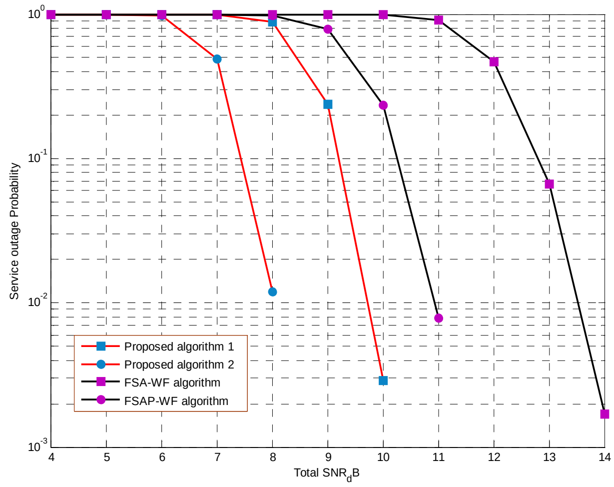

Fig. 7. Service outage probabilities versus total transmit SNR at $R_{DS} = 96$ bits/OFDM symbol.

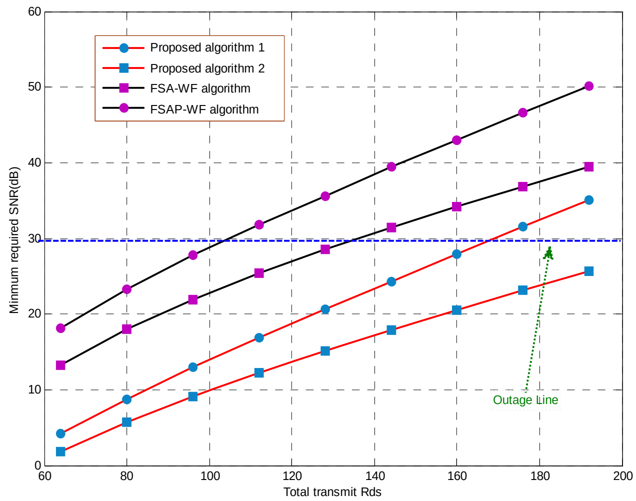

Fig. 8. Minimum required total transmit SNR for different R_{DS}

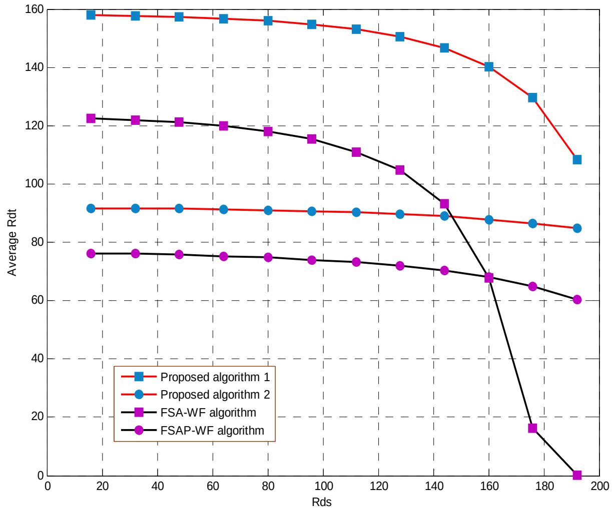

Fig. 9. The basic sum-rate for DS users R_{DS} versus the average sum-rate for DT users R_{DT} at a total transmits SNR of 20dB

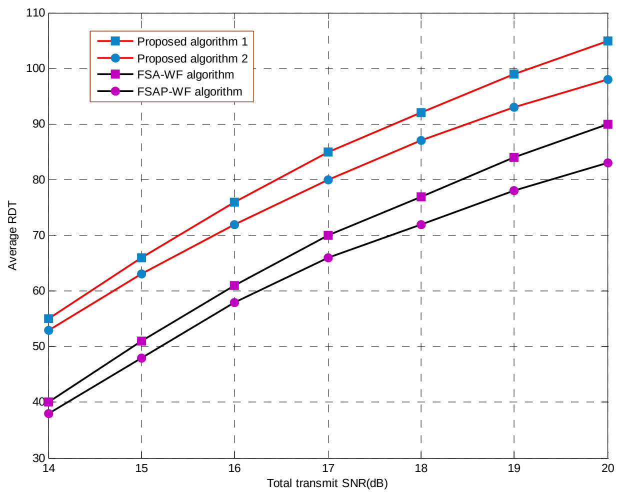

Fig. 10. Average R_{DT} versus total transmit SNR

From **Fig. 7**, it is first observed that the second proposed algorithm has the best performance. This implies that DS users with the priority to choose more sub-carriers will decrease the power required of all DS users P_{DS} , so the service outage probabilities also decrease with it. One can also see that the proposed algorithm 1 is little inferior, because in this algorithm DS users can only choose 32 sub-carriers. However, the two proposed algorithm is with the adaptive sub-carrier allocation, so they are superior to the two schemes of fixed sub-carrier allocation.

Secondly, we plot the minimum required total transmit SNR for different R_{DS} in **Fig. 8** and $P_{total} = 30dB$. It is again observed that the proposed algorithm 2 has a best performance of the total required transmit SNR. It can also see that no matter in low or high transmit rate of DS users, the proposed algorithm is superior to the FSA-WF and FSAP-WF algorithm. **Fig. 8** also show that, as R_{DS} increases, the minimum required total SNR of the proposed adaptive algorithm increases at a much lower speed than that of the two FSA-WF and FSAP-WF schemes.

In **Fig. 9**, we plot the basic sum-rate for DS users R_{DS} versus the average sum-rate for DT users R_{DT} at a total transmit SNR of 20dB. It can be observed that the proposed algorithm 1 has a best performance of the total transmit rate of DT users. Because more sub-carriers are assigned to the DS users in algorithm 2, less resource will be allocated to the DT users. And proposed algorithm 1 is better than FSA-WF algorithm and proposed algorithm 2 is better than FSAP-WF algorithm. We also observed that, as R_{DS} increases, the average sum-rate for DT users R_{DT} decrease at a much lower speed than that of the two FSA-WF and FSAP-WF schemes.

From **Fig. 10**, we can observe that the algorithm with more sub-carrier assigned to *DS* users will get harmful impact on *DT* users. So the proposed algorithm 1 is superior to the proposed algorithm 2 meanwhile FSA algorithm is also superior to FSAP algorithm. Generally speaking, the adaptive algorithms still obtain a better performance than the fixed sub-carrier allocation algorithms.

6. Conclusions

This paper proposed a new heterogeneous cognitive multi-user OFDM system with both delay-sensitive (DS) and delay-tolerant (DT) services. We combine the rate adaptive (RA) and margin adaptive (MA) principle. Specifically, we use a low complexity bit allocation algorithm based on geometric progression for DS users and use a linear water-filling algorithm for DT users. Numerical results show that the performance of the proposed algorithms in terms of service outage probability, achievable transmission rate for users and interference power are superior to the available FSA-WF or FSAP-WF algorithms.

APPENDIX

Problem

The total power of DS user k is

$$P_{DS_k} = \sum_{n=1}^{N_k}\sum_{m=1}^{r_{k,n}} \left|g_{k,n}\right|^2 \Delta p_{k,n,m}$$

$$(\left|g_{k,1}\right|^2 \Delta p_{k,1,1} + \cdots + \left|g_{k,1}\right|^2 \Delta p_{k,1,r_{k,1}}) + \cdots + (\left|g_{k,N_k}\right|^2 \Delta p_{k,N_k,1} + \cdots + \left|g_{k,N_k}\right|^2 \Delta p_{k,N_k,r_{k,N_k}})$$

is minimum,

the sufficient and necessary condition is

$$\Delta P_{DS_k} = \left|g_{k,1}\right|^2 \Delta p_{k,1,r_{k,1}} + \left|g_{k,2}\right|^2 \Delta p_{k,2,r_{k,2}} + \cdots + \left|g_{k,N_k}\right|^2 \Delta p_{k,N_k,r_{k,N_k}}$$

is minimum, here $\Delta p_{k,n,m}, k = 1,2,...,K_1, n = 1,2,...,N_k, m = 1,2,...,r_{k,n}$ is geometric series and the initial terms are $p_{k,n,1}, k = 1,2,...,K_1, n = 1,2,...,N_k$, all the common ratio are 2, $\sum_{i=1}^{N_k} r_{k,n} = R_k$ and R_k is an integer.

Proof of the problem

Necessity:

We suppose that P_{DS_k} is not the minimum. So, we can find some *i* and *j* such that $\left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1} < \left|g_{k,j}\right|^2 \Delta p_{k,j,r_{k,j}}, i,j=1,2,...,N$. This inequality can be rewritten as $\left|g_{k,i}\right|^2 p_{k,i,1}\cdot 2^{r_{k,i}} < \left|g_{k,j}\right|^2 p_{k,j,1}\cdot 2^{r_{k,j}-1}, i,j=1,2,...,N$ for the premise that $\Delta p_{k,n,m}$ is the geometric series. Let us replace $\left|g_{k,j}\right|^2 \Delta p_{k,j,r_{k,j}}$ with $\left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1}$ in P_{DS_k}, so we get

$$\Delta P_{DS_k}{}^* = \left|g_{k,1}\right|^2 \Delta p_{k,1,r_{k,1}} + \cdots \left|g_{k,j-1}\right|^2 \Delta p_{k,j-1,r_{k,j-1}} + \left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1}$$
$$+\cdots+\left|g_{k,N_k}\right|^2 \Delta p_{k,N_k,r_{k,N_k}}$$

And

$$\Delta P_{DS_k} - \Delta P_{DS_k}{}^* = \left|g_{k,j}\right|^2 p_{k,j,1}\cdot 2^{r_{k,j}-1} - \left|g_{k,i}\right|^2 p_{k,i,1}\cdot 2^{r_{k,i}} > 0$$

Here we use the inequality $\left|g_{k,i}\right|^2 p_{k,i,1}\cdot 2^{r_{k,i}} < \left|g_{k,j}\right|^2 p_{k,j,1}\cdot 2^{r_{k,j}-1}$

So we can get $\Delta P_{DS_k} > \Delta P_{DS_k}{}^*$. If P_{DS_k} is not minimum, then ΔP_{DS_k} is also not minimum. Therefore, we have proven the necessity part of the problem.

Adequacy:

Suppose that ΔP_{DS_k} is not the minimum. Then there exist some *i* and *j* such that $\left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1} < \left|g_{k,j}\right|^2 \Delta p_{k,j,r_{k,j}}, i,j=1,2,...,N$. If we replace $\left|g_{k,j}\right|^2 \Delta p_{k,j,r_{k,j}}$ with $\left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1}$ in ΔP_{DS_k}, then we have $\Delta P_{DS_k} > \Delta P_{DS_k}{}^*$. Next we take the replacement in P_{DS_k}, then we get

$$P_{DS_k}{}^* = (\left|g_{k,1}\right|^2 \Delta p_{k,1,1} + \cdots + \left|g_{k,1}\right|^2 \Delta p_{k,1,r_{k,1}})$$
$$+\cdots+(\left|g_{k,j}\right|^2 \Delta p_{k,j,1} + \cdots + \left|g_{k,j}\right|^2 \Delta p_{k,j,r_{k,j-1}})$$
$$+\cdots+(\left|g_{k,i}\right|^2 \Delta p_{k,i,1} + \cdots + \left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1})$$

And

$$+\cdots+(\left|g_{k,N_k}\right|^2 \Delta p_{k,N_k,1} + \cdots + \left|g_{k,N_k}\right|^2 \Delta p_{k,N_k,r_{k,N_k}})$$

$P_{DS_k} - P_{DS_k}{}^* = \left|g_{k,j}\right|^2 \Delta p_{k,j,r_{k,j}} - \left|g_{k,i}\right|^2 \Delta p_{k,i,r_{k,j}+1} > 0$ since we can get $P_{DS_k} > P_{DS_k}{}^*$. By $\Delta P_{DS_k} > \Delta P_{DS_k}{}^*$, we can deduce $P_{DS_k} > P_{DS_k}{}^*$. Therefore, we have proven the adequacy part of the problem.

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