Distributed Space-Time Coded Non-Orthogonal DF Protocols with Source Antenna Switching

Xianglan Jin, Jae-Dong Yang, Jong-Seon No, and Dong-Joon Shin

Abstract: In this paper, a new distributed space-time coded (DS-TCed) non-orthogonal decode-and-forward (NDF) protocol with source antenna switching (SAS) is proposed, where two antennas associated with each radio frequency chain can be alternatively used in the first and second phases. Several DSTC schemes for the NDF with SAS (NDF-SAS) protocol are proposed and their average pairwise error probability for the error-free source-relay (SR) channel is also derived. The simulation results show that the NDF-SAS protocol achieves larger diversity order than the NDF protocol under the error-free and erroneous SR channels.

Index Terms: Distributed space-time code (DSTC), diversity order, non-orthogonal decode-and-forward (NDF), pairwise error probability (PEP), source antenna switching (SAS).

I. INTRODUCTION

In wireless communication systems, the spatial diversity can be achieved by multiple independent paths between multiple antennas at the transmitter and the receiver, possibly in conjunction with space-time block codes (STBCs). If relays are utilized, there also exist additional independent paths between the transmitter (source) and the receiver (destination) through relays. Such a system is called a cooperative network, where the cooperative diversity can be achieved. In a two-phase cooperative protocol as shown in Fig. 1, the source broadcasts signal to the relay and destination in the first phase, and the relay transmits signal to the destination in the second phase. A protocol is called non-orthogonal or orthogonal depending on whether the source continues to transmit in the second phase or not.

Recently, cooperative networks have been extensively studied [1]–[3]. In [1], Laneman and Wornell proposed the repetition and space-time algorithms to achieve cooperative diversity, where the mutual information and outage probability of orthogonal decode-and-forward (ODF) protocol are analyzed. In [2], the design of distributed space-time codes (DSTCs) using

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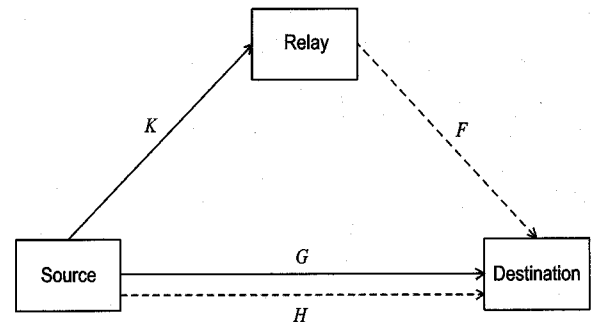
Fig. 1. A cooperative NDF protocol using one relay. The solid line denotes the first phase transmission and the dashed line denotes the second phase transmission.
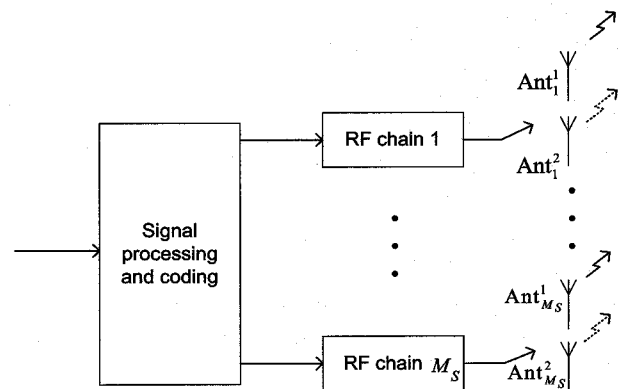

Fig. 2. The transmission at the source for the NDF-SAS protocol. The solid arrow denotes the first phase transmission and the dotted arrow denotes the second phase transmission.

generalized quasi-orthogonal STBC (QOSTBC) is proposed, in which any number of relays can be employed to increase the diversity order. In addition, suboptimal linear decoder can be used to decrease the complexity, while the full diversity order is achievable. Rajan and Rajan [3] proposed generalized non-orthogonal amplify-and-forward (NAF) protocols with single antenna at each of source, relay, and destination. They also proved that three different protocols have the same diversity order $R + 1$, where R is the number of relays and the first and second protocols are similar to the NAF protocol and the orthogonal amplify-and-forward protocol, respectively. The reason why they have the same diversity order is that the signals containing the same information transmitted from the source experience the same fading in the first and second phases.

In order to increase the diversity order of non-orthogonal cooperative networks by utilizing independent paths, two antennas associated with the same radio frequency (RF) chain can be alternatively used in the first and second phases as shown in Fig. 2.

Let M_S be the number of RF chains at the source and each of RF chains has two transmit antennas. Then, M_S transmit antennas corresponding to the first antenna at each RF chain are used in the first phase and the other M_S transmit antennas are used in the second phase. This scheme is called source antenna switching (SAS), which can be easily applied to the case of arbitrary number of antennas at each RF chain. Since RF chain consists of expensive hardware blocks such as analog-to-digital converters, mixers, and high power amplifiers, by adding antennas with negligible additional cost, the SAS scheme can be practically implemented to increase the diversity order. Clearly, the SAS scheme can be used for both NAF and non-orthogonal decode-and-forward (NDF) protocols. However, it is difficult to derive the exact PEPs for NAF and NDF protocols in the erroneous SR channel. Therefore, we will analyze the SAS scheme for NDF protocol under the assumption of error-free source-relay (SR) channel together with the numerical analysis of it for both error-free and erroneous SR channels.

In this paper, some DSTCs using the orthogonal STBC (OS-TBC) [5], coordinate interleaved STBC (CISTBC) [6], and QOSTBC [7] for the NDF with SAS (NDF-SAS) protocol are proposed. In order to compare the achievable diversities of NDF-SAS and NDF protocols, their average pairwise error probabilities (PEPs) for the error-free SR channel are derived. From the average PEP, the union bounds on the average bit error probabilities (BEPs) could also be derived. In order to compare the diversity orders, we simulate the average BEPs of some DSTCs of the NDF-SAS and NDF protocols for both error-free and erroneous SR channels. The numerical results show that the NDF-SAS protocol achieves larger diversity than the conventional NDF protocol. The results also show that the erroneous SR channel case with $\sigma_{SR}^2 \geq \sigma_{SD}^2 = \sigma_{RD}^2$ shows the performance close to that of the error-free SR channel case.

The paper is organized as follows. In Section II, the SAS scheme and the system model are explained. The several DSTC schemes for the NDF-SAS protocol are also proposed. In Section III, the average PEPs of the proposed DSTC schemes of the NDF-SAS and NDF protocols for the error-free SR channel are derived. We also provide the numerical results of the average BEPs for the error-free and erroneous SR channels in Section IV. The conclusion is given in Section V.

The following notations will be used in this paper: Capital letter denotes matrix; I_n denotes the $n \times n$ identity matrix; $\| \cdot \|$ denotes the Frobenius norm of a matrix; $E[\cdot]$ denotes the expectation; the superscripts $(\cdot)^*$ and $(\cdot)^\dagger$ denote the complex conjugation and the complex conjugate transpose, respectively; $\mathbb{C}^{n \times m}$ denotes a set of $n \times m$ complex matrix; For $A \in \mathbb{C}^{n \times m}$, $A \sim \mathcal{CN}(0, \sigma^2 I_{nm})$ denotes that the elements of A are independent and identically distributed (i.i.d.) circularly symmetric Gaussian random variables with zero mean and variance σ^2.

II. NDF-SAS PROTOCOL

A. System Model

A cooperative NDF network with one source, one relay, and one destination is shown in Fig. 1 on the assumption of half duplex transmission. It is assumed that the channels are frequency-flat slow fading and the channel coefficients do not change from

the first phase to the second phase. It is also assumed that the channel state information (CSI) of SR channel is available at the relay and destination and the CSIs of source-destination (SD) and relay-destination (RD) channels are available at the destination.

The source broadcasts signal to the relay and destination during the first phase and in the second phase, the source transmits an STBC and the relay also sends a reencoded STBC using the decoded symbols from the received signal in the first phase. Although the source transmits signal twice through two phases, DSTCs cannot increase the diversity order because the SD channel remains the same for the first and second phases. In order to increase the diversity order, the SAS scheme, i.e., two transmit antennas associated with each RF chain at the source is used as shown in Fig. 2. Then, the channel coefficients of the SD channel in the first and second phases become independent and the achievable diversity can be improved. Next, the system model will be described in detail.

Let L be the number of transmitted data symbols through the first and second phases. Then, in the first phase, the source broadcasts $M_S \times T_1$ codeword $X_1(\mathbf{x})$ constructed from L data symbols $\mathbf{x} = (x_1, x_2, \cdots, x_L)$ to the relay and destination with average transmit power p_1/M_S for each active antenna and in the second phase, the relay constructs a DSTC using the decoded signal $\mathbf{x}_R = (x_1^R, x_2^R, \cdots, x_L^R)$ together with the source and transmits it in the second phase. Let $X_2(\mathbf{x}) \in \mathbb{C}^{M_S \times T_2}$ and $X_3(\mathbf{x}_R) \in \mathbb{C}^{M_R \times T_2}$ be the codeword matrices transmitted from the source and relay with average transmit powers p_2/M_S and p_3/M_R for each active antenna, respectively, where M_R is the number of antennas at the relay. The received signal matrices at the relay and destination in the first phase can be written as

$$Y_R = \sqrt{\frac{p_1}{M_S}} K X_1(\mathbf{x}) + N_R, \qquad (1)$$

$$Y_{D_1} = \sqrt{\frac{p_1}{M_S}} G X_1(\mathbf{x}) + N_{D_1}, \qquad (2)$$

respectively, where $K \in \mathbb{C}^{M_R \times M_S}$ and $G \in \mathbb{C}^{M_D \times M_S}$ consist of channel coefficients of the SR and SD channels in the first phase, which are circularly symmetric Gaussian distributed as $\mathcal{CN}(0, \sigma_{SR}^2 I_{M_R M_S})$ and $\mathcal{CN}(0, \sigma_{SD}^2 I_{M_D M_S})$, respectively, and M_D is the number of antennas at the destination. $N_R \in \mathbb{C}^{M_R \times T_1}$ and $N_{D_1} \in \mathbb{C}^{M_D \times T_1}$ represent the noise matrices with distribution $\mathcal{CN}(0, \sigma^2 I_{M_R T_1})$ and $\mathcal{CN}(0, \sigma^2 I_{M_D T_1})$ at the relay and destination, respectively. $\rho = 1/\sigma^2$ is the parameter linearly proportional to the average transmit signal to noise ratio (SNR).

The received signal at the destination in the second phase is given as

$$Y_{D_2} = \sqrt{\frac{p_2}{M_S}} H X_2(\mathbf{x}) + \sqrt{\frac{p_3}{M_R}} F X_3(\mathbf{x}_R) + N_{D_2}$$

where $H \in \mathbb{C}^{M_D \times M_S}$ and $F \in \mathbb{C}^{M_D \times M_R}$ consist of channel coefficients of the SD and RD channels in the second phase, which are distributed as $\mathcal{CN}(0, \sigma_{SD}^2 I_{M_D M_S})$ and $\mathcal{CN}(0, \sigma_{RD}^2 I_{M_D M_R})$, respectively. $N_{D_2} \in \mathbb{C}^{M_D \times T_2}$ represents the noise matrix with distribution $\mathcal{CN}(0, \sigma^2 I_{M_D T_2})$ in the second phase.

In this paper, the NDF-SAS protocol with independent G and H is compared to the conventional NDF with $G = H$.

B. Distributed Space-Time Codes for NDF-SAS Protocol

In this subsection, several DSTC schemes of the NDF-SAS protocol are proposed for $M_S = M_R = M_D = 1$ and $M_S = M_R = 2$ and $M_D = 1$.

Alamouti code achieves full rate and full diversity for two transmit antennas and CISTBC and QOSTBC with constellation rotation (QOSTBC-CR) achieve full rate and full diversity for four transmit antennas. By using these STBCs, we propose some DSTCs for the NDF-SAS protocol.

Let $A(x_1, x_2) = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$. For $M_S = M_R = M_D = 1$, we use

$$X_1(\mathrm{x}) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

in the first phase and Alamouti scheme in the second phase, i.e.,

$$X_2(\mathrm{x}) = \begin{bmatrix} x_1 & -x_2^* \end{bmatrix},$$
$$X_3(\mathrm{x}_R) = \begin{bmatrix} x_2^R & x_1^{R*} \end{bmatrix}.$$

For $M_S = M_R = 2$ and $M_D = 1$ in the first phase, we use

$$X_1(\mathrm{x}) = \begin{bmatrix} A(x_1, x_2) & A(x_3, x_4) \end{bmatrix}$$

and in the second phase, we use the CISTBC scheme

$$X_2(\mathrm{x}) = \begin{bmatrix} A(\tilde{s}_1, \tilde{s}_2) & 0 \end{bmatrix},$$
$$X_3(\mathrm{x}_R) = \begin{bmatrix} 0 & A(\tilde{s}_3^R, \tilde{s}_4^R) \end{bmatrix},$$

and the QOSTBC-CR scheme

$$X_2(\mathrm{x}) = \begin{bmatrix} A(x_1, x_2) & A(s_3, s_4) \end{bmatrix},$$
$$X_3(\mathrm{x}_R) = \begin{bmatrix} A(s_3^R, s_4^R) & A(x_1^R, x_2^R) \end{bmatrix}$$

where $s_i = x_i e^{j\theta}$, $s_i^R = x_i^R e^{j\theta}$, θ is the rotation angle, $\tilde{z}_i = z_{i,R} + j z_{((i+1) \bmod 4)+1, I}$, for $i = 1, 2, 3, 4$, $z_{i,R}$, and $z_{i,I}$ denote the real and imaginary parts of z_i, respectively, and z_i can be s_i or s_i^R.

III. AVERAGE PAIRWISE ERROR PROBABILITY FOR NDF-SAS PROTOCOL

A. Derivation of Average Pairwise Error Probability

It is very difficult to derive the exact PEPs of the NDF protocol for the erroneous SR channel case. In addition, the erroneous SR channel case with $\sigma_{SR}^2 \geq \sigma_{SD}^2 = \sigma_{RD}^2$ shows the performance close to that of the error-free SR channel case. Thus, in this section, we derive the PEP for the error-free SR channel case, i.e., $\mathrm{x}_R = \mathrm{x}$. Then, the equivalent input-output relation can be summarized as

$$Y_D = H_e X_e(\mathrm{x}) + N_D \qquad (3)$$

where $Y_D = [Y_{D_1} \ Y_{D_2}]$ is the received signal matrix, H_e is the equivalent channel matrix given by

$$H_e = \begin{bmatrix} G & H & F \end{bmatrix},$$

$$X_e(\mathrm{x}) = \begin{bmatrix} \sqrt{\frac{p_1}{M_S}} X_1(\mathrm{x}) & 0 \\ 0 & \sqrt{\frac{p_2}{M_S}} X_2(\mathrm{x}) \\ 0 & \sqrt{\frac{p_3}{M_R}} X_3(\mathrm{x}) \end{bmatrix}$$

is the equivalent codeword matrix, and $N = \begin{bmatrix} N_{D_1} & N_{D_2} \end{bmatrix}$ is the complex Gaussian noise matrix with $\mathcal{CN}(0, \sigma^2 I_{M_D(T_1+T_2)})$.

Similarly to [8], the average PEP of mistaking x for x̌ in (3) will be derived. The maximum likelihood (ML) decoding metric can be defined as $m(Y_D, X_e(\mathrm{x})) = \|Y_D - H_e X_e(\mathrm{x})\|^2$. Then, the PEP can be rewritten as

$$\begin{aligned} P(\mathrm{x} \rightarrow \check{\mathrm{x}}) &= P\big(m(Y_D, X_e(\check{\mathrm{x}})) < m(Y_D, X_e(\mathrm{x}))\big) \\ &= P\big(\|H_e(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))\|^2 + \|N_D\|^2 \\ &\quad + 2\mathrm{Re}\{\mathrm{tr}(H_e(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))N_D^\dagger)\} < \|N_D\|^2\big) \end{aligned}$$

where $\mathrm{Re}(\cdot)$ and $\mathrm{tr}(\cdot)$ denote the real part of a complex value and the trace of a matrix, respectively. Since $2\mathrm{Re}\{\mathrm{tr}(H_e(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))N_D^\dagger)\}$ is a real Gaussian random variable with zero mean and variance $2\sigma^2 \|H_e(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))\|^2$, the PEP is given as

$$Q\left(\sqrt{\frac{1}{2\sigma^2}\|H_e(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))\|^2}\right)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$. Using the Craig's result $Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta$ in [9] and $\rho = 1/\sigma^2$, the average PEP can be derived as

$$\begin{aligned} & \mathsf{E}\big[P(\mathrm{x} \rightarrow \check{\mathrm{x}})\big] \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \mathsf{E}\left[\exp\left\{-\frac{\rho\|H_e(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))\|^2}{4\sin^2\theta}\right\}\right] d\theta \\ &\overset{(a)}{=} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\mathsf{E}\left[\exp\left\{-\frac{\rho\|[H_e]_i(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))\|^2}{4\sin^2\theta}\right\}\right]\right)^{M_D} d\theta \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(M_\Gamma\left(-\frac{\rho}{4\sin^2\theta}\right)\right)^{M_D} d\theta \end{aligned}$$

where $[H_e]_i$ is the ith row of H_e, $\Gamma = \|[H_e]_i(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}}))\|^2$, and $M_\Gamma(s) = \mathsf{E}[\exp(s\Gamma)]$. The equality (a) holds because the rows of H_e have the same statistical properties. By manipulating the moment generating function $M_\Gamma(-\rho/(4\sin^2\theta))$ using the result (14) in [10], the following average PEP can be obtained as

$$\begin{aligned} \mathsf{E}[P(\mathrm{x} \rightarrow \check{\mathrm{x}})] = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \Big| I + \frac{\rho}{4\sin^2\theta} \mathsf{E}\big[[H_e]_i^\dagger[H_e]_i\big] \\ \big(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}})\big)\big(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}})\big)^\dagger \Big|^{-M_D} d\theta. \quad (4) \end{aligned}$$

For high SNR, if the difference matrix $X_e(\mathrm{x}) - X_e(\check{\mathrm{x}})$ has full rank, the rank of $\mathsf{E}\big[[H_e]_i^\dagger[H_e]_i\big]$ will determine the diversity of the average PEP. It is observed that the proposed NDF-SAS protocol can increase the diversity by $M_S M_D$ compared to the NDF protocol.

In the following subsection, the PEPs of some DSTCs for the NDF-SAS protocol proposed in subsection II-B will be derived.

B. Error Probability for NDF-SAS Protocol with DSTCs

B.1 Alamouti Scheme

For Alamouti scheme, we assume $p_1 + p_2 + p_3 = 2$ and therefore ρ represents the average transmit SNR. Since

$$\left(X_e(\mathbf{x}) - X_e(\check{\mathbf{x}})\right)\left(X_e(\mathbf{x}) - X_e(\check{\mathbf{x}})\right)^\dagger$$
$$= \left(|x_1 - \check{x}_1|^2 + |x_2 - \check{x}_2|^2\right)\mathrm{diag}(p_1, p_2, p_3)$$

where $\mathrm{diag}(\cdot)$ denotes the diagonal matrix, this code can be single symbol decoded. Let $X_e(x_k)$ be the codeword X_e by putting $x_i = 0$ for all $i \neq k$. Then, the average PEP can be derived separately. From (4), we obtain the following PEP

$$\mathsf{E}[P(x_k \to \check{x}_k)] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left|I + \frac{\rho}{4\sin^2\theta}\mathsf{E}[H_e^\dagger H_e]\right.$$
$$\left.\left(X_e(x_k) - X_e(\check{x}_k)\right)\left(X_e(x_k) - X_e(\check{x}_k)\right)^\dagger\right|^{-1}d\theta \quad (5)$$

where

$$\mathsf{E}[H_e^\dagger H_e] = \begin{bmatrix} \sigma_{SD}^2 I_2 & 0 \\ 0 & \sigma_{RD}^2 \end{bmatrix}$$

for the NDF-SAS protocol and

$$\mathsf{E}[H_e^\dagger H_e] = \begin{bmatrix} \sigma_{SD}^2 & \sigma_{SD}^2 & 0 \\ \sigma_{SD}^2 & \sigma_{SD}^2 & 0 \\ 0 & 0 & \sigma_{RD}^2 \end{bmatrix}$$

for the NDF protocol. Therefore, the average PEPs for the NDF-SAS protocol and the NDF protocol can be derived as

$$\mathsf{E}[P(x_k \to \check{x}_k)] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1 + \frac{p_1\rho\Delta_{x_k}^2\sigma_{SD}^2}{4\sin^2\theta}\right)^{-1}$$
$$\left(1 + \frac{p_2\rho\Delta_{x_k}^2\sigma_{SD}^2}{4\sin^2\theta}\right)^{-1}\left(1 + \frac{p_3\rho\Delta_{x_k}^2\sigma_{RD}^2}{4\sin^2\theta}\right)^{-1}d\theta \quad (6)$$

and

$$\mathsf{E}[P(x_k \to \check{x}_k)]$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1 + \frac{(p_1+p_2)\rho\Delta_{x_k}^2\sigma_{SD}^2}{4\sin^2\theta}\right)^{-1}\left(1 + \frac{p_3\rho\Delta_{x_k}^2\sigma_{RD}^2}{4\sin^2\theta}\right)^{-1}d\theta, \quad (7)$$

respectively, where $\Delta_{x_k} = |x_k - \check{x}_k|$, $k = 1, 2$ and the closed-form expressions of the right-hand sides of the above two equations can be achieved by referring the results in [10, Appendix 5A]. From (6) and (7), we observe that the NDF-SAS protocol achieves diversity order three which is larger than the diversity order two of the NDF protocol.

From the average PEPs in (6) and (7), the union bound on the average BEPs can be derived as

$$\overline{\mathrm{BEP}} \leq \frac{1}{L}\sum_{k=1}^L\frac{1}{n_b}\sum_{x_k}P(x_k)\sum_{\check{x}_k \neq x_k}\mathsf{E}[P(x_k \to \check{x}_k)]d_H(x_k, \check{x}_k) \quad (8)$$

where $L = 2$ is the number of the transmitted symbols, n_b is the number of bits in each symbol x_k, and $d_H(x_k, \check{x}_k)$ is Hamming distance between x_k and $\check{x}_k$ in their binary expressions.

B.2 CISTBC Scheme

For the CISTBC scheme, we assume $2p_1 + p_2 + p_3 = 4$ and therefore ρ represents the average transmit SNR. Since

$$\left(X_e(\mathbf{x}) - X_e(\check{\mathbf{x}})\right)\left(X_e(\mathbf{x}) - X_e(\check{\mathbf{x}})\right)^\dagger$$
$$= \sum_{k=1}^4\left(X_e(x_k) - X_e(\check{x}_k)\right)\left(X_e(x_k) - X_e(\check{x}_k)\right)^\dagger,$$

the average PEP can be derived separately. Let $\Delta_{s_{k,R}} = |s_{k,R} - \check{s}_{k,R}|$ and $\Delta_{s_{k,I}} = |s_{k,I} - \check{s}_{k,I}|$. Then $\left(X_e(x_k) - X_e(\check{x}_k)\right)\left(X_e(x_k) - X_e(\check{x}_k)\right)^\dagger$ can be written as

$$\left(X_e(x_k) - X_e(\check{x}_k)\right)\left(X_e(x_k) - X_e(\check{x}_k)\right)^\dagger$$
$$= \begin{cases} \frac{1}{2}\mathrm{diag}\left(p_1\Delta_{x_k}^2, p_1\Delta_{x_k}^2, p_2\Delta_{s_{k,R}}^2, p_2\Delta_{s_{k,R}}^2, p_3\Delta_{s_{k,I}}^2, p_3\Delta_{s_{k,I}}^2\right), \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{for } k = 1, 2 \\ \frac{1}{2}\mathrm{diag}\left(p_1\Delta_{x_k}^2, p_1\Delta_{x_k}^2, p_2\Delta_{s_{k,I}}^2, p_2\Delta_{s_{k,I}}^2, p_3\Delta_{s_{k,R}}^2, p_3\Delta_{s_{k,R}}^2\right), \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{for } k = 3, 4. \end{cases} \quad (9)$$

For the NDF-SAS protocol, we have

$$\mathsf{E}[H_e^\dagger H_e] = \begin{bmatrix} \sigma_{SD}^2 I_4 & 0 \\ 0 & \sigma_{RD}^2 I_2 \end{bmatrix} \quad (10)$$

and for the NDF protocol, we have

$$\mathsf{E}[H_e^\dagger H_e] = \begin{bmatrix} \sigma_{SD}^2 I_2 & \sigma_{SD}^2 I_2 & 0 \\ \sigma_{SD}^2 I_2 & \sigma_{SD}^2 I_2 & 0 \\ 0 & 0 & \sigma_{RD}^2 I_2 \end{bmatrix}. \quad (11)$$

Plugging (9) and (10) into (5) and (9) and (11) into (5), the average PEPs for the NDF-SAS and NDF protocols can be derived as

$$\mathsf{E}[P(x_k \to \check{x}_k)]$$
$$= \begin{cases} \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{p_1\rho\Delta_{x_k}^2\sigma_{SD}^2}{8\sin^2\theta}\right)^{-2}\left(1 + \frac{p_2\rho\Delta_{s_{k,R}}^2\sigma_{SD}^2}{8\sin^2\theta}\right)^{-2} \\ \qquad\left(1 + \frac{p_3\rho\Delta_{s_{k,I}}^2\sigma_{RD}^2}{8\sin^2\theta}\right)^{-2}d\theta, \quad \text{for } k = 1, 2 \\ \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{p_1\rho\Delta_{x_k}^2\sigma_{SD}^2}{8\sin^2\theta}\right)^{-2}\left(1 + \frac{p_2\rho\Delta_{s_{k,I}}^2\sigma_{SD}^2}{8\sin^2\theta}\right)^{-2} \\ \qquad\left(1 + \frac{p_3\rho\Delta_{s_{k,R}}^2\sigma_{RD}^2}{8\sin^2\theta}\right)^{-2}d\theta, \quad \text{for } k = 3, 4 \end{cases}$$

and

$$\mathsf{E}[P(x_k \to \check{x}_k)]$$
$$= \begin{cases} \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{(p_1\Delta_{x_k}^2 + p_2\Delta_{s_{k,R}}^2)\rho\sigma_{SD}^2}{8\sin^2\theta}\right)^{-2}\left(1 + \frac{p_3\rho\Delta_{s_{k,J}}^2\sigma_{RD}^2}{8\sin^2\theta}\right)^{-2}d\theta, \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{for } k = 1, 2, \\ \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{(p_1\Delta_{x_k}^2 + p_2\Delta_{s_{k,I}}^2)\rho\sigma_{SD}^2}{8\sin^2\theta}\right)^{-2}\left(1 + \frac{p_3\rho\Delta_{s_{k,R}}^2\sigma_{RD}^2}{8\sin^2\theta}\right)^{-2}d\theta, \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{for } k = 3, 4, \end{cases}$$

respectively. Since $\Delta_{x_k}^2 \neq 0$, $\Delta_{s_{k,R}}^2 \neq 0$, and $\Delta_{s_{k,I}}^2 \neq 0$, the diversity orders of NDF-SAS and NDF protocols are six and four, respectively.

Similarly to the Alamouti scheme, the union bounds on the average BEPs could also be derived by using (8), where $L = 4$.

B.3 QOSTBC-CR Scheme

For the QOSTBC-CR scheme, we assume $p_1 + p_2 + p_3 = 2$ and therefore ρ represents the average transmit SNR. Since

$$\left(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}})\right)\left(X_e(\mathrm{x}) - X_e(\check{\mathrm{x}})\right)^\dagger$$

$$= \left(X_e(x_1, x_3) - X_e(\check{x}_1, \check{x}_3)\right)\left(X_e(x_1, x_3) - X_e(\check{x}_1, \check{x}_3)\right)^\dagger$$

$$+ \left(X_e(x_2, x_4) - X_e(\check{x}_2, \check{x}_4)\right)\left(X_e(x_2, x_4) - X_e(\check{x}_2, \check{x}_4)\right)^\dagger$$

where $X_e(x_k, x_l)$ means the codeword X_e by putting $x_i = 0$ for all $i \neq k, l$, the average PEP can be derived pairwisely for (x_1, x_3) and (x_2, x_4). We will consider $X_e(x_1, x_3)$ to derive the average PEP and use the notation $\mathrm{x} = (x_1, x_3)$, $X(\mathrm{x}) = X_e(x_1, x_3)$, and $X(\check{\mathrm{x}}) = X_e(\check{x}_1, \check{x}_3)$.

From (4), the average PEPs of NDF-SAS and NDF protocols can be derived as

$$E[P(\mathrm{x} \to \check{\mathrm{x}})] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left| I + \frac{\rho}{4\sin^2\theta}E[H_e^\dagger H_e] \right.$$

$$\left. \cdot \left(X(\mathrm{x}) - X(\check{\mathrm{x}})\right)\left(X(\mathrm{x}) - X(\check{\mathrm{x}})\right)^\dagger \right|^{-1} d\theta$$

(12)

where

$$\left(X(\mathrm{x}) - X(\check{\mathrm{x}})\right)\left(X(\mathrm{x}) - X(\check{\mathrm{x}})\right)^\dagger$$

$$= \frac{1}{2}\begin{bmatrix} c_1 p_1 I_2 & 0 & 0 \\ 0 & c_2 p_2 I_2 & d\sqrt{p_2 p_3} I_2 \\ 0 & d\sqrt{p_2 p_3} I_2 & c_2 p_3 I_2 \end{bmatrix},$$

$c_1 = |x_1 - \check{x}_1|^2 + |x_3 - \check{x}_3|^2$, $c_2 = |x_1 - \check{x}_1|^2 + |s_3 - \check{s}_3|^2$, and $d = 2\mathrm{Re}\{(x_1 - \check{x}_1)(s_3 - \check{s}_3)^*\}$. Since $s_k = x_k e^{j\phi}$, we have $c_1 = c_2$. Then the average PEP for the NDF-SAS and NDF protocols can be derived as

$$E[P(\mathrm{x} \to \check{\mathrm{x}})] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + \frac{p_1 \rho c_2 \sigma_{SD}^2}{8\sin^2\theta}\right)^{-2}$$

$$\cdot \left(1 + \frac{p_2 \rho c_2 \sigma_{SD}^2}{8\sin^2\theta} + \frac{p_3 \rho c_2 \sigma_{RD}^2}{8\sin^2\theta} + \frac{p_2 p_3 \rho^2 \sigma_{SD}^2 \sigma_{RD}^2}{64\sin^4\theta}(c_2^2 - d^2)\right)^{-2} d\theta$$

and

$$E[P(\mathrm{x} \to \check{\mathrm{x}})]$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + \frac{(p_1 + p_2)\rho c_2 \sigma_{SD}^2}{8\sin^2\theta} + \frac{p_3 \rho c_2 \sigma_{RD}^2}{8\sin^2\theta}\right.$$

$$\left. + \frac{p_1 p_3 \rho^2 c_2^2 \sigma_{SD}^2 \sigma_{RD}^2}{64\sin^4\theta} + \frac{p_2 p_3 \rho^2 \sigma_{SD}^2 \sigma_{RD}^2}{64\sin^4\theta}(c_2^2 - d^2)\right)^{-2} d\theta,$$

respectively, where $c_2 = |x_1 - \check{x}_1|^2 + |s_3 - \check{s}_3|^2$, $d = 2\mathrm{Re}\{(x_1 - \check{x}_1)(s_3 - \check{s}_3)^*\}$, and $c_2^2 - d^2 > 0$.

Let $\mathrm{x}' = (x_2, x_4)$. Then the average PEPs for NDF-SAS and NDF protocols could be derived by using the similar steps.

Finally, it is observed that the diversity orders of NDF-SAS and NDF protocols are six and four, respectively. In this case, double-symbol decoding is possible and the union bounds on the average BEPs could be derived as

$$\overline{BEP} \leq \frac{1}{2}\frac{1}{2n_b}\sum_{\mathrm{x}} P(\mathrm{x})\sum_{\check{\mathrm{x}} \neq \mathrm{x}} E[P(\mathrm{x} \to \check{\mathrm{x}})]d_H(\mathrm{x}, \check{\mathrm{x}})$$

$$+ \frac{1}{2}\frac{1}{2n_b}\sum_{\mathrm{x}'} P(\mathrm{x}')\sum_{\check{\mathrm{x}}' \neq \mathrm{x}'} E[P(\mathrm{x}' \to \check{\mathrm{x}}')]d_H(\mathrm{x}', \check{\mathrm{x}}')$$
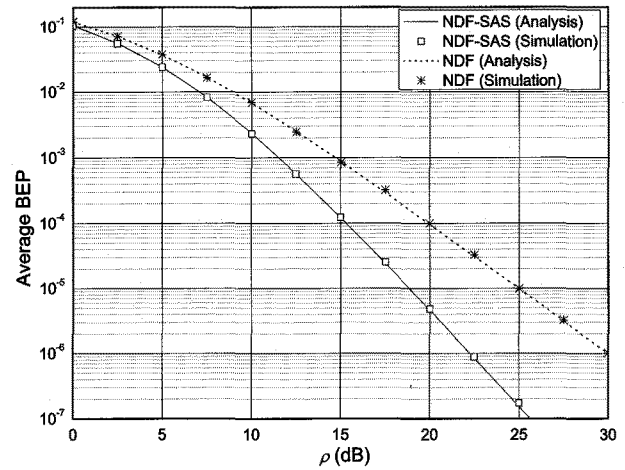
(13)

Fig. 3. Performance comparison of Alamouti scheme of the NDF-SAS and NDF protocols with $M_S = M_R = M_D = 1$ for $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$ in the error-free SR channel.
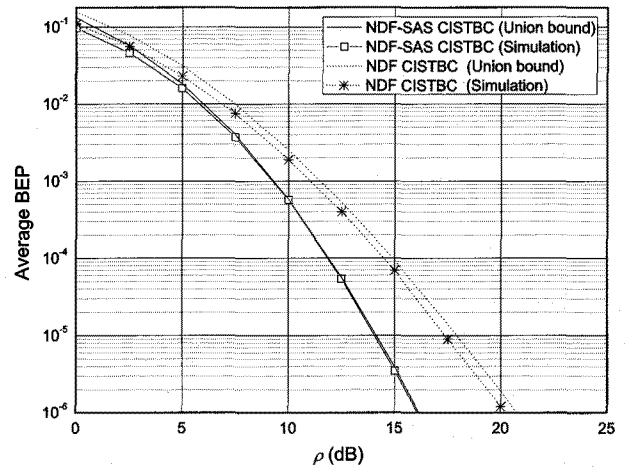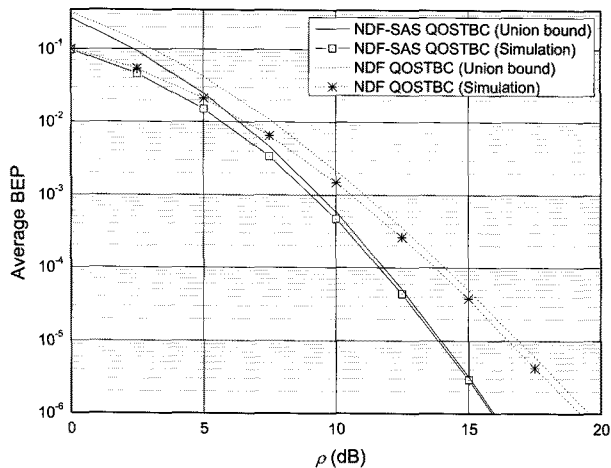

Fig. 4. Performance comparison of the NDF-SAS and NDF protocols for CISTBC scheme with $M_S = M_R = 2$ and $M_D = 1$ and $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$ in the error-free SR channel.

where $d_H(\mathrm{x}, \check{\mathrm{x}})$ and $d_H(\mathrm{x}', \check{\mathrm{x}}')$ are Hamming distances between x and $\check{\mathrm{x}}$, and x' and $\check{\mathrm{x}}'$ in their binary expressions, respectively.

IV. NUMERICAL RESULTS

For the simulation, it is assumed that the total transmit powers in the first and second phases are the same, where the power is equally allocated to each transmit antenna. Therefore, for $M_S = M_R = M_D = 1$, it is assumed that $p_1 = 1$, $p_2 = 0.5$, and $p_3 = 0.5$ for both NDF-SAS and NDF protocols. For $M_S = M_R = 2$ and $M_D = 1$, it is also assumed that $p_1 = 1$, $p_2 = 1$, and $p_3 = 1$ for the CISTBC scheme and $p_1 = 1$, $p_2 = 0.5$, and $p_3 = 0.5$ for the QOSTBC-CR scheme to make the total transmit powers equal. For QPSK modulation, we use the optimal rotation angle $\theta = -31.7175°$ [6] for the CISTBC scheme and $\theta = 45°$ [7] for the QOSTBC-CR scheme.

For the error-free SR channel case, we compare the average BEPs of the NDF-SAS and NDF protocols for Alamouti scheme with $M_S = M_R = M_D = 1$ and CISTBC and QOSTBC-CR

$$\hat{x} = \arg\max_{x \in \mathcal{A}^L} \left[-\frac{\left\| Y_{D_1} - \sqrt{\frac{p_1}{M_S}} G X_1(x) \right\|^2}{\sigma^2} + \ln \sum_{\hat{x}_R \in \mathcal{A}^L} \exp\left(\frac{-\left\| Y_{D_2} - \sqrt{\frac{p_2}{M_S}} H X_2(x) - \sqrt{\frac{p_3}{M_R}} F X_3(\hat{x}_R) \right\|^2 + \sigma^2 \ln P_{SR}(\hat{x}_R|x)}{\sigma^2} \right) \right] \quad (14)$$

Fig. 5. Performance comparison of the NDF-SAS and NDF protocols for QOSTBC scheme with $M_S = M_R = 2$ and $M_D = 1$ and $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$ in the error-free SR channel.
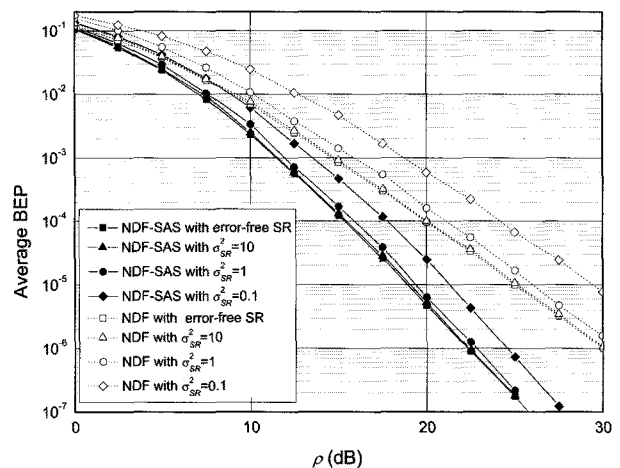

Fig. 6. Performance comparison of Alamouti scheme of the NDF-SAS and NDF protocols with $M_S = M_R = M_D = 1$ for $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$ in the erroneous SR channel.

schemes with $M_S = M_R = 2$ and $M_D = 1$. Specially, since the Alamouti scheme is one-dimensional symbol (the real part or imaginary part of symbol) decodable, the exact average BEP can be derived from the average PEP between one-dimensional symbols for QPSK. Fig. 3 shows the BEPs of Alamouti scheme for NDF-SAS and NDF protocols in the case of $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$, where the 'Analysis' means the exact average BEP derived from the average PEP. Figs. 4 and 5 show the BEPs of CISTBC and QOSTBC-CR schemes, respectively, for the NDF-SAS and NDF protocols in the case of $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$. From Figs. 3–5, we observe that the diversity order of the NDF-SAS protocol is larger than that of the NDF protocol and the union bounds of the average BEPs in (8) and (13) derived from the average PEPs are close to the simulation results in the error-free SR channel. It is also observed that the NDF-SAS protocol improves the performance about 5 dB and 2.5 dB than the conventional NDF protocol in the average BEP 10^{-4} for the cases of $M_S = M_R = M_D = 1$ and $M_S = M_R = 2, M_D = 1$, respectively.

For the erroneous SR channel case, the ML decoding scheme as shown in [12] is used as (14), at the top of this page, where $\mathcal{A}$ is the signal set for the M-ary signal constellation and $P_{SR}(\hat{x}_R|x)$ is the probability that the relay decodes the received signal to $\hat{x}_R$ when the source transmits x in the first phase. We compare the average BEPs of the NDF-SAS and NDF protocols with ML decoder for $M_S = M_R = 1$ and $M_S = M_R = 2$. Fig. 6 compares the average BEPs of Alamouti scheme of the NDF-SAS and NDF protocols for $M_S = M_R = M_D = 1$ and Fig. 7 compares the average BEP of the CISTBC scheme of the NDF-SAS and NDF protocols with $M_S = M_R = 2$ and $M_D = 1$ for the cases of error-free SR channel, $\sigma_{SR}^2 = 10$, $\sigma_{SR}^2 = 1$, and $\sigma_{SR}^2 = 0.1$. From Figs. 6 and 7, we observe that for $M_D = 1$,
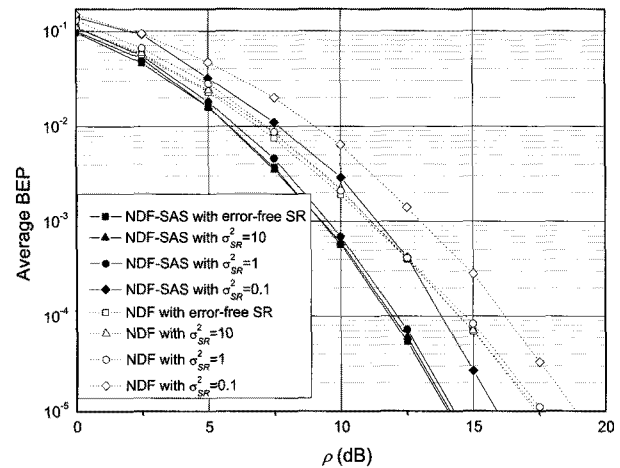

Fig. 7. Performance comparison of the CISTBC scheme of the NDF-SAS and NDF protocols with $M_S = M_R = 2$ and $M_D = 1$ for $\sigma_{SD}^2 = \sigma_{RD}^2 = 1$ in the erroneous SR channel.

the average BEP of the erroneous SR channel case is similar to that of the error-free SR channel case for $\sigma_{SR}^2 \geq 1$. We can also observe that although the average BEP for $\sigma_{SR}^2 = 0.1$ has a gap with the average BEP for the error-free SR channel, they have the same diversity order for both NDF-SAS and NDF protocols, i.e., the NDF-SAS protocol has diversity improvement compared to the NDF protocol in the erroneous SR channel.
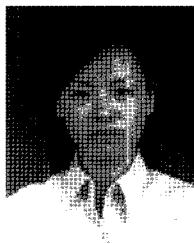
V. CONCLUSION

In this paper, the NDF-SAS protocol is proposed, which increases the diversity order compared to the conventional NDF protocol. Several DSTC schemes using the OSTBC, CISTBC, and QOSTBC for the NDF-SAS protocol are proposed and

their average PEPs for the error-free SR channel are also derived. From the numerical results for both error-free and erroneous SR channels, it is observed that the NDF-SAS protocol increases the diversity order compared to the NDF protocol. They also show the NDF-SAS protocol improves the performance about 5dB and 2.5dB than the conventional NDF protocol at the BEP 10^{-4} for the cases of $M_S = M_R = M_D = 1$ and $M_S = M_R = 2, M_D = 1$, respectively.

In practice, even though there is enough space between antennas, the correlation between channel coefficients of antennas also exists. The study on the NDF-SAS protocol with correlated G and H will be very interesting as a further work.

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