상호 코렌트로피를 이용한 복소 채널 블라인드 등화

Complex-Channel Blind Equalization Using Cross-Correntropy

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요 약

두 랜덤 변수의 상호 코렌트로피 최대화 기준은 최소자승오차 기준에 비해 탁월한 성능을 보인다. 이 논문에서 제안하는 블라인드 등화 방식은, 등화기 출력 PDF와 자기 생성 심볼 집합의 파전 PDF를 두 랜덤 변수로 하는 상호 코렌트로피에 기준을 두어 QAM과 복소 채널 환경을 위해 설계되었다. 복소 채널 통신 환경의 시뮬레이션 결과에서 상 위상 회전이 없이 크게 향상된 심볼 포인트 집중 성능을 나타냈다.

ABSTRACT

The criterionmaximizing cross-correntropy (MCC) of two different random variables has yielded superior performance comparing to mean squared error criterion. In this paper we present a complex-valued blind equalizer algorithm for QAM and complex channel environments based on cross-correntropy criterion which uses, as two variables, equalizer output PDF and Parzen PDF estimate of a self-generated symbol set. Simulation results show significantly enhanced performance of symbol-point concentration with no phase rotation in complex-channel communication.

☞ KeyWords : Cross-correntropy, Complex-valued blind equalizer, QAM;Complex channel, Parzen PDF, A self-generated symbol set, 상호-코렌트로피, 복소 블라인드 등화, 복소 채널, 자가 생성 심볼열

1. Introduction

In computer communication networks including the Internet and the wireless/mobile networks, blind equalizers to counteract multi-path effects are very useful since they do not require a training sequence. Constant modulus algorithm(CMA) is a well known blind equalization algorithm that is based on mean squared error(MSE) criterion[1].

Unlike the MSE criterion, information theoretic learning(ITL) methods introduced by Princepe have been demonstrated that the ITL-trained systems produce the output distribution closer to the distribution of desired signals compared to MSE criterion[2,3].

Recently, ITL method has developed from entropy, Euclidian distance(ED) to the extension of the fundamental definition of correlation function for random processes. The generalized correlation function is called correntropy[4] which contains higher order moments of the PDF, and cross-correntropy handles the probability that how similar two different real random variables are[5].

In some applications, signals are complex-valued such as QAM signal space. Then some concealed problems in real signal processing such as symbol-phase rotation are exposed.

Neither the criterion of ED of Probability distribution nor correntropy criterion has been applied and dealt with the complex channel problems including symbol-phase rotation.

In this paper, we introduce the cross-correntropy

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concept to complex channel blind equalization method for QAM modulation schemes in phase rotating complex channel environments. This paper is organized as follows. In Section 2, we briefly describe the conventional constant modulus blind equalizer algorithm which is based on MSE criterion. In Section 3, based on the ED minimization method using CME, its complex-version is introduced. The concept of cross-correntopy is explained briefly in Section 4. Using this cross-correntopy concept, in Section 5, the complex-valued blind equalization algorithm with a self-generated symbol set is proposed. In Section 6 simulation results obtained in severely distorted complex channels through MSE convergence and constellation comparison are presented with discussions. Finally, we presented concluding remarks in Section 7.

2. Constant Modulus Algorithm

The cost function P_{CMA} of CMA minimizes constant modulus error based on MSE criterion.

$$P_{CMA} = E[e_{CME}^2] \tag{1}$$

The constant modulus error e_{CME} for the equalizer output y_k and source signal constant modulus R_2, is defined as

$$e_{CME} = |y_k|^2 - R_2 \tag{2}$$

With weight vector W_k composed of L weights and input vector $X_k = [x_k, x_{k-1}, ..., x_{k-L+1}]^T$, the output at symbol time k can be produced as $y_k = W_k^T X_k$. Then the well known CMA[1] becomes

$$W_{k+1} = W_k - 2\mu_{CMA} \cdot X_k^* y_k (|y_k|^2 - R_2) \tag{3}$$

Employing M-ary PAM signaling systems, the level value A_m takes the following discrete values

$$A_m = 2m - 1 - M \ , \ \ m = 1,2,...,M \tag{4}$$

Then the constant modulus R_2 becomes

$$R_2 = E[|A_m|^4] / E[|A_m|^2] \tag{5}$$

3. Complex - Valued Blind Equalization Based On Ed Minimization And Cme

According to the research[6], minimizing the Euclidian distance between the two PDFs, the error signal PDF $f_E(e_{CME})$ and Dirac-delta function $\delta(e_{CME})$, the error PDF forms a sharp impulse shape located at the origin. The ED for CME is

$$ED[f_E(e_{CME}), \delta(e_{CME})] \\ = \int f_E^2(\xi)d\xi + \int \delta^2(\xi)d\xi - 2\int f_E(\xi)\delta(\xi)d\xi \tag{6}$$

where the term $\int f_E^2(\xi)d\xi$ is defined as information potential[5] IP_{CME} for CME signal, and $\int \delta^2(\xi)d\xi$ does not depend on the weights of the adaptive system. Then the cost function ED_{CME} for CME can be expressed as

$$ED_{CME} = IP_{CME} - 2f_E(e_{CME} = 0). \tag{7}$$

For convenience sake, $f_E(e_{CME}=0)$ will be referred to as PE in this paper.

In order to calculate the error PDF $f_E(e_{CME})$ non-parametrically, we need the Parzen estimator[2] using Gaussian kernel and a block of N CME samples as follows

$$f_E(e_{CME}) = \frac{1}{N}\sum_{i=1}^{N} G_\sigma(e_{CME} - e_{CME\,i})$$
$$= \frac{1}{N}\sum_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-(e_{CME} - e_{CME\,i})^2}{2\sigma^2}] \tag{8}$$

By (8) and $e_{CME} = |y_k|^2 - R_2$, the two terms in (7) becomes

$$IP_{CME} = \int f_E^2(\xi)d\xi = \frac{1}{N}\sum_{l=1}^{N}\sum_{i=1}^{N} G_{\sigma\sqrt{2}}(|y_i|^2 - |y_l|^2) \tag{9}$$

$$PE = \frac{1}{N}\sum_{i=1}^{N} G_\sigma(|y_i|^2 - R_2) \tag{10}$$

Then we obtain the following complex-valued gradient by differentiating ED_{CME} with respect to W.

$$\frac{\partial ED_{CME}}{\partial W} = \nabla_{IP_{CME},\mathrm{Re}} + j\nabla_{IP_{CME},\mathrm{Im}} - 2(\nabla_{PE,\mathrm{Re}} + j\nabla_{PE,\mathrm{Im}}) \tag{11}$$

where subscripts Re and Im indicate real part and imaginary part of a complex number.

$$\nabla_{IP_{CME},\mathrm{Re}} = \frac{1}{2N^2\sigma^3\sqrt{\pi}}\sum_{l=1}^{N}\sum_{i=1}^{N} \exp[\frac{(|y_i|^2 - |y_l|^2)}{-4\sigma^2}]$$
$$(|y_i|^2 - |y_l|^2)[y_{l,\mathrm{Re}}X_{l,\mathrm{Re}} + y_{l,\mathrm{Im}}X_{l,\mathrm{Im}}$$
$$-(y_{i,\mathrm{Re}}X_{i,\mathrm{Re}} + y_{i,\mathrm{Im}}X_{i,\mathrm{Im}})] \tag{12}$$

$$\nabla_{IP_{CME},\mathrm{Im}} \frac{1}{2N^2\sigma^3\sqrt{\pi}}\sum_{l=1}^{N}\sum_{i=1}^{N} \exp[\frac{(|y_i|^2 - |y_l|^2)}{-4\sigma^2}]$$
$$(|y_i|^2 - |y_l|^2)[y_{l,\mathrm{Im}}X_{l,\mathrm{Re}} - y_{l,\mathrm{Re}}X_{l,\mathrm{Im}}$$
$$-(y_{i,\mathrm{Im}}X_{i,\mathrm{Re}} - y_{i,\mathrm{Re}}X_{i,\mathrm{Im}})] \tag{13}$$

$$\nabla_{PE,\mathrm{Re}} = \frac{2}{N\sigma^3\sqrt{2\pi}}\sum_{i=1}^{N} \exp[\frac{(y_{i,\mathrm{Re}}^2 + y_{i,\mathrm{Im}}^2 - R_2)}{-2\sigma^2}]$$
$$(y_{i,\mathrm{Re}}^2 + y_{i,\mathrm{Im}}^2 - R_2)(y_{i,\mathrm{Re}}X_{i,\mathrm{Re}} + y_{i,\mathrm{Im}}X_{i,\mathrm{Im}}) \tag{14}$$

$$\nabla_{PE,\mathrm{Im}} = \frac{2}{N\sigma^3\sqrt{2\pi}}\sum_{i=1}^{N} \exp[\frac{(y_{i,\mathrm{Re}}^2 + y_{i,\mathrm{Im}}^2 - R_2)}{-2\sigma^2}]$$
$$(y_{i,\mathrm{Re}}^2 + y_{i,\mathrm{Im}}^2 - R_2)(y_{i,\mathrm{Im}}X_{i,\mathrm{Re}} - y_{i,\mathrm{Re}}X_{i,\mathrm{Im}}) \tag{15}$$

Employing steepest descent method and replacing index i with time index k-i+1, we can update the weights of the complex blind equalizer (we will call this MED-CME in this paper).

$$W_{k+1} = W_k - \mu_{MED-CME}\frac{\partial ED_{CME}}{\partial W} \tag{16}$$

This approach has been proposed and studied in depth in the work of [6]. As mentioned in that paper, this complex-version algorithm shows very poor performance because the cost function pushes output samples to a constant power value. This characteristic keeps the blind equalizer from dealing with channel phase distortion like rotation. To cope with this drawback, we will briefly introduce correntropy concept in the following section, and in section 5, we will propose complex channel blind equalization method based on the correntropy concept for QAM modulation schemes in phase rotating complex channel environments.

4. Cross-correntropy of two Independent PDFs

Let a nonlinear mapping Φ transform the data toan infinite dimensional reproducing kernel Hilbert space F. If two scalar random variables X and Y are statistically independent, cross-correntropy[5] is defined by

$$V(X,Y) = < E[\Phi(X)], E[\Phi(Y)] >_F \tag{17}$$

where $E[\cdot]$ and $<\cdot,\cdot>_F$ denote statistical expectation and inner product in F, respectively. From the view point of kernel methods, $f_X(\xi)=E[\Phi(X)]$ and $f_Y(\xi)=E[\Phi(Y)]$ are two points in the RKHS, and then the cross-correntropy becomes the inner product between two PDFs, that is, $\int f_X(\xi)\cdot f_Y(\xi)d\xi$. Therefore, maximizing cross correntropy (MCC) of two independent PDFs is equivalent to maximization of the inner product of two independent PDFs.

Based on this cross-correntropy criterion, we propose a new complex-valued blind equalizer algorithm for phase-distorted complex channels in the following section 5.

5. Complex-Valued Blind Equalization Algorithm Based on MCC Criterion

Normally, modulation schemes are known to receivers. Furthermore most transmitters use independent and identically distributed symbols. Under these considerations, we propose to employ MCC criterion in complex channel blind equalization using a self-generated symbol set.

For 16QAM scheme, the receiver generates 16 constellation symbol points $d_i=d_{i,\mathrm{Re}}+jd_{i,\mathrm{Im}}$ which are equally likely as

$$d_{i,\mathrm{Re}}=\begin{cases}+3: i=1,2,3,......................., N/4\\ +1: i=N/4+1, N/4+2,..., N/2\\ -1: i=N/2+1, N/2+2,...,3N/4\\ -3: i=3N/4+1,3N/4+2,....., N\end{cases}$$

$$d_{i,\mathrm{Im}}=\begin{cases}+3: i=1,2,3,......................., N/4\\ +1: i=N/4+1, N/4+2,..., N/2\\ -1: i=N/2+1, N/2+2,...,3N/4\\ -3: i=3N/4+1,3N/4+2,....., N\end{cases} \tag{18}$$

Now we maximize the cross correntropy between the two PDFs, $f_D(\xi)$ and $f_Y(\xi)$. This implies that the similarity between the PDF of desired symbols and that of system output PDF becomes maximized where $f_D(\xi)$ is constructed from a self-generated symbol set at the receiver. The MCC for this approach can be expressed as

$$\max_W CC_{DY}=\max_W \int f_D(\xi)f_Y(\xi)d\xi \tag{19}$$

Now CC_{DY} is rewritten non-parametrically using the Parzen method as

$$CC_{DY}=\frac{1}{N^2}\sum_{i=1}^{N}\sum_{l=1}^{N}G_{\sigma\sqrt{2}}(d_i-y_l) \tag{20}$$

For maximization of CC_{DY} the steepest descent method can be applied as

$$W_{new}=W_{old}+\mu_{MCC}\frac{\partial CC_{DY}}{\partial W} \tag{21}$$

The complex-valued gradient can be obtained as follows:

$$\frac{\partial CC_{DY}}{\partial W}=\nabla_{CC_{DY},\mathrm{Re}}+j\nabla_{CC_{DY},\mathrm{Im}} \tag{22}$$

where

$$\nabla_{CC_{DY},\mathrm{Re}}=\frac{1}{4N^2\sigma^3\sqrt{\pi}}$$
$$\sum_{l=1}^{N}\sum_{i=1}^{N}\exp[\frac{(d_{i,\mathrm{Re}}-y_{l,\mathrm{Re}})^2+(d_{i,\mathrm{Im}}-y_{l,\mathrm{Im}})^2}{-4\sigma^2}]$$
$$[(d_{i,\mathrm{Re}}-y_{l,\mathrm{Re}})(X_{l,\mathrm{Re}}-X_{i,\mathrm{Re}})$$
$$+(d_{i,\mathrm{Im}}-y_{l,\mathrm{Im}})(X_{l,\mathrm{Im}}-X_{i,\mathrm{Im}})] \tag{23}$$

$$\nabla_{CC_{DY},\mathrm{Im}}=\frac{1}{4N^2\sigma^3\sqrt{\pi}}$$
$$\sum_{l=1}^{N}\sum_{i=1}^{N}\exp[\frac{(d_{i,\mathrm{Re}}-y_{l,\mathrm{Re}})^2+(d_{i,\mathrm{Im}}-y_{l,\mathrm{Im}})^2}{-4\sigma^2}]$$
$$[(d_{i,\mathrm{Im}}-y_{l,\mathrm{Im}})(X_{l,\mathrm{Re}}-X_{i,\mathrm{Re}})$$
$$+(d_{i,\mathrm{Re}}-y_{l,\mathrm{Re}})(X_{l,\mathrm{Im}}-X_{i,\mathrm{Im}})] \tag{24}$$

For convenience sake, this method shall be referred to here as complex-valued MCC(CMCC) algorithm.

To investigate the robustness of the proposed algorithm CMCC to channel phase distortions over CMA, we rewrite the term CC_{DY} as a set of partitioned functions. Considering16QAM signaling, as used in our simulation in section VI, the set of outputs y_i can be partitioned according to the transmitted symbol set $A_m = \{\pm 1 \pm j, \pm 3 \pm j, \pm 1 \pm 3j, \pm 3 \pm 3j\}$ into 16 subsets as

$$R^{(+1,+j)} = \{y_i, A_m = 1+j\}, R^{(+1,-j)} = \{y_i, A_m = 1-j\}, \cdots,$$
$$R^{(-3,+3j)} = \{y_i, A_m = -3+3j\},$$
$$R^{(-3,-3j)} = \{y_i, \ A_m = -3-3j\} \quad (25)$$

Then the cross correntropy CC_{DY} with 16 subsets can be expressed as

$$CC_{DY} = \sum_{i \in R^{(+1,+j)}} G_{\sigma\sqrt{2}}(1+j-y_i) + \sum_{i \in R^{(+1,-j)}} G_{\sigma\sqrt{2}}(1-j-y_i)$$
$$+ \dots$$
$$+ \sum_{i \in R^{(-3,+3j)}} G_{\sigma\sqrt{2}}(-3+3j-y_i) + \sum_{i \in R^{(-3,-3j)}} G_{\sigma\sqrt{2}}(-3-3j-y_i) \quad (26)$$

Clearly each term in (26) is maximized when $y_i = 1+j$ for $i \in R^{(+1,+j)}$, $y_i = 1-j$ for $i \in R^{(+1,-j)}$, $\cdots$, $y_i = -3-3j$ for $i \in R^{(-3,-3j)}$, respectively. This can be interpreted that the cost function forces the output signal to have correct symbol values through adjusting weights to compensate amplitude and phase distortion induced from channel.

On the other hand, the CMA cost function (1) can be partitioned using a sample mean estimator as

$$P_{CMA} = \sum_{i \in R^{(+1+j)}} (R_2 - |y_i|^2)^2 + \sum_{i \in R^{(+1-j)}} (R_2 - |y_i|^2)^2$$
$$+ \dots + \sum_{i \in R^{(-3+3j)}} (R_2 - |y_i|^2)^2 + \sum_{i \in R^{(-3-3j)}} (R_2 - |y_i|^2)^2 \quad (27)$$

where $R_2 = E[|A_m|^4] / E[|A_m|^2] = 13.2$. Clearly each term in (28) is minimized when $|y_i|^2 = 13.2$ for all symbol regions: $i \in R^{(+1+j)}, i \in R^{(+1-j)}, \dots, i \in R^{(-3-3j)}$. This implies that the cost function of CMA pushes output samples to have a constant power 13.2 regardless of symbol classes. This means that the phase shift can not be detected in CME-based algorithms. From this analysis, the intrinsic difference of characteristics between the proposed CMCC algorithm and CME-based blind algorithms.

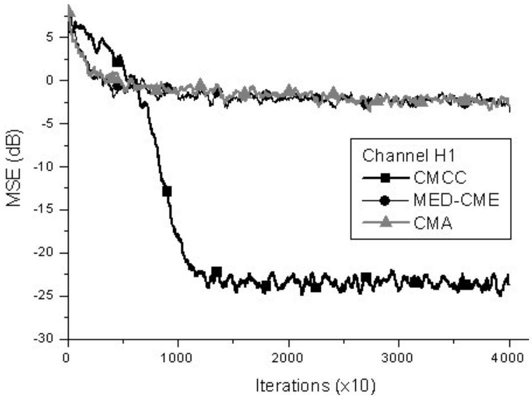
6. Results and Discussion

In this section the performance of complex-blind algorithms is investigated in 16QAM and complex channel environment. The three complex blind algorithms are considered: the CMA in (3), MED-CME in (16), CMCC in (21). Two complex channel models $H_1(z)$[7] and $H_2(z)$[8] are considered in this simulation and a zero mean whiteGaussian noise sequence with variance 0.001 was added to yield the channel output.

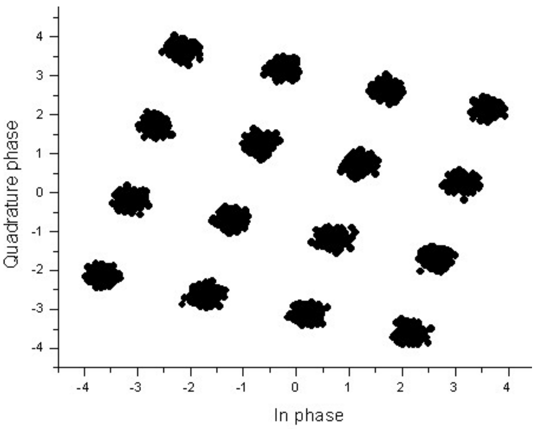
$$H_{1,\mathrm{Re}}(z) = -0.005 + 0.009z^{-1} - 0.024z^{-2}$$
$$+ 0.854z^{-3} - 0.218z^{-4} - 0.049z^{-5} - 0.016z^{-6}$$
$$H_{1,\mathrm{Im}}(z) = -0.004 + 0.030z^{-1} - 0.104z^{-2}$$
$$+ 0.520z^{-3} + 0.273z^{-4} - 0.074z^{-5} + 0.020z^{-6} \quad (28)$$

$$H_{2,\mathrm{Re}}(z) = -0.141z^{-1} + 0.95z^{-2}$$
$$+ 0.27z^{-3} - 0.078z^{-4}$$
$$H_{2,\mathrm{Im}}(z) = -0.004z^{-1} - 0.919z^{-2}$$
$$+ 0.37z^{-3} - 0.089z^{-4} \quad (29)$$

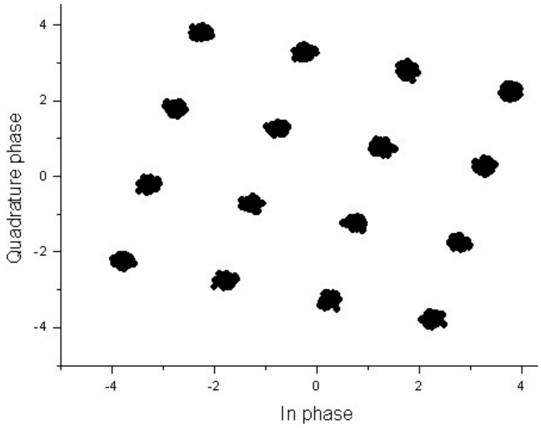
The convergence parameters are set as: μ_{MCC}=0.0000005 for CMA, $\mu_{MED\text{-}CME}$=0.005 for



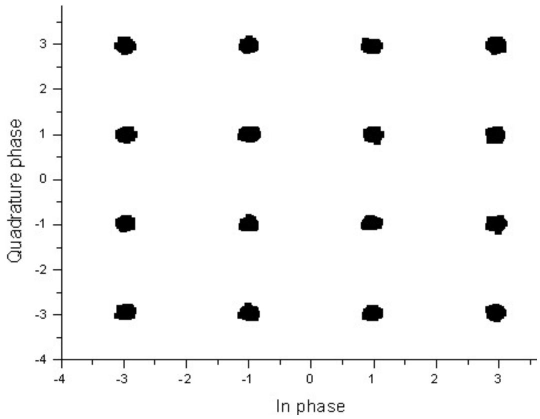
(Fig. 1) MSE convergence comparison for channel H1.



(Fig. 2) Constellation performance of CMA for H1.

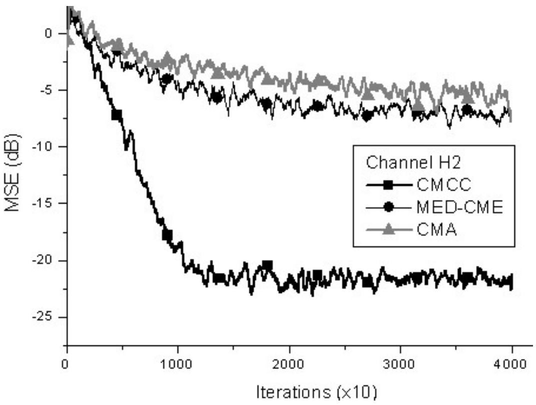


(Fig. 3) Constellation performance of MED-CME for H1.

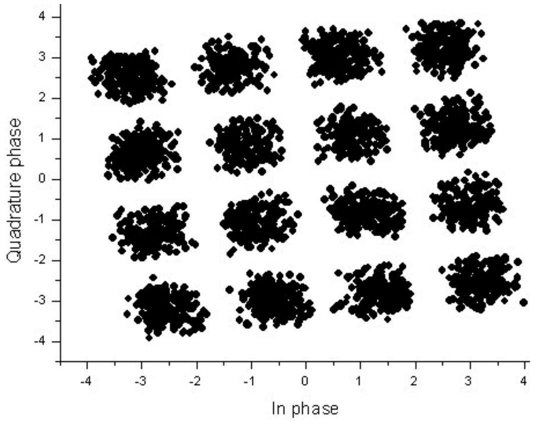


(Fig. 4) Constellation performance of CMCC for H1.

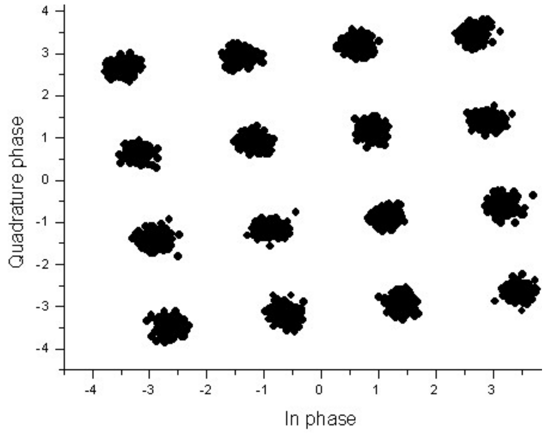
MED-CME, and μ_{MCC}=0.001 for CMCC, respectively. The kernel size σ for ITL algorithms is 15.0 for MED-CME and 0.5 for CMCC. The convergence results are illustrated in Fig. 1 for channel model H1, and in Fig. 5 for channel model H2. For H1 in Fig. 1, the CME based CMA and MED-CME have produced very poor minimum MSE performance but CMCC shows significant convergence performance. Compared to CME based algorithms, CMCC yields a minimum MSE value lower by over 20 dB. This enhancement can be clearly understood by examining their constellation in Fig. 2-4. CMA in Fig. 2 and MED-CME in Fig. 3 can not solve the problem of channel phase distortion, but CMCC produces output points that are well concentrated to the exact constellation symbol points in Fig. 4. In the severer channel model H2, these advantages of CMCC are observed clearly. The CME-based algorithms shows inferior MSE learning performance as in Fig. 5, and MED-CME based on ITL is better than CMA by about 3 dB. Most of all, CMCC yields performance superiority of 17dB gain compared to CMA and 15 dB to MED-CME. The constellation performance depicted



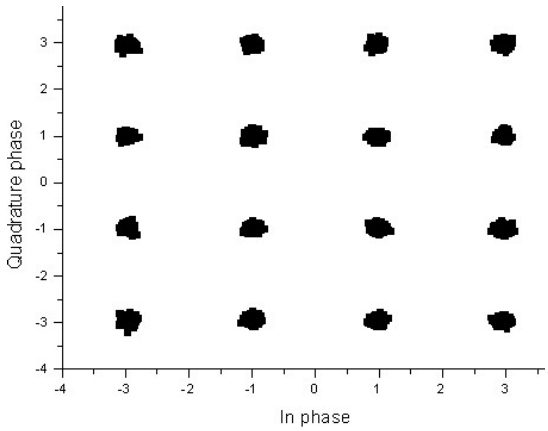
(Fig. 5) MSE convergence comparison for channel H2



(Fig. 6) Constellation performance of CMA for H2.



(Fig. 7) Constellation performance of MED-CME for H2.



(Fig. 8) Constellation performance of CMCC for H2.

in Fig. 6-8 gives more insight to the performance enhancement of CMCC. In Fig. 6 for CMA I, very dispersed and phase-distorted constellation is observed. The ED-based MED-CME shows more concentrated constellation than CMA but it still does not solve the channel phase problem. The constellation result of CMCC based on correntropy and a self-generated symbol set for the channel model H2 proves its robustness to channel severity by producing output points that are closely concentrated to the exact constellation symbol points.

7. Conclusion

In this paper, the maximization of cross-correntropy concept has been applied to complex channel blind equalization for QAM systems. The criterion is to maximize the similarity between the output PDF and the PDF of a self-generated symbol set at the receiver. In the analysis of the robustness of the cross-correntropy-based complex-valued blind algorithm to channel phase distortions, it is revealed that the cost function forces the output signal to have correct symbol values and compensate amplitude and phase distortion simultaneously without any phase compensation process, whereas the cost functions based on constant modulus push output samples to have a constant power regardless of symbol classes.

Simulation results for severely distorted complex channels proved that the cross-correntropy based complex channel blind algorithm yields closely concentrated output symbols to the exact constellation symbol points of QAM systems with no need of phase compensation. This makes us conclude that the cross-correntropy-based complex channel blind method can be a good candidate for blind equalization for complex channel and QAM applications.

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