Clipping Value Estimate for Iterative Tree Search Detection

Jianping Zheng, Baoming Bai, and Ying Li

Abstract: The clipping value, defined as the log-likelihood ratio (LLR) in the case wherein all the list of candidates have the same binary value, is investigated, and an effective method to estimate it is presented for iterative tree search detection. The basic principle behind the method is that the clipping value of a channel bit is equal to the LLR of the maximum probability of correct decision of the bit to the corresponding probability of erroneous decision. In conjunction with multilevel bit mappings, the clipping value can be calculated with the parameters of the number of transmit antennas, N_t ; number of bits per constellation point, M_c ; and variance of the channel noise, σ^2 , per real dimension in the Rayleigh fading channel. Analyses and simulations show that the bit error performance of the proposed method is better than that of the conventional fixed-value method.

Index Terms: Clipping value, iterative tree search (ITS), multiinput multi-output (MIMO), multilevel bit mappings.

I. INTRODUCTION

It has been shown that under favorable circumstances such as rapid channel fading, space-time bit-interleaved coded modulation (ST-BICM) systems that employ turbo coding and iterative processing at the receiver reach their ergodic multi-input multi-output (MIMO) capacity limit [1]–[4]. To achieve the high-rate potential in practice, low-complexity and high-performance soft MIMO detection is vital. A soft-input soft-output MIMO detection scheme, referred to as the iterative tree search (ITS), was introduced in reference [4]. When compared to the list variant of the sphere decoder in reference [3], the per-bit complexity of ITS detection is constant, and it is asymptotically linear in the number of transmit antennas, N_t , and insensitive to the number of bits per constellation point, M_c .

In ITS detection, the extrinsic information on the channel bits is computed and expressed as a log-likelihood ratio (LLR), as in reference [3].

$$L_E(x_{n,k}) = \log \frac{\sum_{\mathbf{x} \in \mathbf{L} \cap \mathbf{X}_{n,k}^{+1}} \exp(\mu(\mathbf{s}))}{\sum_{\mathbf{x} \in \mathbf{L} \cap \mathbf{X}_{n,k}^{-1}} \exp(\mu(\mathbf{s}))} - L_A(x_{n,k}). \quad (1)$$

In this expression, $\mathbf{s} = [s_1, s_2, \dots, s_{N_t}]^T$ is the transmitted symbol vector, $x_{n,k}$ denotes the kth bit mapped to s_n , $L_A(x_{n,k})$ de-

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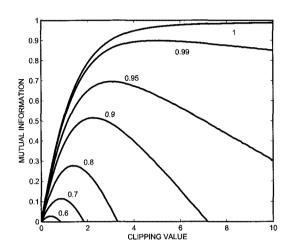


Fig. 1. Mutual information $I(L_E(x_{n,k});x_{n,k})$ versus clipping value $L_c(x_{n,k})$, for different values of $\Pr(x_{n,k}L_E(x_{n,k})>0)$.

notes the LLR expression of the *a priori* information of $x_{n,k}$, L denotes the list of M candidate bit sequences, which is obtained from the M-algorithm, and $\mathbf{X}_{n,k}^{+1}$ and $\mathbf{X}_{n,k}^{-1}$ are the sets of all possible bit sequences \mathbf{x} for which $x_{n,k}$ is +1 and -1, respectively, i.e., $\mathbf{X}_{n,k}^{\pm 1} = \{\mathbf{x} | x_{n,k} = \pm 1\}$. The metric $\mu(\mathbf{s})$ in (1) is given by

$$\mu(\mathbf{s}) = \frac{1}{\sigma^2} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 + \sum_{i=1}^{N_t} \sum_{i \in \mathbf{J}_i} L_A(x_{i,j})$$
 (2)

where **H** is the channel matrix, σ^2 is the variance of the channel noise per real dimension, and $J_i = \{j = 1, \dots, M_c | x_{i,j} = +1\}$.

In (1), it is possible for all the bit sequences in ${\bf L}$ to have the same binary value in some special positions, especially when ${\bf L}$ is small. In such a case, $L_E(x_{n,k})$ is then assigned a negative or positive constant value $(\mp L_c)$. It was found that the performance of ITS detection is sensitive to the magnitude of this value. When the clipping value is significantly lower than the optimal value, there would be a degrade in its error-correction effectiveness, whereas when the clipping value is significantly higher than the optimal value, it would result in error propagation.

Ideally, L_c should be different for each channel bit $x_{n,k}$, such that it maximizes the mutual information

$$I(L_E(x_{n,k}); x_{n,k}) = 1 - E\{\log_2(1 + e^{-x_{n,k}L_E(x_{n,k})})\}$$
 (3)

which is dependent on the unknown probability that $L_E(x_{n,k})$ has the correct sign, i.e., on the probability of correct decision of $x_{n,k}$. Fig. 1 shows the curve of mutual information

 $I(L_E(x_{n,k}); x_{n,k})$ versus clipping value $L_c(x_{n,k})$ for different values of $\Pr(x_{n,k}L_E(x_{n,k}) > 0)$.

It is interesting to note that in Fig. 1, the mutual information is maximum when the clipping value is equal to the LLR of the indicated value of $\Pr(x_{n,k}L_E(x_{n,k})>0)$ to $1-\Pr(x_{n,k}L_E(x_{n,k})>0)$. Note that the value of $\Pr(x_{n,k}L_E(x_{n,k})>0)$ is the probability of correct decision of the channel bit $x_{n,k}$. On the basis of this observation, we infer that the clipping value could be the LLR of the probability of correct decision to the probability of erroneous decision. In particular, in this paper, in conjunction with the multilevel bit mappings and the statistic character of the MIMO Raleigh fading channel, the clipping value is analytically tractable. Our method of estimating the clipping value results in a better performance when compared to the conventional fixed-value method.

Recently, a signal-to-noise ratio (SNR)-aware LLR clipping (SLC) method has been proposed [5] and a clipping value estimate for mixed hard/soft output detection has been reported [6]. Although the estimation of the clipping value in both the methods is similar to that in our study, our method is different from these methods from the aspect of conception and realization. In particular, the list number, which is taken into account in the SLC method, is not taken into account in our method. In the SLC method, the clipping value is determined on the basis of the instantaneous channel condition. However, in our method, the clipping value is determined only from the statistic character of the channel condition and on the basis of the assumption that the detection at each layer is perfect, for which the list number is not considered. Further, the criterion in reference [6] is to minimize the probability of error at the decoder output, whereas in our study, the criterion is to maximize the mutual information.

II. CLIPPING VALUE ESTIMATE

The following notations are used throughout the paper. We use boldface (e.g., A) to denote matrices and vectors. For a matrix A, the variables A^T , A^H , and A^{-1} denote its transpose, conjugate transpose, and inverse, respectively. The notation $E(\cdot)$ stands for the expectation of its argument.

It is popular to model **H** as a random matrix with independent and identically distributed (i.i.d.) zero-mean unit-variance complex Gaussian entries, denoted by $\mathcal{CN}(0,1)$, in wireless communications. For QR-decomposition of the channel matrix, i.e., $\mathbf{H} = \mathbf{QR}$, the elements of the upper triangular matrix **R** are distributed independently according to $r_{ii}^2 \sim \varsigma(N_t+1-i)$ and $|r_{ij}|^2 \sim \varsigma(1)$ for i < j [7], where $\varsigma(n)$ denotes the standard Gamma distribution with the density function $f(x) = \frac{1}{\Gamma(n)}x^{n-1}\exp(-x)$ and $\Gamma(n)$ denotes the Gamma function. Note that from $\mathbf{H}^H\mathbf{H} = (\mathbf{QR})^H\mathbf{QR} = \mathbf{R}^H\mathbf{R}$ through QR decomposition and $\mathbf{H}^H\mathbf{H} = \mathbf{L}^H\mathbf{L}$ through Cholesky decomposition, we have $\mathbf{L} = \mathbf{R}$ in ITS detection. This implies that $l_{ii}^2 \sim \varsigma(N_t+1-i)$ and $E(l_{ii}^2) = N_t+1-i$.

Subsequently, after a series of preprocessing steps, the inputoutput relation of the MIMO channel can be written as follows [4],

$$\bar{\mathbf{y}} = \mathbf{L}\mathbf{s} + \bar{\mathbf{n}} \tag{4}$$

where $\bar{\mathbf{y}} = \mathbf{L}^{-H}\mathbf{H}^H\mathbf{y}$ and $\bar{\mathbf{n}} \sim \mathcal{CN}(0, 2\sigma^2\mathbf{I})$. In the ideal case

that there exists no error propagation between layers, ITS detection can be decomposed into a series of scalar decisions given by

$$\tilde{y}_i = l_{ii}s_i + \bar{n}_i, \quad i = 1, \dots, N_t \tag{5}$$

with $\tilde{y_i} = \bar{y_i} - \sum_{k=i+1}^{N_t} l_{ik} s_k$. Since l_{ii} is real and positive, a set of $2N_t$ equations can be obtained from (5) as

$$\Re \tilde{y_i} = l_{ii} \Re s_i + \Re \bar{n}_i, \Im \tilde{y_i} = l_{ii} \Im s_i + \Im \bar{n}_i, \quad i = 1, \dots, N_t$$
 (6)

where \Re and \Im denote the real and imaginary parts of a complex quantity, respectively.

Let us denote the $M_c/2$ -dimensional vectors $\mathbf{a}_i = (a_i [1], \cdots, a_i [M_c/2])^T$ and $\mathbf{b}_i = (b_i [1], \cdots, b_i [M_c/2])^T$ as the binary representation of $\Re s_i$ and $\Im s_i$, respectively, with $a_i [t], b_i [t] \in \{\pm 1\}, t = 1, \cdots, M_c/2$. Then, (6) can be rewritten as

$$\Re y_i' = \sum_{k=1}^{M_c/2} a_i [k] + \Re n_i', \Im y_i' = \sum_{k=1}^{M_c/2} a_i [k] + \Im n_i', i = 1, \dots, N_t$$
(7)

where $\Re y_i' = \Re \tilde{y}_i/l_{ii}$, $\Im y_i' = \Im \tilde{y}_i/l_{ii}$, $\Re n_i' = \Re \bar{n}_i/l_{ii}$ and $\Im n_i' = \Im \bar{n}_i/l_{ii}$. The standard deviation for both $\Re n_i'$ and $\Im n_i'$ is $\sigma' = \sigma/l_{ii}$. Thus, the detection of the real and imaginary parts will yield an identical bit error probability expression. In the following equations, only the detection of the real part is considered and the prefix \Re is omitted for notational simplicity.

Then, (7) can be expressed as

$$y_{i}'[k] = \frac{1}{2^{M_{c}/2-k}} \left(y_{i}' - \sum_{j \neq k} 2^{M_{c}/2-j} \tilde{a_{i}}[j] \right) = a_{i}[k] + n_{i}''$$

for $k=1,\cdots,M_c/2, i=1,\cdots,N_t$. Here, $n_i''=n_i'/2^{M_c/2-k}$, and $\tilde{a_i}[j]$ is the estimate of $a_i[j]$ from the preceding iteration. We assume that the estimate is correct and has no error propagation between layers, and this assumption becomes an asymptotic fact when the iteration converges. Then, (8) is an equivalent binary phase-shift keying (BPSK) modulated additive white Gaussian noise (AWGN) channel with unit amplitude and a standard deviation of $\sigma_{eff}=\sigma'/2^{M_c/2-k}=\sigma/(2^{M_c/2-k}l_{ii})$. Therefore, the bit error probability is $\Pr(a_i[k],e)=Q(1/\sigma_{eff})$ [8] and the corresponding probability of correct decision is $\Pr(a_i[k],c)=1-Q(1/\sigma_{eff})$, where the Gaussian tail probability is $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$.

Let us consider the probability of correct decisions to be $\Pr(a_i[k],c)$. It is easy to find that this probability is a increasing function of l_{ii} . Recall that l_{ii} is real and positive and $l_{ii}^2 \sim \varsigma(N_t+1-i)$. A suitable upper bound β_i of l_{ii}^2 should be chosen such that the probability that the samples of l_{ii}^2 are in the region $(0,\beta_i)$ is high. Thus, β_i can be chosen probabilistically so that

$$\int_0^{\beta_i} f(x) \, dx = \int_0^{\beta_i} \frac{1}{\Gamma(N_t + 1 - i)} x^{N_t - i} \exp(-x) \, dx = 1 - \epsilon$$
(9)

where $\epsilon \ll 1$. Then l_{ii}^2 , $i=N_t,\cdots,1$ in (5)–(8) is to be replaced with the corresponding β_i for calculating the clipping value.

Table 1. Maximum probabilities of correct decision and corresponding clipping values of channel bits from antennas 2 to 1 in a 2×2 configuration with (upper) QPSK modulation and (lower) 16-QAM modulation, respectively.

E_b/N_0 (dB)	$\Pr(x_i, c)$	L_c
1.5	0.986 0.995	4.25 5.29
2.0	0.990 0.997	4.60 5.81
2.5	0.993 0.998	4.95 6.21
3.0	0.996 0.999	5.52 6.91
4.5	0.988 0.994	4.41 5.11
5.0	0.991 0.996	4.70 5.52
5.5	0.993 0.997	4.95 5.81
6.0	0.995 0.998	5.29 6.21
6.5	0.997 0.999	5.81 6.91

In ITS detection, each constellation point can be estimated recursively with the aid of multilevel bit mappings. The estimate of s_i is first made from $s_i^{(1)}$; it is, then, refined by $s_i^{(2)}, \cdots, s_i^{(M_c/2)}$ until all M_c bits are known with $s^{M_c/2} = s_i$. Therefore, the probabilities of the M_c bits are not independent and we assume that all the M_c bits $x_{i,j}, j=1, \cdots, M_c$ have the same probability of correct decision, shown below

$$\Pr(x_{i,j}, c) = \Pr(x_{i}, c) = (\Pr(s_{i}, c))^{\frac{1}{M_{c}}}$$

$$= \left(\prod_{k=1}^{M_{c}/2} \Pr(a_{i}[k], c) \Pr(b_{i}[k], c)\right)^{\frac{1}{M_{c}}}, \quad (10)$$
for $k = 1, \dots, M_{c}/2, \quad i = 1, \dots, N_{t}.$

At the same time, the estimations of different constellation points from different transmit antennas in ITS detection are mutually independent. Recall that the mutual information reaches its maximum value when the clipping value is equal to the LLR of $\Pr(x_{n,k}L_E(x_{n,k})>0)$ to $1-\Pr(x_{n,k}L_E(x_{n,k})>0)$, and the probability of correct decision is $\Pr(x_{n,k},c)=\Pr(x_{n,k}L_E(x_{n,k})>0)$. Therefore, the clipping values of the channel bits in ITS detection can be calculated as

$$L_c(x_{i,j}) = L_c(x_i) = \ln \frac{\Pr(x_{i,j}, c)}{1 - \Pr(x_{i,j}, c)}$$
(11)

for $j = 1, \dots, M_c/2, i = 1, \dots, N_t$.

Tables 1 and 2 list the maximum probabilities of correct decision and the corresponding clipping values of channel bits in each transmit antenna for certain unique SNR regions in ITS detection with $N_r=N_t=2$ and $N_r=N_t=4$, respectively. Here, N_r represents the number of receive antennas.

We conclude this section with the following remarks.

In the aforementioned derivation, we assume that there exists no error propagation between layers; this assumption is true when the iteration converges. Further, in the initial iterations, the probability of correct decision is lower and the probability of erroneous decision is higher when compared to the corresponding probabilities when the iteration converges as a result of error propagation, which makes the probability

Table 2. Maximum probabilities of correct decision and corresponding clipping values of channel bits from antennas 4 to 1 in a 4×4 configuration with (upper) QPSK modulation and (lower) 16-QAM modulation, respectively.

E_b/N_0 (dB)	$\Pr(x_i, c)$	L_c
1.5	0.940 0.966 0.977 0.984	2.75 3.35 3.76 4.16
2.0	0.950 0.971 0.982 0.988	2.92 3.57 4.03 4.45
2.5	0.960 0.978 0.987 0.992	3.18 3.80 4.33 4.82
3.0	0.968 0.983 0.991 0.995	3.40 4.06 4.73 5.25
5.0	0.963 0.977 0.985 0.989	3.25 3.75 4.18 4.45
5.5	0.969 0.983 0.989 0.992	3.44 4.06 4.45 4.82
6.0	0.974 0.986 0.991 0.994	3.64 4.25 4.70 5.11
6.5	0.980 0.989 0.993 0.996	3.89 4.50 5.03 5.52
7.0	0.984 0.992 0.996 0.997	4.12 4.82 5.52 5.81

- that either of the two sets $\mathbf{L} \cap \mathbf{X}_{n,k}^{+1}$ or $\mathbf{L} \cap \mathbf{X}_{n,k}^{-1}$ is empty close to zero when the list number M in ITS detection is sufficiently large. Therefore, the performance degradation due to the use of the clipping values from (11) in initial iterations is negligible.
- 2) The calculation of the clipping value in (11) does not take the list number M into account. This is because, in our method, perfect detection at each layer is assumed, and this assumption is especially true when the list number M is large. In fact, the simulation results show that the effect of M on the clipping value is insignificant and hence can be neglected even if M is moderate, e.g., M=32 in the 4×4 MIMO system.
- 3) ϵ is an important parameter for estimating the clipping value, and its value is to be carefully computed. A significantly higher value will result in a decrease in the probability that the samples of l_{ii}^2 are lower than β_i and degrade the effect of clipping value as the peak value, whereas a significantly lower value will result in a false upper bound and an impractically high clipping value. Unfortunately, a method to compute the optimal value of ϵ has not yet been developed. In our simulations, we set $\epsilon=0.001$, which shows a good performance. We do not have a good explanation for choosing this value of ϵ , and leave this as an open problem.
- 4) Note that the clipping values remain constant at a certain value of the SNR. Therefore, the additional complexity introduced by the calculation of the clipping value is negligible when compared to the global complexity.

III. SIMULATION RESULTS

We assume the channel to be fast fading, with the elements of the channel being chosen from a rich-scattering Rayleigh MIMO model. The channel code is a turbo code of rate R=1/2, with feedforward and feedback generators 5 and 7 (in octal notation), respectively. Frames of 9216 information bits are fed to the turbo encoder. There are four iterations in the detector/decoder loop and eight iterations in the turbo decoder. All interleavers are pseudorandom, and no attempt was made to optimize their design.

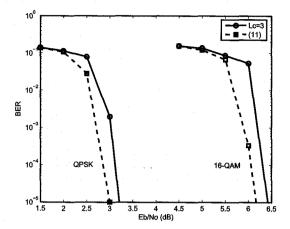


Fig. 2. Error performance of a 2×2 ST-BICM MIMO system employing ITS detection with M=32 and clipping values from (11). Channel code is a turbo code of rate 1/2 and memory 2. Performance with $L_c=3$ is shown as a reference.

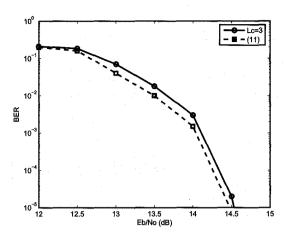


Fig. 4. Error performance of an 8×8 ST-BICM MIMO system employing ITS detection with M=64 and clipping values from (11). Channel code is a turbo code of rate 3/4 and memory 2. Performance with $L_c=3$ is shown as a reference.

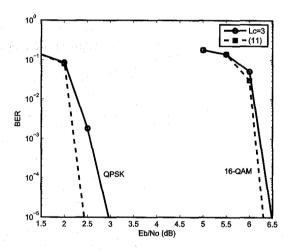


Fig. 3. Error performance of a 4×4 ST-BICM MIMO system employing ITS detection with M=32 and clipping values from (11). Channel code is a turbo code of rate 1/2 and memory 2. Performance with $L_c=3$ is shown as a reference.

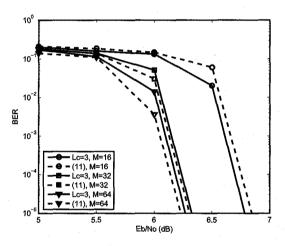


Fig. 5. Error performance of a 4×4 ST-BICM MIMO system employing ITS detection with M=16,32,64 and clipping values from (11). Channel code is a turbo code of rate 1/2 and memory 2, and 16-QAM constellation is used. Performance with $L_c=3$ is shown as a reference.

The SNR is defined as

$$\frac{E_b}{N_0} = \frac{N_t E_s}{2\sigma^2 R M_c} \tag{12}$$

where E_s is the average signal energy for each transmit PSK or QAM constellation symbol.

Figs. 2 and 3 show the simulated bit error rate (BER) performance for 2×2 and 4×4 channels, respectively, with ITS detection and M=32. Results for both the cases of QPSK and 16-QAM modulations are given. The simulated performance with a fixed clipping value of $L_c=3$ from reference [4] is also shown as a reference. It is obvious from the figures that the performance of our method is better when compared to that of the fixed-value method.

In Fig. 4, the channel code is a turbo code of rate R=3/4 and memory 2 (a punctured version of the code of rate R=1/2

used in the previous examples), and this results in a considerably high spectral efficiency of $RN_tM_c=36$ bits per channel use (64-QAM on an 8×8 channel) with ITS detection and M=64. It is easy to find that, in this case, our method still has a performance gain compared to the fixed-valued method.

Fig. 5 shows the performance curves of our method for different list number M. In this simulation, we consider the 4×4 channel and 16-QAM modulation. From this figure, it can be seen that our method is only effective when the list number is moderate to large, e.g., M=32 or 64 in this case. When the list number is small, e.g., M=16, the performance of our method is worsen than that of the fixed-value method. This can be explained by the fact that our method is based on the assumption of perfect detection at each layer; however, this assumption is not true when the list number is small.

IV. CONCLUSION

An effective method for estimating the clipping value for ITS detection is proposed. This method is based on the principle that the clipping value of a channel bit is equal to the LLR of the maximum probability of correct decision of the bit to the corresponding probability of erroneous decision. Computer simulations are carried out, and the results confirm that the proposed method is more effective than the conventional fixed-value method.

REFERENCES

- S. L. Ariyavisitakul, "Turbo space-time processing to improve wireless channel capacity," *IEEE Trans. Commun.*, vol. 48, pp. 1347–1359, Aug. 2000.
- [2] A. Stefanov and T. M. Duman, "Turbo-coded modulation for systems with transmit and receive antenna diversity over block fading channels: System model, decoding approaches, and practical considerations," *IEEE J. Sel. Areas Commun.*, vol. 19, pp. 958–968, May 2001.
- [3] B. M. Hochwald and S. T. Brink, "Achieving near-capacity on a multipleantenna channel," *IEEE Trans. Commun.*, vol. 51, pp. 389–399, Mar. 2003.
- [4] YLC. de Jong and T. J. Willink, "Iterative tree search detection for MIMO wireless systems," *IEEE Trans. Commun.*, vol. 53, pp. 930–935, June 2005.
- [5] D. L. Milliner, E. Zimmermann, J. R. Barry, and G. Fettweis, "Channel state information based LLR clipping in list MIMO detection," in *Proc. IEEE PIMRC*, Cannes, France, Sept. 2008.
- [6] E. Zimmermann, D. L. Milliner, J. R. Barry, and G. Fettweis, "Optimal LLR clipping levels for mixed hard/soft output detection," in *Proc. IEEE GLOBECOM*, New Orleans, USA, Nov. 2008, pp. 1–5.
- [7] R. J. Muirhead, Aspects of Multivariate Statistical Theory. New York: Wiley, 1982.
- [8] J. G. Proakis, Digital Communications, 4th Edition. McGraw-Hill Companies, Inc., 2001.



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