충격성 잡음 채널의 블라인드 등화를 위한 최대 영-확률 알고리듬에 대한 성능 분석

Performance Analysis of Maximum Zero-Error Probability Algorithm for Blind Equalization in Impulsive Noise Channels

김 남 용*
Namyong Kim

요 약

이 논문은 충격성 잡음 환경에 대해 상수 모듈러스 오차(CME)와 가우시안 커널에 근거한 블라인드 등화 알고리듬의 성능 분석을 보이고 있다. CME와 평균 자승 오차(MSE)에 근거한 상수 모듈러스 알고리듬(CMA)는 충격성 잡음 환경에서 수렴에 실패한다. 이런 충격성 잡음에 대한 내항성을 위해 최근에 소개된 코렌트로피 블라인드 등화 알고리듬도 PAM 변조 방식에서는 만족할 만한 결과를 보이지 못한다. 원래 가우시안 잡음 환경을 위해 제안되었던 최대 영-확률 블라인드 알고리듬(MZEP-CME)이 충격성 잡음 환경에서도 탁월한 성능을 보인다는 것이 이 논문의 이론적, 그리고 시뮬레이션을 통한 분석에 의해 입증된다. MZEP-CME 알고리듬의 가우시안 커널은 충격성 잡음에 의해 발생하는 출력 신호 전력과 CME 사이의 큰 차이에 민감하게 반응하지 못하게 하는 강한 영향력을 발휘한다.

ABSTRACT

This paper presentsthe performance study of blind equalizer algorithms for impulsive-noise environments based on Gaussian kernel and constant modulus error(CME). Constant modulus algorithm(CMA) based on CME and mean squared error(MSE) criterion fails in impulsive noise environment. Correntropy blind method recently introduced for impulsive-noise resistance has shown in PAM system not very satisfying results. It is revealed in theoretical and simulation analysis that the maximization of zero-error probability based on CME(MZEP-CME) originally proposed for Gaussian noise environments produces superior performance in impulsive noise channels as well. Gaussian kernel of MZEP-CME has a strong effect of becoming insensitive to the large differences between the power of impulse-infected outputs and the constant modulus value.

☞ KeyWords : Impulsive noise, blind equalization, maximization of zero-error probability, constant modulus error, correntropy, CMA. 충격성잡음 블라인드 등화, 영확률 최대화, 상수 모듈러스 오차, 코렌트로피

1. Introduction

Communication systems are interfered with not only Gaussian noise but also impulsive noise from a variety of impulse noise sources[1,2]. Impulsive noise induces large instantaneous system output and error which makes the system fail to produce desirable performance. In applications of multipoint communication including the Internet, the ATM, broadcast and the wireless/mobile networks[3], blind equalizers to counteract multipath effects and severe channel noise are very useful since they do not require a training sequence[4,5]. The most commonly used constant modulus algorithm(CMA) for blind equalization is based on mean squared error(MSE) criterion[6]. The CMA is designed to minimize the average of the constant modulus error(CME) between equalizer output power and constant modulus. It is known that CMA based on MSE often fails to converge in impulsive noise environment.

* 정 회 원 : 강원대학교 전자정보통신공학부 교수
namyong@kangwon.ac.kr
[2010/04/05 투고 - 2010/04/12 심사 - 2010/08/02 심사완료]

Unlike MSE criterion that utilizes error energy, the information-theoretic learning(ITL) method, based on a combination of a nonparametric probability density function(PDF) estimator and a procedure to compute entropy, has been introduced and well developed[7]. As a robust ITL algorithm, minimization of errorentropy(MEE) has been developed by Principe, Erdogmus and co-workers[8]. In their work it has also been shown that Renyi's quadratic entropy expression with Parzen PDF estimator is negatively proportional to the logarithmic value of the information potential(IP) of error samples. Since logarithm is a monotonic function, IP is maximized in MEE instead of minimizing Renyi's entropy. Therefore, MEE criterion can be considered as maximization of information potential(MIP). MEE has shown superior performance as an alternative to MSE in supervised channel equalization applications[9]. In our previous work[10], as an alternative to MEE algorithm only for supervised training, maximizing zero-error probability for blind equalization by employing CME(MZEP-CME) has been proposed for Gaussian noise environments.

To cope with impulsivenoise problem, a blind method for partial response system based on correntropy has been introduced[11]. This method has robustness against impulsive noise but has not shown satisfying results in PAM systems in our research works. So in this paper we analyze that how the MZEP-CME algorithm is robust against impulsive noise theoretically and prove it to be very superior in that situation by computer simulations.

2. Impulsive Noise Model

There exist the background Gaussian noise and impulsive noise together in common impulsive-noise added communication channels. The background noise is AWGN, of which variance is σ^2_{GN}. The impulsive noise occurs according to a Poisson process and the average number of Poisson occurrence impulses per information symbol duration is defined as ε. The amplitude distribution of impulsive noise has a Gaussian with variance σ^2_{IN}. This noise model is widely used as an impulsive noise model in[11,12].

The PDF of the background AWGN is expressed as

$$f_{GN}(\xi)=\frac{1}{\sigma_{GN}\sqrt{2\pi}}\exp[\frac{-\xi^2}{2\sigma^2_{GN}}] \qquad (1)$$

The impulsive noise with Gaussian amplitude has the PDF expression as

$$f_{IN}(\xi)=(1-\varepsilon)\cdot\delta(\xi)+\frac{\varepsilon}{\sigma_{IN}\sqrt{2\pi}}\exp[\frac{-\xi^2}{2\sigma^2_{IN}}] \qquad (2)$$

The total noise is a sum of the two random processes and the PDF form of the total noise is obtained by taking the convolution of (1) and (2). From the convolution process we obtain the following total noise PDF expression.

$$f_{NOISE}(\xi)=\frac{1-\varepsilon}{\sigma_1\sqrt{2\pi}}\exp[\frac{-\xi^2}{2\sigma_1^2}]+\frac{\varepsilon}{\sigma_2\sqrt{2\pi}}\exp[\frac{-\xi^2}{2\sigma_2^2}] \quad (3)$$

where $\varepsilon<1$, $\sigma_2=\sqrt{\sigma^2_{GN}+\sigma^2_{IN}}$, $\sigma_1=\sigma_{GN}$, and $\sigma_1^2<<\sigma_2^2$.

3. Correntropy Algorithm

For a tapped delay line (TDL) equalizer with weight vector W of L elements in training-aided equalization, error sample e_k between the desired training symbol d_k and output y_k are produced by $e_k=d_k-y_k=d_k-W_k^TX_k$ at time k where the equalizer input vector is $X_k = [x_k, x_{k-1}, x_{k-2}, ..., x_{k-L+1}]^T$. Channel equalization without the aid of a training sequence d_k is referred to as blind channel equalization. As a new blind signal processing approach for the linear TDL filter structure, correntropy concept has been introduced by Santamaria [11].

Correntropy is a similarity measure that has the analogy with the autocorrelation of two random processes. For a discrete-time stationary stochastic process, the correntropy function is defined as

$$V_X[m] = E[G_\sigma(X_k - X_{k-m})] \quad (4)$$

where $E[\cdot]$ denotes statistical expectation and $G_\sigma(\cdot)$ is a zero-mean Gaussian kernel with standard deviation σ as

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-x^2}{2\sigma^2}] \quad (5)$$

And through the sample mean, it can be estimated using N samples as

$$V_X[m] = \frac{1}{N-m+1} \sum_{k=m}^{N} G_\sigma(X_k - X_{k-m}) \quad (6)$$

Since the correntropy function conveys information about the PDF and correlation of the signal, the authors in [11] proposed the following cost function.

$$P_{CE} = \sum_{m=1}^{M} (V_S[m] - V_Y[m])^2 \quad (7)$$

where $V_S[m]$ is the source correntropy, $V_Y[m]$ is the equalizer output correntropy, and M is the number of lags. The cost function can be minimized by using a gradient descent approach, and then the correntropy blind algorithm is obtained as

$$W_{k+1} = W_k - \mu_{CE} \frac{1}{(N-m+1)\sigma^2} \sum_{m=1}^{M} \sum_{i=k-N+m}^{k} (V_S[m] - V_Y[m]) \cdot G_\sigma(y_i - y_{i-m}) \cdot (y_i - y_{i-m})(X_i - X_{i-m}) \quad (8)$$

For a given correlated source signal such as in a partial response system introduced to make an efficient use of the available bandwidth, the theoretical correntropy function is obtained first. In adaptation process, output correntropy function is calculated at each iteration and weight vector is adjusted to minimize difference between the two correntropy functions. When the source signal is i.i.d. as in most modulation schemes, weights are updated to force output signal correntropy to follow the source signal correntropy so that blind equalization is carried out.

4. Maximization of Zero-Error Probability for Impulsive Noise Channels

In this section we introduce the criterion of Zero-Error Probability for Blind Algorithm [10] which has been proposed for Gaussian noise environments and analyze it theoretically how adequate the algorithm is in impulsive noise channels. The criterion has been developed from a supervised method that tries to create a concentration of error samples near zero, so the Euclidian distance $D[f_E(e), \delta(e)]$ between the two PDFs, the error signal PDF $f_E(e)$ and Dirac-delta function $\delta(e)$ is firstly minimized.

$$D[f_E(e), \delta(e)] = \int f_E^2(\xi)d\xi + \int \delta^2(\xi)d\xi - 2\int f_E(\xi)\delta(\xi)d\xi \quad (9)$$

Substituting V_e for $\int f_E^2(\xi)d\xi$ in (1), where V_e is defined as information potential in[7], we obtain

$$D[f_E(e),\delta(e)] = V_e + C - 2f_E(0) \tag{10}$$

where $\int \delta^2(\xi)d\xi$ is considered as a constant C since it has nothing to do with system weights. Minimizing $D[f_E(e), \delta(e)]$ leads to minimization of V_e and maximization of $f_E(0)$, simultaneously. It is noticeable that minimization of V_e, which indicates maximization of error entropy, forces the error samples to achieve a uniform distribution. This is in discord with MEE criterion that maximizes V_e to force the error samples to be near zero[8].

To avoid this conflict, the authors in[10] proposed only to maximize the third term $f_E(0)$ while omitting the error information potential V_e from (9). The constant term can also be removed from (9) because it is not controllable with equalizer weight W. From this procedure the new criterion for errors, maximum zero-error probability (MZEP) criterion, can be obtained as follows:

$$\max_W f_E(0) \tag{11}$$

Using Gaussian kernel and a block of N error samples as

$$f_E(e) = \frac{1}{N}\sum_{i=1}^{N} G_\sigma(e-e_i) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\sigma\sqrt{2\pi}}\exp[\frac{-(e-e_i)^2}{2\sigma^2}] \tag{12}$$

With e=0, the zero-error probability $f_E(0)$ reduces to

$$f_E(0) = \frac{1}{N}\sum_{i=1}^{N} G_\sigma(-e_i) \tag{13}$$

The gradient is evaluated from

$$\frac{\partial f_E(0)}{\partial W} = \frac{1}{\sigma^2 N}\sum_{i=1}^{N} e_i \cdot G_\sigma(-e_i)\cdot\frac{\partial y_i}{\partial W} \tag{14}$$

Then, the supervised MZEP algorithm becomes

$$W_{k+1} = W_k + \frac{\mu}{\sigma^2 N}\sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(-e_i)\cdot X_i \tag{15}$$

where μ is the step-size for adaptation control.

In comparison with MEE algorithm in [8] as

$$W_{k+1} = W_k + \frac{\mu}{2\sigma^2 N^2}\sum_{i=k-N+1}^{k}\sum_{j=k-N+1}^{k}(e_j - e_i) \cdot G_{\sigma\sqrt{2}}(e_j - e_i)[X_j - X_i] \tag{16}$$

We see that MEE in (16) is computationally cumbersome due to the $O(N^2)$ complexity but that of MZEP in (15) requires only $O(N)$.

Now we develop the supervised algorithm into the unsupervised (blind) one. Many of blind equalization algorithms employ CMA that minimizes constant modulus error $e_{CMA} = |y_k|^2 - R_2$ based on MSE criterion [6] such as

$$P_{CMA} = E[(|y_k|^2 - R_2)^2] \tag{17}$$

where $R_2 = E[|d_k|^4]/E[|d_k|^2]$.

By minimizing the cost function P_{CMA} with the step-size parameter μ_{CMA}, we obtain the following CMA algorithm:

$$W_{k+1} = W_k - 2\mu_{CMA} \cdot X_k^* \cdot y_k \cdot (|y_k|^2 - R_2) \tag{18}$$

For Gaussian noise cases, averaging CMEs taken from different time instants discards the effects of the Gaussian noise, but a single large, impulsive noise sample can dominate these sums and defeat the averaging. In (18), a single big impulsive noise sample can produce a large output sample y_k and the weight update process (18) may become very unstable. To avoid this significant flaw induced from outliers, we insert $e_{CMA}=|y_k|^2-R_2$ into (12) and using a block of past output samples, we have

$$f_E(e_{CMA})=\frac{1}{N}\sum_{i=0}^{N-1}G_\sigma(e_{CMA}-[|y_{k-i}|^2-R_2])$$
$$=\frac{1}{N}\sum_{i=0}^{N-1}\frac{1}{\sigma\sqrt{2\pi}}\exp[\frac{-(e_{CMA}-[|y_{k-i}|^2-R_2])^2}{2\sigma^2}] \quad (19)$$

Letting e_{CMA} be zero, the probability $f_E(e_{CMA}=0)$ reduces to

$$f_E(e_{CMA}=0)=\frac{1}{N}\sum_{i=0}^{N-1}G_\sigma(-[|y_{k-i}|^2-R_2]) \quad (20)$$

Compared to MSE-based cost function (17), we can notice that outliers induced from impulse-infected large output samples are cut out by the Gaussian kernel function. This proves that the MZEP cost function has a superior robustness against impulsive noise in blind channel equalization applications.

Now we derive a gradient ascent method for the maximization of the cost function (20) and obtain the following MZEP-CME algorithm.

$$W_{k+1}=W_k+\mu_{MZEPCME}\cdot\frac{2}{\sigma^2 N}\sum_{i=0}^{N-1}G_\sigma(|y_{k-i}|^2-R_2)$$
$$\cdot(R_2-|y_{k-i}|^2)\cdot y_{k-i}\cdot X_{k-i}^*, \quad (21)$$

In this weight update equation we can notice that MZEP-CME has a feature of cutting out outliers by employing Gaussian kernel for CME that could be very large at any instant due to impulsive noise.

Gaussian kernel has the key to mitigation of impulsive noise thanks to the cutting out effect on very large outliers. Now the second key factor can be what the outliers consist of. The outliers of MZEP-CME in (20) and (21) are excessively large output powers from the constant modulus.

On the other hand, correntropy blind algorithm (8) cuts out only large differences between outputs themselves. These large differences could occur not only from impulsive noise but also between two outputs corresponding to the largest positive transmitted symbol and the largest negative output symbol, so correntropy blind algorithm can not guarantee that it efficiently cuts out outliers come only from impulsive noise.

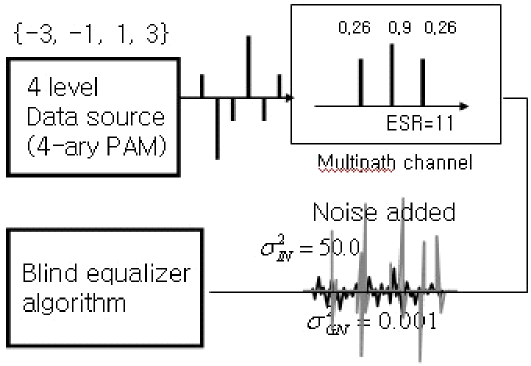
5. Simulation Results and Discussion

In this section, we test the convergence performance of three blind equalizer algorithms in impulsive-noise added multipath fading channel environment. The CMA, blind correntropy and the MZEP-CME are compared in their MSE convergence and error distribution performance. PAM modulation scheme is considered and the all four levels (-3, -1, 1, 3) are equally likely to be transmitted. The transmitted signal is distorted by an multipath channel: $H(z)$=0.26+ 0.93z^{-1}+0.26z^{-2}, then added with a zero-mean white impulsive noise n_k, generated according to the following Gaussian mixture model as described in previous sectionas in Fig. 1:

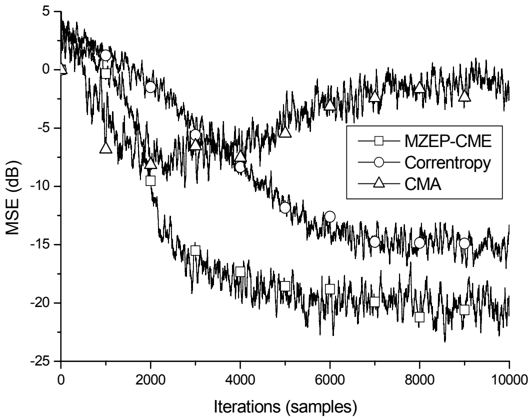
$$f_{NOISE}(n_k)=\frac{1-\varepsilon}{\sigma_1\sqrt{2\pi}}\exp[\frac{-n_k^2}{2\sigma_1^2}]+\frac{\varepsilon}{\sigma_2\sqrt{2\pi}}\exp[\frac{-n_k^2}{2\sigma_2^2}] \quad (22)$$

where we use ε=0.03, $\sigma_1^2=\sigma_{GN}^2=0.001$ and $\sigma_2^2=\sigma_{GN}^2+\sigma_{IN}^2$=50.001.

Fig. 2 shows the MSE convergence curves. A 11-tap TDL equalizer was used and initialized with the center weight set to unity and the rest to zero. The step-size for MZEP-CME is 0.02. The step-size



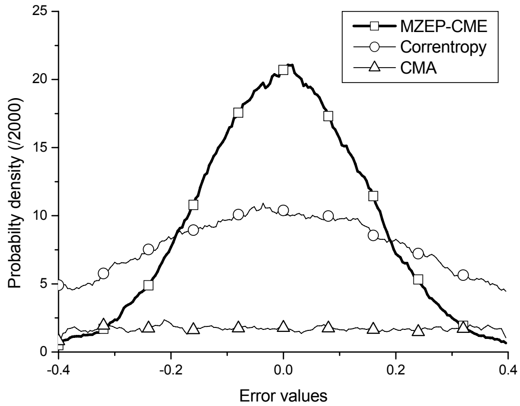
(Fig. 1) Simualtion system with impulsive noise addition



(Fig. 2) MSE convergence performance under impulsive noise.

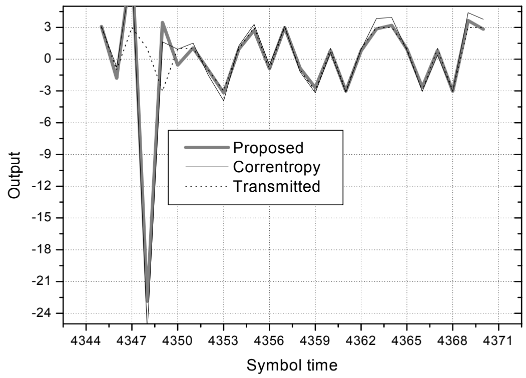
for blind correntropy algorithm and CMA are μ_{CE}=0.01 and μ_{CMA}=0.000001, respectively. The number of lags is M=20 and data-block size N=30. And kernel sizes σ for MZEP-CME and correntropy are 6 and 2.8, respectively.

We see that CMA fails to converge even for the small step-size. On the other hand, the blind algorithms based on Gaussian kernel method converge well. Comparedto the correntropy algorithm, the MSE curve for MZEP-CME reaches lower steady state MSE and the minimum MSE performance enhancement is above 5 dB. Fig. 3

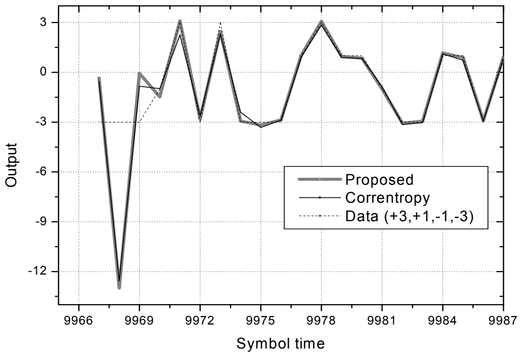


(Fig. 3) Probability density for errors under impulsive noise.

depicts the estimated error probability densities of the algorithms. Their performance differences are shown more clearly. The error values of CMA appear not to be gathered well around zero, but the correntropy and MZEP-CME produce error distribution still concentrated around zero. Clearly, MZEP-CME yields superior equalizer-error PDF performance. In accordance with the theoretical analysis of the robustness to impulsive noise of the blind algorithms, Gaussian kernel based blind algorithms show good performance, and particularly, MZEP-CME employing Gaussian kernel for CME and retaining the ability to cut efficiently out outliers caused by impulsive noise reveals conspicuously successful error performance under impulsive noise. To investigate equalizer output behavior right after being afflicted with an impulsive noise sample, two cases of output comparison are depicted in Fig. 4 and 5. In the middle of convergence, in Fig. 4, after a big impulsive noise, the output samples of correntropy algorithm are shown to be more sensitive and produce incorrect output values as at the symbol time 4353, 4363 and 4364 than the proposed algorithm. Even after convergence we observe similar performance difference as in Fig. 5. The



(Fig. 4) Output comparison in the middle of convergence process.



(Fig. 5) Output comparison after convergence.

output samples of the proposed algorithm at symbol time 9974 and 9975 yield almost correct symbol value, -3, but those of correntropy algorithm do not, though the performance inferiority after convergence is milder than in the middle of convergence process.

6. Conclusion

This paper presented the performance study of Gaussian kernel based blind equalizer algorithms for impulsive-noise added multipath communication channel environments. CMA commonly used for blind equalization and based on MSE criterion is revealed to fail in impulsive noise environment in theoretical and simulation analysis. However, blind methods using Gaussian kernel have the net effect of reducing the contribution of samples that are far away from the mean value of the error distribution and so behave insensitive to the large value errors.

Correntropy blind methodrecently introduced for impulsive-noise resistance has shown acceptable robustness against impulsive noise, but in PAM communication system model with impulsive noise environments, the correntropy blind method has not shown very satisfying results. On theother hand, MZEP-CME algorithm produces significant performance. Unlike the correntropy method, Gaussian kernel of MZEP-CME has a strong effect of becoming insensitive to the large differences between the power of impulse-infected outputs and constant modulus value.

From these analysis and observations, we conclude that MZEP-CME can surprisingly outperform the CMA and correntropy blind algorithm in impulsive noise environments.

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◐ 저 자 소 개 ◑

김 남 용 (Namyong Kim)

1986년 연세대학교 전자공학과 졸업(학사)
1988년 연세대학교 대학원 전자공학과 졸업(석사)
1991년 연세대학교 대학원 전자공학과 졸업(박사)
1992년~1998년 관동대학교 전자통신공학과 부교수
1998년~현재 강원대학교 전자정보통신공학부 교수
관심분야 : Adaptive equalization, RBFN algorithms, ITL algorithms, Odor sensing systems etc.
E-mail : namyong@kangwon.ac.kr