Distributed Uplink Resource Allocation in Multi-Cell Wireless Data Networks

Soomin Ko, Hojoong Kwon, and Byeong Gi Lee

Abstract: **In this paper, we present a distributed resource allocation algorithm for multi-cell uplink systems that increases the weighted sum of the average data rates over the entire network under the average transmit power constraint of each mobile station. For the distributed operation, we arrange each base station (BS) to allocate the resource such that its own utility gets maximized in a non-cooperative way. We define the utility such that it incorporates both the weighted sum of the average rates in each cell and the induced interference to other cells, which helps to instigate implicit cooperation among the cells. Since the data rates of different cells are coupled through inter-cell interferences, the resource allocation taken by each BS evolves over iterations. We establish that the resource allocation converges to a unique fixed point under reasonable assumptions. We demonstrate through computer simulations that the proposed algorithm can improve the weighted sum of the average rates substantially without requiring any coordination among the base stations.**

Index Terms: **Distributed algorithm, multi-cell, power control, pricing, uplink, utility.**

I. INTRODUCTION

In accordance with the recent growth of high-speed wireless Internet access, wireless data networks have evolved to provide high data-rate access. The conventional data networks such as 1xEV-DO and high speed packet access (HSPA) [1], [2] used to be structured asymmetric with fat downlink and skinny uplink channels, conforming to the asymmetric data traffic such as web browsing and file downloads [3]. Recently, the demand for data upload services like user-created content (UCC) has increased rapidly, so the emerging networks such as the mobile WiMAX network [4] have been designed to allow dynamic control of bandwidth allocation between the downlink and uplink channels. This trend requires to develop efficient resource management techniques for the uplink channel.

There have been reported a good amount of research works on uplink resource management [3], [5]–[7] but most of them mainly focused on the case of isolated, single cell without external interferences. In practical environments, however, wireless data networks have a cellular structure that divides the service area into multiple cells with a base station (BS) deployed in each cell and reuses the given frequency spectrum repeatedly in each cell. This enables to expand the service area arbitrarily large depending on the given frequency bandwidth and transmit power. However, it makes the resource management more challenging, as the transmitted signal power in one cell may possibly interfere the communications in its neighboring cells. Thus, it is important to design the resource management scheme such that it can manage the interaction among the cells efficiently in multi-cell environment.

As far as the uplink resource management schemes for multi-cell systems are concerned, there have been reported several algorithms that improve performance in terms of data rate and instantaneous transmission power. Hande *et al.* [8] proposed a distributed algorithm that maximizes the sum of data rates with an instantaneous maximum power constraint of each mobile station (MS). Saraydar *et al.* [9] proposed an algorithm that enhances the ratio of the correctly received bits to the energy consumption. Moretti *et al.* [10] proposed an algorithm that decreases the instantaneous total power consumption while satisfying the instantaneous rate requirement of each MS in a distributed way. However, all those schemes did not consider the constraints in the average power consumption of each MS. In practical uplink systems, however, each MS has a limited battery power and thus requires to keep the average power consumption below a permitted limit.

In this paper, we are going to propose a resource allocation algorithm that increases the weighted sum of the average data rates over the entire network under an average transmit power constraint of each MS. The proposed algorithm is designed to operate in a distributed way (i.e., each BS manages the resource allocation of the MSs within its own cell only, without coordination among the BSs), since a centralized approach requires a hierarchical network architecture that is not scalable and not cost-effective. For such distributed operation, we arrange each BS to maximize its own utility in a non-cooperative way. The utility function is designed to incorporate both the weighted sum of the average rates in each cell and the induced interference to other cells. This instigates implicit cooperation among the cells even though each BS cares about only its own utility, and consequently enhances the network-wise performance. Since the data rates of different cells are coupled through inter-cell interferences and each BS behaves in a distributed way, the proposed algorithm allocates the resource in an iterative manner. We establish that the algorithm converges to a unique fixed point under some reasonable assumptions. We demonstrate through computer simulations that the proposed algorithm can improve the weighted sum of the average rates substantially without requiring any coordination among the BSs.

The rest part of the paper is organized as follows. In

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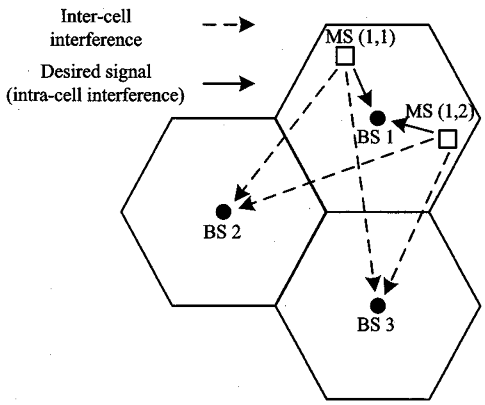

Fig. 1. An example of multi-cellular network.

Section II, we describe the system model. In Section III, we propose a resource allocation algorithm that operates in a distributed way. Then, in Section IV, we discuss the convergence property of the proposed algorithm, and finally, in Section V, we examine the simulation results of the proposed algorithm.

II. SYSTEM MODEL

We consider an uplink system with N cells serving $K(\equiv \sum_{n=1}^{N} K_n)$ MSs that are randomly located over the wireless network, where K_n denotes the number of MSs within the nth cell, as illustrated in Fig. 1. We assume that the total frequency band forms a single channel and the channel is shared by all the cells in the network.[1] We assume that the multiple MSs in each cell transmit signals simultaneously and the receiving BS performs a successive intra-cell interference cancellation [11] to estimate the original signals of the transmitting MSs within its own cell.

We adopt the time-division duplex (TDD) as the duplexing technique. In the TDD system, time is divided into periodic frames and each frame consists of a downlink data slot followed by an uplink data slot. We assume a frequency-flat fading channel with the coherence time being large enough when compared with the frame interval. We denote by an index pair (n, k) the kth MS associated with the nth BS and by $g^m_{(n,k)}$ the channel gain between MS (n, k) and BS m. Then, we define the cell gain matrix associated with BS n, $\mathbf{G}^n$, by

$$\mathbf{G}^n = \begin{pmatrix} g^1_{(n,1)} & g^2_{(n,1)} & \cdots & g^N_{(n,1)} \\ g^1_{(n,2)} & g^2_{(n,2)} & \cdots & g^N_{(n,2)} \\ \vdots & \vdots & \vdots & \vdots \\ g^1_{(n,K_n)} & g^2_{(n,K_n)} & \cdots & g^N_{(n,K_n)} \end{pmatrix}. \quad (1)$$

We also define the network gain matrix to be $\mathbf{G} \equiv [(\mathbf{G}^1)^T\ (\mathbf{G}^2)^T\ \cdots\ (\mathbf{G}^N)^T]^T$, where $[\cdot]^T$ denotes the matrix transpose operation.

We denote by $\mathbf{p}^n(\mathbf{G}) \equiv [p_{(n,1)}(\mathbf{G})\ p_{(n,2)}(\mathbf{G})\ \cdots\ p_{(n,K_n)}(\mathbf{G})]^T$ the cell power vector associated with cell n, where $p_{(n,k)}(\mathbf{G})$ denotes the transmission power of MS (n, k) for a given network gain matrix $\mathbf{G}$. Based on this, we define the network power matrix to be $\mathbf{P}(\mathbf{G}) \equiv [\mathbf{p}^1(\mathbf{G})\ \mathbf{p}^2(\mathbf{G})\ \cdots\ \mathbf{p}^N(\mathbf{G})]$. We also denote by $s^n(\mathbf{G}) \equiv \sum_{k=1}^{K_n} p_{(n,k)}(\mathbf{G})$ the cell total power associated with cell n, and define the network total power vector to be $\mathbf{s}(\mathbf{G}) \equiv [s^1(\mathbf{G})\ s^2(\mathbf{G})\ \cdots\ s^N(\mathbf{G})]$. We denote by $\mathbf{s}^{-n}(\mathbf{G})$ the vector that remains after separating out $s^n(\mathbf{G})$ from $\mathbf{s}(\mathbf{G})$, and call it the external total power vector as it represents the transmission powers of the MSs generating interferences to cell n. We assume that the average transmission power of each MS is constrained to be less than $P^{\max}_{(n,k)}$ (i.e., $\mathrm{E}_{\mathbf{G}}[p_{(n,k)}(\mathbf{G})] \leq P^{\max}_{(n,k)}$).[2]

As BS n decodes the signals of the multiple MSs by performing intra-cell interference cancellation, the signal-to-noise-and-interference ratio (SINR) of the signals depends on the order of decoding (i.e., the SINR increases if decoded later). We denote by $\mathbf{d}^n(\mathbf{G})$ the cell decoding order vector associated with cell n that is given by the permutation of the vector $[(n,1)\ (n,2)\ \cdots\ (n,K_n)]^T$, which means that BS n decodes MS $[\mathbf{d}^n(\mathbf{G})]_1$ first, MS $[\mathbf{d}^n(\mathbf{G})]_2$ second, and so on.[3] We also define the network decoding order matrix $\mathbf{D}(\mathbf{G}) = [\mathbf{d}^1(\mathbf{G})\ \mathbf{d}^2(\mathbf{G})\ \cdots\ \mathbf{d}^N(\mathbf{G})]$. If BS n decides to decode the signal from MS (n, k) in the jth order, the SINR of MS (n, k)'s signal at BS n can be expressed by

$$\gamma^n_{(n,k)}(\mathbf{P}(\mathbf{G}), \mathbf{D}(\mathbf{G})) = \frac{g^n_{(n,k)} p_{(n,k)}(\mathbf{G})}{\sum_{i=j+1}^{K_n} g^n_{[\mathbf{d}^n(\mathbf{G})]_i} p_{[\mathbf{d}^n(\mathbf{G})]_i}(\mathbf{G}) + I^n_{\text{inter}}(\mathbf{P}(\mathbf{G})) + \sigma^2},$$
$$I^n_{\text{inter}}(\mathbf{P}(\mathbf{G})) = \sum_{l=1, l\neq n}^{N} \sum_{k=1}^{K_l} g^n_{(l,k)} p_{(l,k)}(\mathbf{G}) \quad (2)$$

for the given network gain matrix $\mathbf{G}$, the network power matrix $\mathbf{P}(\mathbf{G})$, and the network decoding order matrix $\mathbf{D}(\mathbf{G})$. In the equation the first and the second terms in the denominator represent the intra-cell interference received from the MSs within cell n and the inter-cell interference received from the MSs within the other cells, respectively, and σ^2 is the noise power. Then, the achievable data rate of MS (n, k)'s signal at BS n is given by

$$R^n_{(n,k)}(\mathbf{P}(\mathbf{G}), \mathbf{D}(\mathbf{G})) = B \log\left(1 + \gamma^n_{(n,k)}(\mathbf{P}(\mathbf{G}), \mathbf{D}(\mathbf{G}))\right) \quad (3)$$

for the total bandwidth B. Thus the weighted sum of the average data rates in the overall network is given by $\mathrm{E}_{\mathbf{G}}[\sum_{n=1}^{N} \sum_{k=1}^{K_n} \mu_{(n,k)} R^n_{(n,k)}(\mathbf{P}(\mathbf{G}), \mathbf{D}(\mathbf{G}))]$, where $\mu_{(n,k)}$ is the MS-dependent weighting factor. The weighting factors are chosen by each BS considering the quality of service (QoS) requirements of the users inside a cell.[4] We may assume, without loss of generality, that $\mu_{(n,1)} \geq \mu_{(n,2)} \geq \cdots \geq \mu_{(n,K_n)}$ and call the MSs with the same weighting factor a MS group.

[1] However, it can be easily extended to the case of multiple channels where the distribution of channel condition is the same for all channels. The objective and constraint of the problem that we deal with are the weighted sum of the 'average' data rates and the 'average' transmit power, respectively. Therefore, the solution of the multi-channel problem is to apply the solution of the single-channel problem to each channel individually.

[2] $\mathrm{E}_A[\cdot]$ indicates the expectation with respect to random variable A.

[3] $[\mathbf{a}]_k$ indicates the kth element of vector $\mathbf{a}$.

[4] It is an important issue how to choose the weighting factors to satisfy the QoS requirements of each user, but it is not within the scope of this paper. We are interested in improving the weighted sum of the average data rates for the given weighting factors.

III. DISTRIBUTED RESOURCE ALLOCATION (DRA) ALGORITHM

We investigate how to maximize the weighted sum of the average data rates in the overall network under the average power constraint of each MS. We can formulate the optimization problem as

$$\begin{aligned}\max_{\mathbf{P(G)},\mathbf{D(G)}} \quad & \mathrm{E}_{\mathbf{G}}\left[\sum_{n=1}^{N}\sum_{k=1}^{K_n}\mu_{(n,k)}R^{n}_{(n,k)}(\mathbf{P}(\mathbf{G}),\mathbf{D}(\mathbf{G}))\right]\\ \text{subject to} \quad & \mathrm{E}_{\mathbf{G}}[p_{(n,k)}(\mathbf{G})] \le P^{\max}_{(n,k)},\\ & \text{for } n=1,2,\cdots,N,\ k=1,2,\cdots,K_n,\\ & p_{(n,k)}(\mathbf{G}) \ge 0,\\ & \text{for } n=1,2,\cdots,N,\ k=1,2,\cdots,K_n. \end{aligned} \quad (4)$$

In the case of single-cell (i.e., $N = 1$), the above problem becomes a convex optimization problem and the optimal solution can be determined by exploiting the convexity as proposed by Knopp *et al.* [12] and Tse *et al.* [5]. In multi-cell environment, however, it is extremely hard to determine the optimal solution because, in general, the problem is no longer a convex optimization problem when $N > 1$. Moreover, even if we could find the optimal solution, the optimization requires a hierarchical network architecture consisting of BSs and a base station controller that has the information about the channel gains of all the BS-MS pairs. Since such hierarchical network architecture is not scalable and not cost-effective, the next generation systems are expected to be built on all-IP horizontal network architecture [13]. In this case, the BS has to perform radio resource management without help of the central controller, so we devise a distributed suboptimal algorithm called distributed resource allocation (DRA) that is compatible with the horizontal network architecture.

For the distributed operation, we design the DRA algorithm such that it assigns a utility to each cell individually and makes each cell determine its strategy (i.e., the cell power vector and the cell decoding order vector) to maximize its utility. Since the data rates of different cells are coupled through inter-cell interferences and each cell behaves in a distributed way, the DRA algorithm makes each cell maximize its own utility in an iterative manner.[5] As will be proved in the next section, the DRA algorithm can determine an equilibrium point in a finite number of iterations where no cell can improve its utility by changing its strategy unilaterally. To support such iterative operation, we consider the frame structure shown in Fig. 2(a) in which a training period is inserted between the downlink and uplink data slots. We design the DRA algorithm such that it first determines the equilibrium point by evolving the strategy iteratively during the training period, and then transmits data signal using the strategy corresponding to the equilibrium point.

In distributed operation, each BS can use only the information that it can obtain for itself without any additional signal exchange among BSs. Thus, we first consider what information can be obtained by BS n locally. First, BS n can obtain the cell gain matrix $\mathbf{G}^n$. MS (n,k) can measure the kth row of matrix $\mathbf{G}^n$ through the pilot signals from the neighboring BSs because the coherence time is large enough when compared with the length of a frame in the system model. MSs feed the measured information back to BS n and then BS n composes $\mathbf{G}^n$ using the feedback information. Second, BS n can measure the value $x^n \equiv I^{n}_{\text{inter}}(\mathbf{P}(\mathbf{G})) + \sigma^2$, which is dictated by the strategies determined by the neighboring cells in the previous iteration. We call this value the external force associated with BS n. Based on those local information $(\mathbf{G}^n, x^n)$, BS n controls the cell power vector $\mathbf{p}^n$ and the cell decoding order vector $\mathbf{d}^n$.

[5]Let us consider a two-cell case. If cell A changes its strategy, cell B will change its strategy because the interference from cell A varies. Then, the interference from cell B will also vary, so cell A will change its strategy again.

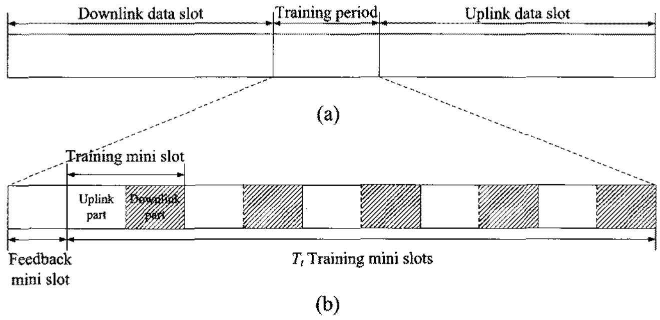

Fig. 2. Frame structure: (a) A frame consisting of a downlink data slot, a training period, and an uplink data slot and (b) a training period consisting of a feedback mini slot and T_t training mini slots.

Now, we discuss how to define the utility function. In multi-cell networks, the data rate of a cell increases if the transmission powers of the MSs within the cell increase but it induces an increased inter-cell interference in the other cells too. Thus, a strategy that strives to maximize the weighted sum of the average rates in one cell may result poor performance in the network-wise sense due to the inter-cell interference. So we define the induced interference of BS n, $I^{n}_{\text{induced}}(\mathbf{p}^n(\mathbf{G}^n, x^n))$, or the interference induced by the transmission of the MSs associated with BS n to the other cells, by

$$I^{n}_{\text{induced}}(\mathbf{p}^n(\mathbf{G}^n, x^n)) \equiv \sum_{k=1}^{K_n} h_{(n,k)}p_{(n,k)}(\mathbf{G}^n, x^n) \quad (5)$$

where $h_{(n,k)} = \sum_{l=1,l\neq n}^{N} g^{l}_{(n,k)}$ is the interfering gain of MS (n,k). Then, we define the utility function associated with BS n by

$$\begin{aligned}&u_n(\mathbf{p}^n(\mathbf{G}^n, x^n), \mathbf{d}^n(\mathbf{G}^n, x^n)) =\\ &\mathrm{E}_{\mathbf{G}^n,x^n}\left[\sum_{k=1}^{K_n}\mu_{(n,k)}R^{n}_{(n,k)}(\mathbf{p}^n(\mathbf{G}^n, x^n), \mathbf{d}^n(\mathbf{G}^n, x^n))\right]\\ &- c\mathrm{E}_{\mathbf{G}^n,x^n}\left[I^{n}_{\text{induced}}(\mathbf{p}^n(\mathbf{G}^n, x^n))\right] \end{aligned} \quad (6)$$

where the term $R^{n}_{(n,k)}(\mathbf{p}^n(\mathbf{G}^n, x^n), \mathbf{d}^n(\mathbf{G}^n, x^n))$ emphasizes that each BS controls only its own strategy based on the local information, and c denotes the price per unit induced interference in bps/W. The interference price represents the cost imposed on each cell for the co-channel interference generated by the cell. We will discuss how to choose a proper value of the interference price in Section V. With this definition of utility, each BS controls the transmission power of its MSs considering not only the channel gains between it and the MSs but also the interfering gains between the neighboring BSs and the MSs. Intuitively,

1. During the downlink data slot: Each MS measures the channel gains between it and all the BSs.
2. During the training period:
   During the feedback mini slot: Each MS feeds back the measured channel gains to its associated BS.
   For training mini slot = 1 to T_t
      During the uplink part:
      If training mini slot = 1
         Each MS transmits the training signal with the pre-determined transmission power.
      Else
         Each MS transmits the training signal according to the power determined in the previous training mini slot.
      End
      During the downlink part:
      Each BS measures the external force.
      Each BS determines the optimal cell decoding order vector and the cell power vector for the given cell gain matrix and external force.*
      Each BS makes the associated MSs know their transmission power.
   End
3. During the uplink data slot: Each MS transmits its data signal according to the power determined in the T_tth training mini slot.

* The optimal strategy is discussed in subsections III-A and III-B.

Fig. 3. Description of the DRA algorithm.

the BS tends to permit high transmission power to the MS that has a high channel gain and a low interfering gain. Therefore, the utility defined above renders a measure to instigate implicit cooperation among BSs, which contributes to enhancing the performance in network-wise sense.

Based on the above discussions, we arrange the training period to consist of a feedback mini slot followed by T_t training mini slots as detailed in Fig. 2(b), and define the DRA algorithm to take the procedure described in Fig. 3. Note that at each training mini slot, each BS maximizes its own utility using the local information that it can receive from the associated MSs or can measure for itself. This explains how the DRA operates in a distributed manner. As the training signal is intended to determine the equilibrium point, the training mini slot may be made much shorter in length than the data slot. We will discuss how to choose a proper number of training mini slots in Section V. In the next two subsections, we discuss the algorithms that determine the optimal strategy to maximize the above utility in the single and the multiple user cases.

A. Single-User Case

We first consider the case when single user resides in each cell (i.e., $K_n = 1$), thereby getting insight for handling the multi-user case. In this case, the utility maximization problem for BS n is formulated by

$$\begin{aligned}\max_{p_{(n,1)}(\mathbf{G}^n,x^n)} \quad & \mathrm{E}_{\mathbf{G}^n,x^n}\left[R^n_{(n,1)}(p_{(n,1)}(\mathbf{G}^n,x^n))\right] \\ & - c\mathrm{E}_{\mathbf{G}^n,x^n}\left[I^n_{\text{induced}}(p_{(n,1)}(\mathbf{G}^n,x^n))\right] \\ \text{subject to} \quad & \mathrm{E}_{\mathbf{G}^n,x^n}[p_{(n,1)}(\mathbf{G}^n,x^n)] \le P^{\max}_{(n,1)}, \\ & p_{(n,1)}(\mathbf{G}^n,x^n) \ge 0. \end{aligned} \tag{7}$$

Proposition 1: When a single user $(n,1)$ exists in cell n, the optimal solution to the problem in (7) is given by

$$p^*_{(n,1)}(\mathbf{G}^n,x^n) = \left[\frac{B}{\lambda^*_{(n,1)} + ch_{(n,1)}} - \frac{x^n}{g^n_{(n,1)}}\right]^+, \tag{8}$$

$$\lambda^*_{(n,1)}\left(\mathrm{E}_{\mathbf{G}^n,x^n}[p_{(n,1)}(\mathbf{G}^n,x^n)] - P^{\max}_{(n,1)}\right) = 0,\ \lambda^*_{(n,1)} \ge 0 \tag{9}$$

where $\lambda^*_{(n,1)}$ is the Lagrange multiplier for the average power constraint.[6]

Proof: See Appendix A. □

The proposition indicates that the optimal strategy of BS n is the water-filling allocation with the water level determined by both the interference price and the average power constraint. $\lambda^*_{(n,1)}$ is determined to be the value satisfying the average power constraint of MS $(n,1)$ with equality or to be zero if such value does not exist. We can determine $\lambda^*_{(n,1)}$ that satisfies the average power constraint with equality by solving the equation

$$P^{\max}_{(n,1)} = \int_{x=0}^{\infty}\int_{h=0}^{\infty}\int_{g=0}^{\infty}\left[\frac{B}{\lambda^*_{(n,1)} + ch} - \frac{x}{g}\right]^+ f_{g^n_{(n,1)}}(g)\, \mathrm{d}g f_{h_{(n,1)}}(h)\mathrm{d}h f_{x^n}(x)\mathrm{d}x \tag{10}$$

where $f_a(\cdot)$ denotes the probability density function (PDF) of a random variable a. If the solution to the above equation is negative, we set $\lambda^*_{(n,1)}$ to zero.

B. Multi-User Case

Now we consider the case when multiple users reside in each cell. In this situation, the problem that BS n has to solve can be formulated by[7]

$$\begin{aligned}\max_{\mathbf{p}^n(\mathbf{G}^n,x^n),\mathbf{d}^n(\mathbf{G}^n,x^n)} \quad & u_n(\mathbf{p}^n(\mathbf{G}^n,x^n),\mathbf{d}^n(\mathbf{G}^n,x^n)) \\ \text{subject to} \quad & \mathrm{E}_{\mathbf{G}^n,x^n}[p_{(n,k)}(\mathbf{G}^n,x^n)] \le P^{\max}_{(n,k)}, \\ & \text{for } k = 1,2,\cdots,K_n, \\ & \mathbf{p}^n(\mathbf{G}^n,x^n) \succeq 0. \end{aligned} \tag{11}$$

Proposition 2: When there are K_n users in cell n, and $\mu_{(n,1)} \ge \mu_{(n,2)} \ge \cdots \ge \mu_{(n,K_n)}$, the optimal strategy of BS n is given by[8]

$$\mathbf{d}^{*n}(\mathbf{G}^n,x^n) = [(n,K_n)\ (n,K_{n-1})\ \cdots\ (n,1)], \tag{12}$$

$$\mathbf{p}^{*n}(\mathbf{G}^n,x^n) = \mathbf{z}^{*n}(\mathbf{G}^n,x^n)/[\mathbf{G}^n]_{:,n} \tag{13}$$

where $\mathbf{z}^{*n}(\mathbf{G}^n,x^n) = [z^*_{(n,1)}(\mathbf{G}^n,x^n)\ z^*_{(n,2)}(\mathbf{G}^n,x^n)\ \cdots\ z^*_{(n,K_n)}(\mathbf{G}^n,x^n)]^T$ is the optimal vector determined by solving the problem

$$\max_{\mathbf{z}^n(\mathbf{G}^n,x^n)} \sum_{k=1}^{K_n}\int_{\sum_{i=0}^{k-1} z_{(n,i)}(\mathbf{G}^n,x^n)}^{\sum_{i=0}^{k} z_{(n,i)}(\mathbf{G}^n,x^n)}$$

[6]Note that $[x]^+ = x$ if $x \ge 0$ and 0 otherwise.

[7]$\succeq$ indicates the element-wise vector inequality.

[8]$\mathbf{x}/\mathbf{y}$ denotes component-wise vector division between $\mathbf{x}$ and $\mathbf{y}$ and $[\mathbf{X}]_{:,j}$ denotes the jth column vector of matrix $\mathbf{X}$.

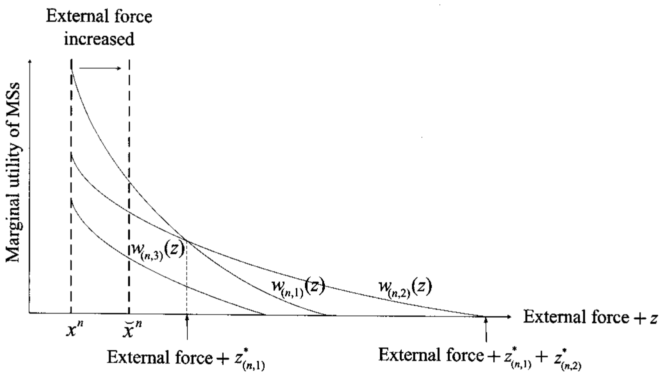

Fig. 4. Illustration of the greedy power allocation for three-MS case.

$$\left(\frac{\mu_{(n,k)}B}{x^n+z} - \frac{\lambda^*{}_{(n,k)} + ch_{(n,k)}}{g^n_{(n,k)}}\right) \mathrm{d}z$$
$$\text{subject to } z_{(n,0)}(\mathbf{G}^n, x^n) = 0,$$
$$\mathbf{z}^n(\mathbf{G}^n, x^n) \succeq 0,$$
$$\lambda^*{}_{(n,k)}\left(\mathrm{E}_{\mathbf{G}^n,x^n}[p_{(n,k)}(\mathbf{G}^n, x^n)] - P^{\max}_{(n,k)}\right)$$
$$= 0, \text{ for } k = 1, 2, \cdots, K_n,$$
$$\lambda^*{}_{(n,k)} \geq 0, \text{ for } k = 1, 2, \cdots, K_n \tag{14}$$

where $\lambda^*{}_{(n,k)}$ is the Lagrange multiplier for the average power constraint.

Proof: See Appendix B. □

The optimal solution to the problem in (14) can be determined by a simple greedy algorithm, in a similar way to that proposed in [5]. The greedy algorithm sequentially determines $z^*{}_{(n,k)}(\mathbf{G}^n, x^n)$ for all k in the descending order of the weighting factors of the MSs (i.e., from MS $(n, 1)$ to MS (n, K_n)) as follows: For the kth MS, it increases $z_{(n,k)}(\mathbf{G}^n, x^n)$ from zero until $w_{(n,k)}(\sum_{i=0}^{k} z_{(n,i)}(\mathbf{G}^n, x^n))$ becomes less than $[\max_j w_{(n,j)}(\sum_{i=0}^{k} z_{(n,i)}(\mathbf{G}^n, x^n))]^+$, where $w_{(n,k)}(z)$ denotes the marginal utility of MS (n, k) expressed by $w_{(n,k)}(z) = \frac{\mu_{(n,k)}B}{x^n+z} - \frac{\lambda^*{}_{(n,k)}+ch_{(n,k)}}{g^n_{(n,k)}}$.

Fig. 4 illustrates the greedy algorithm for a three-MS case with $\mu_{(n,1)} > \mu_{(n,2)} = \mu_{(n,3)}$. The x-axis represents the total interference, which is the summation of the external force and the intra-cell interference and the y-axis represents the marginal utility. The crossing point of the marginal utilities of MS $(n, 1)$ and $(n, 2)$, and that of the marginal utility of MS $(n, 2)$ and the x-axis determines $z^*_{(n,1)}$ and $z^*_{(n,2)}$, respectively. In the optimal solution, at most one MS, specifically, the MS with minimum $\frac{\lambda^*{}_{(n,k)}+ch_{(n,k)}}{g^n_{(n,k)}}$, can use positive transmit power in each MS group. Thus, $z^*_{(n,3)} = 0$ in this example.

We can determine $\Lambda^{*n} = [\lambda^*{}_{(n,1)}\ \lambda^*{}_{(n,2)} \cdots \lambda^*{}_{(n,K_n)}]$ satisfying the average power constraint with equality by solving the equation

$$P^{\max}_{(n,k)} = \int_{z=0}^{\infty} \mathrm{E}_{\mathbf{G}^n,x^n}\left[\frac{I[w_{(n,k)}(z) \geq [\max_j w_{(n,j)}(z)]^+]}{g^n_{(n,k)}}\right] \mathrm{d}z,$$
$$\text{for } k = 1,\ 2, \cdots,\ K_n \tag{15}$$

where $I[\cdot]$ is an indicator function that is 1 if the entity is true, and 0 otherwise. We can rewrite the right hand side of (15) as shown in (16), where $F_a(\cdot)$ denotes the cumulative density function (CDF) of a random variable a. If the solution to the above equation is negative, we set $\lambda^*{}_{(n,k)}$ to zero. We will discuss a practical way of calculating the Lagrange multiplier vector in Section V.

C. Extension to Practical Systems

So far we assumed no constraint on the instantaneous power. We also assumed that each BS can receive signals from multiple MSs simultaneously within its own cell aided by the successive intra-cell interference canceller. In practical systems, however, MSs cannot increase the transmission power infinitely and the implementation of successive interference cancellation is highly complicated. Therefore, we need to extend the DRA algorithm to more practical environments as demonstrated below.

First, when the maximum instantaneous power constraint is imposed, each BS solves the problem similar to (11) except for an additional constraint on the maximum instantaneous power. Since the problem is still convex, the optimal strategy can be obtained by solving (12), (13) and a modified version of (14) which contains the additional maximum instantaneous power constraint. To solve the problem in (14) with the additional maximum instantaneous power constraint, we can use a combinatorial greedy algorithm presented in [5] with the number of steps bounded by $2K_n$.

Secondly, when at most one MS can transmit signal at each time, an additional constraint is needed on the number of the transmitting MSs. Unfortunately, the resulting new problem is not convex any longer, so it is difficult to determine the optimal solution. However, considering the fact that a Lagrange dual function yields an upper bound of the optimal value, we may use a sub-optimal power allocation algorithm that modifies the power allocation proposed in Proposition 2. Specifically, we select MS (n, k) which yields the maximum $u_n(\mathbf{p}^n(\mathbf{G}^n, x^n), \mathbf{d}^n(\mathbf{G}^n, x^n))$ when the power of MS (n, k) is set to $\left[\frac{\mu_{(n,k)}B}{\lambda^*{}_{(n,k)}+ch_{(n,k)}} - \frac{x^n}{g^n_{(n,k)}}\right]^+$ and the powers of the other MSs are set to zero, and then permit the selected MS to use the power $\left[\frac{\mu_{(n,k)}B}{\lambda^*{}_{(n,k)}+ch_{(n,k)}} - \frac{x^n}{g^n_{(n,k)}}\right]^+$.

IV. CONVERGENCE OF THE DRA ALGORITHM

In order to check the validity of the DRA algorithm, we need to prove that it converges to a unique equilibrium point after iterations. We denote by (n, k), $k = 1, 2, \cdots, M$, the kth MS within cell n that is allowed to transmit signal. As it is clear that the allowed users belong to different MS groups, we assume that $\mu_{(n,1)} > \mu_{(n,2)} > \cdots > \mu_{(n,M)}$ without loss of generality.

Proposition 3: Given that the network gain matrix $\mathbf{G}$ remains fixed, the cell total power associated with cell n, $s^n(\mathbf{G}^n, x^n)$, decreases as the external force of BS n increases, and the decreasing ratio is smaller than $\max_{(n,k)\in\mathcal{A}} 1/g^n_{(n,k)}$, where $\mathcal{A}$ indicates the set of the MSs allowed to transmit at present time.

Proof: See Appendix C. □

$$\int_{z=0}^{\infty}\int_{x=0}^{\infty}\int_{h_{K_n}=0}^{\infty}\cdots\int_{h_1=0}^{\infty}\int_{g=\frac{(\lambda^*_{(n,k)}+ch_k)(x+z)}{\mu_{(n,k)}B}}^{\infty} t f_{g^n_{(n,k)}}(g)\mathrm{d}g\prod_{j=1}^{K_n}(f_{h_{(n,j)}}(h_j)\mathrm{d}h_j)f_{x^n}(x)\mathrm{d}x\mathrm{d}z,$$
$$t=\frac{1}{g}\prod_{j=1,j\neq k}^{K_n}F_{g^n_{(n,j)}}\left(\frac{g(\lambda^*_{(n,j)}+ch_j)(x+z)}{(\lambda^*_{(n,k)}+ch_k)(x+z)+gB(\mu_{(n,j)}-\mu_{(n,k)})}\right). \quad (16)$$

Based on the above proposition, we can establish the existence and uniqueness of the equilibrium point as follows

Theorem 1: The DRA has a unique fixed equilibrium point under the following assumptions:

i) Even when only one MS group exists in cell n, there exists an MS (n,k) such that $p_{(n,k)}(\mathbf{G}^n, x^n) > 0$ for any network power matrix $\mathbf{P}(\mathbf{G})$.[9]

ii) The noise power σ^2 is much smaller than the interference.[10]

Proof: We prove the theorem by proving that the power allocation rule of the DRA is type-II standard [15] in the sense of cell total power, that is, it has the following properties:

a) Positivity: $s^n(\mathbf{G}^n, x^n) > 0$ for any external total power vector $\mathbf{s}^{-n}(\mathbf{G})$.

b) Type-II monotonicity: If $\mathbf{s}^{-n}(\mathbf{G}) \prec \hat{\mathbf{s}}^{-n}(\mathbf{G})$, then $s^n(\mathbf{G}^n, x^n) > s^n(\mathbf{G}^n, \hat{x}^n)$, where $\hat{x}^n$ denotes the external force for the external total power vector $\hat{\mathbf{s}}^{-n}(\mathbf{G})$.

c) Type-II scalability: For all $\alpha > 1$, $(1/\alpha)s^n(\mathbf{G}^n, x^n) < s^n(\mathbf{G}^n, \breve{x}^n)$, where $\breve{x}^n$ denotes the external force for the external total power vector $\alpha\mathbf{s}^{-n}(\mathbf{G})$.

The positivity property is obtained directly from the first assumption and the definition of the cell total power. We can prove the type-II monotonicity property by applying Proposition 3.

For the type-II scalability property, we first prove that the property holds in the case that only one MS group exists in cell n. Proposition 3 implies that increasing the cell total power by α times does not mean increasing the transmit power of every MS within the cell by α times individually. Thus, the external force $\breve{x}^n$ for the external total power vector $\alpha\mathbf{s}^{-n}(\mathbf{G})$ may not be equal to αx^n. However, we can prove that $\breve{x}^n$ is upper bounded by $\alpha\beta x^n$ for a finite value β. See Appendix D for the proof of this argument. Noting the above argument, we assume that $\breve{x}^n = \alpha\beta x^n$. As we assume that only one MS group exists in cell n, the reactions against x^n and $\breve{x}^n$ are expressed respectively by

$$s^n(\mathbf{G}^n, x^n) = a - b > 0,$$
$$a = \frac{\mu_{(n,k)}B}{\lambda^*_{(n,k)} + ch_{(n,k)}},\ b = \frac{x^n}{g^n_{(n,k)}}, \quad (17)$$

$$s^n(\mathbf{G}^n, \breve{x}^n) = a - \alpha\beta b > 0 \quad (18)$$

where MS (n,k) is the MS that has the largest marginal utility in the MS group and the inequalities are valid due to the positivity property. In addition, for any $\alpha > 1$, we get

$$a - \frac{\alpha^2\beta - 1}{\alpha - 1}b > 0 \quad (19)$$

due to the positivity property, since $(\alpha^2\beta - 1)/(\alpha - 1)$ is finite for any $\alpha > 1$. Using (19), we can derive the inequality

$$a - \alpha\beta b > \frac{1}{\alpha}(a - b) > 0. \quad (20)$$

Thus, the scalability holds when $\breve{x}^n = \alpha\beta x^n$. When $\breve{x}^n$ is smaller than $\alpha\beta x^n$, the transmission power decrease with respect to $\breve{x}^n$ is smaller than that with respect to $\alpha\beta x^n$, so the scalability still holds.

In reality, there may be multiple MS groups in cell n. In this case, the ratio by which the transmission power decreases when the external force increases is smaller than that in the single-group case by Proposition 3. Therefore, we can conclude that the scalability property also holds in the multi-group case.

It is readily proven in [15] that a type-II standard power allocation rule converges to the unique fixed point without regard to the initial value, so the network total power vector converges to an equilibrium for any initial values. Then, one can easily prove that the network power matrix also converges to an equilibrium as the network total power vector converges to an equilibrium. □

V. NUMERICAL RESULTS

In order to confirm the convergence and the performance of the proposed DRA algorithm, we conducted computer simulations over the wireless uplink system composed of 27 cells and 270 MSs, with a BS with omni-directional antenna residing at the center of each cell. We considered a hexagonal cell structure with a cell diameter of 1 km. The 270 MSs are randomly located in the 27 cell regions by uniform distribution. Each MS is connected to the BS to which the average channel gain, determined by the path loss and the shadowing, is the maximum among its neighboring BSs. We adopted the Rayleigh fading model and assumed that the channel gain remains fixed for one frame and changes independently from frame to frame. We set the values of the involved parameters as follows: the total bandwidth, 1Hz; the maximum average transmission powers of all the MSs, 30 dBm; the path loss exponent, 3.76; the standard deviation of log-normal shadowing, 8 dB; and the noise power, -120 dBm. We divided the MSs in each cell into two groups according to their average channel gain: The top 50% of MSs belong to group B and the other MSs belong to group A. We set the weighting factors of the MSs in group A to 1, and varied the weighting factors of the MSs in group B in the simulations. We set the price of the induced interference to 1×10^{13} (bps/W) unless specified otherwise.

As the calculation process proposed in (16) requires the $(3+K_n)$th order integration and full knowledge about the distribution of the external force, it is hard to obtain the optimal

[9]This assumption is not unrealistic as there would exist at least one MS in each MS group that has a very high ratio of channel gain to interference gain.

[10]This assumption is reasonable in interference-limited systems.

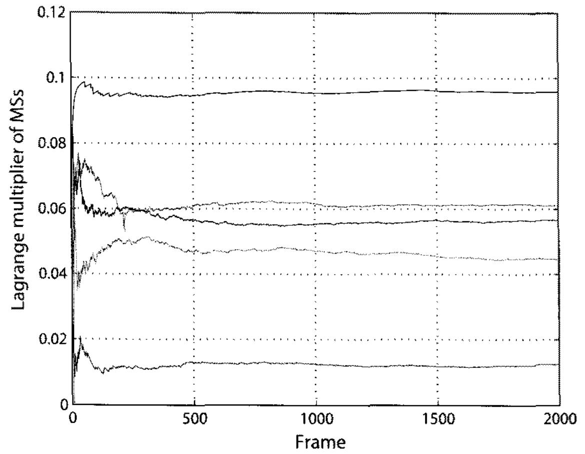

Fig. 5. Convergence of the Lagrange multipliers of randomly selected 5 MSs with respect to the number of frames.

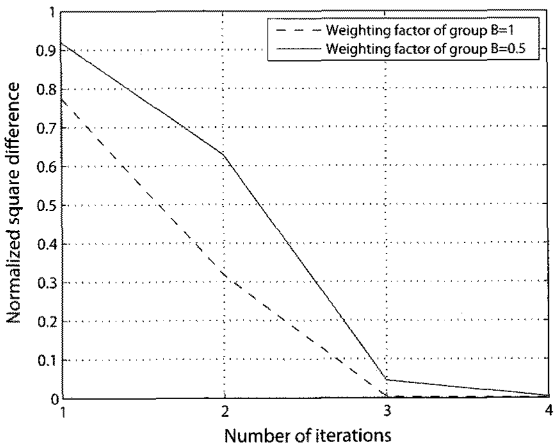

Fig. 6. Normalized square differences with respect to the number of iterations.

Lagrange multiplier vector by using (16). Thus, for computer simulations, we used the history of the cell gain matrix and the external force instead of the PDF of them. At the end of each frame, we recorded the cell gain matrix $\mathbf{G}^n$ and the external force x^n associated with the frame, and updated the optimal Lagrange multiplier vector Λ^{*n} to satisfy the average power constraint of each MS with equality by using the history of $\mathbf{G}^n$ and x^n. After enough number of frames are elapsed, we confirmed the convergence of Λ^{*n}. Fig. 5 plots the Lagrange multipliers of randomly selected 5 MSs with respect to the number of frames. It is the result for the case when the weighting factors of all the MSs are set to 1. We observe that each Lagrange multiplier of those MSs converges close to a fixed value after about 500 frames.

In order to test the convergence of the DRA algorithm, we defined an empirical index called the normalized square difference (NSD) by

$$\text{NSD} = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{s^n(\mathbf{G}^n, x^n)_{\text{curr}} - s^n(\mathbf{G}^n, x^n)_{\text{pre}}}{s^n(\mathbf{G}^n, x^n)_{\text{curr}} + s^n(\mathbf{G}^n, x^n)_{\text{pre}}}\right)^2 \tag{21}$$

where $s^n(\mathbf{G}^n, x^n)_{\text{curr}}$ and $s^n(\mathbf{G}^n, x^n)_{\text{pre}}$ means the cell total power determined in the current training mini slot and that in the previous training mini slot, respectively. We measured the NSD through iterations for two different weighting factors of the MSs in group B, 1 and 0.5. Fig. 6 depicts the resulting NSDs with respect to the number of iteration. We observe that for both cases, the DRA algorithm converges to the equilibrium within about 3 iterations. Based on this result, we set the number of the training mini slots, T_t to 5 in the subsequent simulations.

We then examined the network performance of the DRA algorithm. For performance comparison, we conducted simulations on two other reference algorithms as well, namely selfish distributed resource allocation (SDRA) and Hande's algorithm [8]. In the case of the SDRA algorithm, each BS strives to maximize the weighted sum of the average rates in its cell (i.e., the price of the induced interference is set to zero) and other operations are similar to the DRA.[11] The SDRA algorithm may be regarded as a primitive multi-cell version of the resource allocation algorithm which is optimal in the single-cell case proposed in [5]. In the case of the Hande's algorithm, each BS assigns a target SINR for each MS and each MS decides its transmission power through the Foschini-Miljanic method [16] to achieve the target SINR. Each BS finds the optimal target SINR vector to maximize the objective function in an iterative manner. The Hande's algorithm is optimal for maximizing the weighted sum of instantaneous data rates with the instantaneous power constraint of each MS. As the Hande's algorithm only considers the constraints on the instantaneous power, we determined the maximum instantaneous power of each MS that satisfies the average power constraint by repeating simulations.

Fig. 7 depicts the resulting average rates of the MSs in groups A and B for the three algorithms, with the weighting factors of the MSs in group B varied from 1 to 0. We varied the price of interference for the DRA algorithm from 1×10^{11} to 1×10^{15}. From this figure, we observe three notable results:

1) The DRA algorithm performs best for the price of 1×10^{13} and the performance degrades when the price goes above or below that value. The marginal utility of MS (n,k), $w_{(n,k)}(z)$, which is introduced in subsection III-B explains the reason of this phenomenon. If the price is too low, the Lagrange multiplier predominates over the product of price and interfering gain (i.e., $\lambda^*_{(n,k)} \gg ch_{(n,k)}$). Thus, the DRA algorithm cannot consider the interfering gain of each user properly. In contrast, if the price is too high, the DRA algorithm cannot reflect the effect of the average power constraint of each user properly. A rule of thumb to determine a proper value of the price is to make the Lagrange multiplier averaged over the MSs in the cell equal to the product of price and interfering gain averaged over the channel realizations and the MSs. In the simulation setting, the magnitude of the price obtained using the above rule is about 1×10^{13}.
2) The trade-off curves are similar to each other for the prices 1×10^{12}, 1×10^{13} and 1×10^{14}. This means that although the price affects the performance of the DRA algorithm, the performance is insensitive to the price. Therefore, the DRA algorithm can work well for a very wide range of prices.

[11]The term 'selfish' comes from the fact that even no implicit cooperation exists among the BSs in the SDRA algorithm.

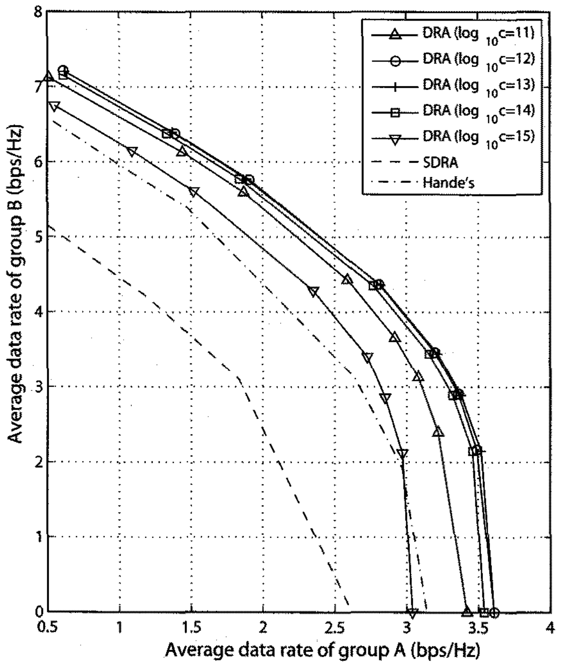

Fig. 7. Average rates of the MSs in group A and group B, with the weighting factors of the MSs in group B varying from 1 (left-upper side) to 0 (right-lower side).

3) The DRA algorithm outperforms the SDRA algorithm and the Hande's algorithm for an appropriately chosen interference price. For example, when the average rate of group B is fixed to 4 bps/Hz, the average rates of group A for SDRA, Hande's, and DRA algorithm with the price of 1×10^{13} are 1.3, 2.2, and 3.0, respectively. Thus, the DRA algorithm gets a performance improvement of 130% and 36% respectively over the SDRA and the Hande's algorithms.

VI. CONCLUSIONS

In this paper, we have presented a new resource allocation algorithm called DRA for multi-cell uplink systems that enhances the weighted sum of the average rates in the whole network under the average power constraint of each MS. We designed the DRA algorithm such that it requires no coordination among the BSs. In the DRA algorithm, each BS implicitly cooperates with each other by iteratively selecting the best strategy that maximizes the weighted sum of the average data rate while minimizing the induced interference to other cell. Such behavior of the BSs turned out to improve the average data rate significantly in the network-wise sense when compared with the selfish behavior of BSs not considering the induced interference.

As the cost of the distributed operation, the DRA algorithm requires some iterative calculations of the transmission power vector. However, the transmission power vector attained by the DRA is proven to converge to a unique fixed point for any initial values, under some reasonable assumptions. Computer simulations revealed that the required number of iterations to converge to the fixed point is very small, about 3.

We have adopted the concept of interference price which is imposed on the interference experienced by other cells and determines the degree of concerns of other cells. It is important to set the interference price appropriately to obtain an efficient operation of the DRA algorithm. We have proposed a rule of thumb of determining a proper value of the price, but the determination of the optimal value that maximizes the overall performance of the entire network remains as a future work.

APPENDIX

A. Proof of Proposition 1

The problem is the same as that considered in [5] except that the induced interference term is added to the objective function. So we can determine the optimal solution by taking an approach similar to that in [5]. Hence, we only provide an outline of the solution.

The objective function can be rewritten as

$$E_{\mathbf{G}^n,x^n}\left[B\log\left(1+\frac{g^n_{(n,1)}p_{(n,1)}(\mathbf{G}^n,x^n)}{x^n}\right) - ch_{(n,1)}p_{(n,1)}(\mathbf{G}^n,x^n)\right]. \quad (22)$$

It is clear that the problem is a convex optimization problem, so we can solve the problem by solving the Lagrange dual problem [14, p. 223]

$$\min_{\lambda_{(n,1)}} \; L(\lambda_{(n,1)}) \quad \text{subject to } \lambda_{(n,1)} \geq 0 \quad (23)$$

where $L(\lambda_{(n,1)})$ denotes the Lagrange dual function

$$L(\lambda_{(n,1)}) = \max_{p_{(n,1)}(\mathbf{G}^n,x^n)\geq 0} E_{\mathbf{G}^n,x^n}\left[B\log\left(1+\frac{g^n_{(n,1)}p_{(n,1)}(\mathbf{G}^n,x^n)}{x^n}\right) - (\lambda_{(n,1)}+ch_{(n,1)})p_{(n,1)}(\mathbf{G}^n,x^n)\right]. \quad (24)$$

The problem of maximizing $L(\lambda_{(n,1)})$ for a given $\lambda_{(n,1)}$ is equivalent to the problem of maximizing the entity in the expectation operator in (24) for every realization of $\mathbf{G}^n$ and x^n, i.e.,

$$\max_{p_{(n,1)}(\mathbf{G}^n,x^n)} B\log\left(1+\frac{g^n_{(n,1)}p_{(n,1)}(\mathbf{G}^n,x^n)}{x^n}\right) - (ch_{(n,1)}+\lambda_{(n,1)})p_{(n,1)}(\mathbf{G}^n,x^n)$$
$$\text{subject to } p_{(n,1)}(\mathbf{G}^n,x^n) \geq 0. \quad (25)$$

By defining the received interference at BS n due to the transmission of MS $(n,1)$ by $z_{(n,1)}(\mathbf{G}^n,x^n) \equiv g^n_{(n,1)}p_{(n,1)}(\mathbf{G}^n,x^n)$, we can rewrite the above problem as

$$\max_{z_{(n,1)}(\mathbf{G}^n,x^n)} \int_0^{z_{(n,1)}(\mathbf{G}^n,x^n)} \left(\frac{B}{x^n+z} - \frac{\lambda_{(n,1)}+ch_{(n,1)}}{g^n_{(n,1)}}\right) dz$$

subject to $z_{(n,1)}(\mathbf{G}^n, x^n) \geq 0$. (26)

We call the entity in the integral the marginal utility of MS $(n, 1)$ and denote it by $w_{(n,1)}(z)$. Since $w_{(n,1)}(z)$ is a decreasing function of z, the optimal solution is to increase $z_{(n,1)}(\mathbf{G}^n, x^n)$ from zero until $w_{(n,1)}(z_{(n,1)}(\mathbf{G}^n, x^n))$ becomes negative. Then, the resulting power allocation is given by

$$p_{(n,1)} = \left[\frac{B}{\lambda_{(n,1)} + ch_{(n,1)}} - \frac{x^n}{g^n_{(n,1)}}\right]^+. \quad (27)$$

We can determine $\lambda_{(n,1)}$ that minimizes the Lagrange dual function by applying the Karush-Khun-Tucker (KKT) condition [14, p. 243], which corresponds to (9).

B. Proof of Proposition 2

According to the results derived in [5], for any given power control, the weighted sum of the average rates is maximized when the cell decoding order vector is sorted in the ascending order of the weighting factor (i.e., the MS with the smallest weighting factor is decoded first, and the one with the largest weighting factor is decoded last). This cell decoding order vector is optimal to the problem in (11) since $I^n_{\text{induced}}(\mathbf{p}^n(\mathbf{G}^n, x^n))$ is independent of the decoding order. As $\mu_{(n,1)} \geq \mu_{(n,2)} \geq \cdots \geq \mu_{(n,K_n)}$, the optimal cell decoding order vector becomes $\mathbf{d}^{*n}(\mathbf{G}^n, x^n) = [(n, K_n)\ (n, K_{n-1}) \cdots (n, 1)]$.

Then, the objective function of (11) can be rewritten as shown in (28), where $g^n_{(n,0)}p_{(n,0)}(\mathbf{G}^n, x^n) = 0$. We can easily prove that the objective is a concave function, so the problem is also a convex optimization problem. Thus, we can solve the problem by solving the Lagrange dual problem

$$\min_{\Lambda^n} L(\Lambda^n)$$
$$\text{subject to } \Lambda^n \succeq 0 \quad (29)$$

where $\Lambda^n = [\lambda_{(n,1)}\ \lambda_{(n,2)} \cdots \lambda_{(n,K_n)}]$ and the Lagrange dual function $L(\Lambda^n)$ is calculated as shown in (30). The remaining steps are similar to that in the proof of Proposition 1. In order to maximize $L(\Lambda^n)$ for a given Λ^n, BS n determines the cell power vector that maximizes the entity in the expectation operator for every realization of $\mathbf{G}^n$ and x^n. The optimal power allocation is given by the solution to the problem in (14). In addition, the optimal vector Λ^{*n} can be determined to be the value satisfying the average power constraints of all the MSs associated with BS n with equality or to be zero.

C. Proof of Proposition 3

We consider the three-MS example illustrated in Fig. 4. Since $\mathbf{G}^n$ remains fixed, the increase of the external force only causes the shift of the bold dashed vertical line but the crossing points of the marginal utilities do not move. The transmit power of MS $(n, 1)$ is determined by the interval between the bold dashed line and the crossing point of the utility of MS $(n, 1)$ and that of MS $(n, 2)$, so only MS $(n, 1)$ decreases its power while MS $(n, 2)$ does not, until the line crosses the crossing point. As $p_{(n,1)} = z_{(n,1)}/g^n_{(n,1)}$, the decreasing ratio is $1/g^n_{(n,1)}$. After the bold dashed line passes over the crossing point of MS $(n, 1)$ and MS $(n, 2)$, then it is clear that $p_{(n,1)} = 0$ and MS $(n, 2)$ decreases its power with the ratio $1/g^n_{(n,2)}$. In this example, $(n, 3) \notin \mathcal{A}$ since MS $(n, 3)$ is not allowed to transmit, as mentioned in subsection III-B.

It is clear that the above discussion can be generalized to M-allowed MS case: MSs $(n, 1), (n, 2), \cdots, (n, M)$ decrease their powers sequentially as the external force increases and thus the decreasing rate is smaller than $\max_{(n,k)\in\mathcal{A}} 1/g^n_{(n,k)}$.

D. Proof of the Bounded External Force

We consider a cell m that makes an interference of amount $\sum_{k=1}^{K_m} g^n_{(m,k)}p_{(m,k)}$ on cell n. We assume that the cell total power of BS m is increased by α times, i.e., $\breve{s}^m = \sum_{k=1}^{K_m} \breve{p}_{(m,k)} = \sum_{k=1}^{K_m}(p_{(m,k)} + \delta_{(m,k)}) = \alpha\sum_{k=1}^{K_m} p_{(m,k)} = \alpha s^m$, where $\delta_{(m,k)} \geq 0$ denotes the power increment of MS (m, k) for $k = 1, 2, \cdots, K_m$. Then, we prove that there exists a finite value β such that

$$\sum_{k=1}^{K_m} g^n_{(m,k)}(p_{(m,k)} + \delta_{(m,k)}) \leq \alpha\beta\sum_{k=1}^{K_m} g^n_{(m,k)}p_{(m,k)}. \quad (31)$$

By the definition of the power increment, $\delta_{(m,k)}$, the left hand side of (31) is upper bounded as

$$\sum_{k=1}^{K_m} g^n_{(m,k)}(p_{(m,k)} + \delta_{(m,k)}) \leq \alpha\max_j g^n_{(m,j)}\sum_{k=1}^{K_m} p_{(m,k)}. \quad (32)$$

Now, we decide β to be

$$\beta = \frac{\max_j g^n_{(m,j)}}{\min_j g^n_{(m,j)}}. \quad (33)$$

Then, we get

$$\alpha\max_j g^n_{(m,j)}\sum_{k=1}^{K_m} p_{(m,k)} - \alpha\beta\sum_{k=1}^{K_m} g^n_{(m,k)}p_{(m,k)}$$
$$= \alpha\beta\sum_{k=1}^{K_m}(\min_j g^n_{(m,j)} - g^n_{(m,k)})p_{(m,k)} \leq 0. \quad (34)$$

Thus, the interference caused by the increased transmission power of cell m is bounded by $\alpha\beta$ times that of its previous interference. Since the same process is applicable to the other cells, $\breve{x}^n$ is upper bounded by $\alpha\beta x^n$ for a finite value β.

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$$\mathrm{E}_{\mathbf{G}^n,x^n}\left[\sum_{k=1}^{K_n}\mu_{(n,k)}B\log\left(1+\frac{g^n_{(n,k)}p_{(n,k)}(\mathbf{G}^n,x^n)}{x^n+\sum_{i=0}^{k-1}g^n_{(n,i)}p_{(n,i)}(\mathbf{G}^n,x^n)}\right)-\sum_{k=1}^{K_n}ch_{(n,k)}p_{(n,k)}(\mathbf{G}^n,x^n)\right] \tag{28}$$

$$L(\Lambda^n)=\max_{\mathbf{p}^n(\mathbf{G}^n,x^n)\succeq 0}\ \mathrm{E}_{\mathbf{G}^n,x^n}\left[\sum_{k=1}^{K_n}\mu_{(n,k)}B\log\left(1+\frac{g^n_{(n,k)}p_{(n,k)}(\mathbf{G}^n,x^n)}{x^n+\sum_{i=0}^{k-1}g^n_{(n,i)}p_{(n,i)}(\mathbf{G}^n,x^n)}\right)\right.$$
$$\left.-\sum_{k=1}^{K_n}(\lambda_{(n,k)}+ch_{(n,k)}p_{(n,k)})(\mathbf{G}^n,x^n)\right]. \tag{30}$$

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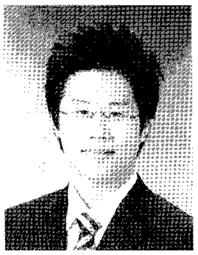

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