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On Fuzzy Weak r-minimal Continuity Between Fuzzy Minimal Spaces and Fuzzy Topological Spaces

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Abstract

In this paper, we introduce the concept of fuzzy weakly r-minimal continuous function between a fuzzy r-minimal space and a fuzzy topological space. We investigate characterizations and some properties for the continuity.

Key Words : r-minimal structure, fuzzy r-minimal continuous, fuzzy weakly r-minimal continuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [12]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3], Chattopadhyay, Hazra and Samanta introduced a smooth topological space which is a generalization of fuzzy topological space. In [11], Yoo et al. introduced the concept of fuzzy r-minimal space which is an extension of the smooth topological space. The concept of fuzzy r-M continuity was also introduced and investigated in [11]. In [9], the author introduced the concepts of fuzzy r-minimal continuous function and fuzzy r-minimal open function between fuzzy r-minimal spaces and fuzzy topological spaces, and investigate characterizations for such functions. The purpose of this paper is to generalize the concept of fuzzy r-minimal continuous function. So, in this paper, we introduce the concept of fuzzy weakly r-minimal continuous function between a fuzzy r-minimal space and a fuzzy topological space. We investigate characterizations and some properties for the continuity.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a fuzzy set of X. By $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$ we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{\mathbf{1}} - A$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

A fuzzy point x_α is said to belong to a fuzzy set A in X, denoted by $x_\alpha \in A$, if $\alpha \leq A(x)$ for $x \in X$. A fuzzy set A in X is the union of all fuzzy points which belong to A.

Let $f : X \to Y$ be a function and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

for $y \in Y$, and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)), x \in X$.

A *fuzzy topology* (or *smooth topology*) [3, 10] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:
(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.
(2) $\mathcal{T}(A_1 \cap A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$ for $A_1, A_2 \in I^X$.
(3) $\mathcal{T}(\cup A_i) \geq \wedge \mathcal{T}(A_i)$ for $A_i \in I^X$.

The pair $(X, \mathcal{T})$ is called a *fuzzy topological space* [10]. $A \in I^X$ is said to be *fuzzy r-open* (resp., *fuzzy r-closed*) [6] if $\mathcal{T}(A) \geq r$ (resp., $\mathcal{T}(A^c) \geq r$).

The *r-closure* of A, denoted by $cl(A,r)$, is defined as $cl(A,r) = \cap\{B \in I^X : A \subseteq B \text{and } B \text{is fuzzy } r\text{-closed}\}$.

The *r-interior* of A, denoted by $int(A,r)$, is defined as $int(A,r) = \cup\{B \in I^X : B \subseteq A \text{and } B \text{is fuzzy } r\text{-open}\}$.

Definition 2.1. Let A be a fuzzy set in a fuzzy topological space (X, σ). Then A is said to be
(1) *fuzzy r-semiopen* [7] if there is a fuzzy r-open set B in X such that $B \subseteq A \subseteq cl(B,r)$,
(2) *fuzzy r-preopen* [6] if $A \subseteq int(cl(A,r),r)$,
(3) *fuzzy r-regular open* (resp., *fuzzy r-regular closed*) [8] if $A = int(cl(A,r),r)$ (resp., $A = cl(int(A,r),r)$),
(4) *fuzzy r-β-open* [1] if $A \subseteq cl(int(cl(A,r),r),r)$.

Definition 2.2 ([11]). Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* if the family

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

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Then the $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* (simply r-FMS) if $\mathcal{M}$ has a fuzzy r-minimal structure. Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the complement of A (simply, A^c) is a fuzzy r-minimal open set.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure and the fuzzy r-minimal interior of A [11], denoted by $mC(A, r)$ and $mI(A, r)$, respectively, are defined as

$mC(A, r) = \cap\{B \in I^X : B^c \in \mathcal{M}_r \text{and } A \subseteq B\}$,
$mI(A, r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{and } B \subseteq A\}$.

Theorem 2.3 ([11]). Let $(X, \mathcal{M})$ be an r-FMS and A, B in I^X.

(1) $mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.

(2) $A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.

(3) If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.

(4) $mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.

(5) $mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.

(6) $\tilde{\mathbf{1}} - mC(A, r) = mI(\tilde{\mathbf{1}} - A, r)$ and $\tilde{\mathbf{1}} - mI(A, r) = mC(\tilde{\mathbf{1}} - A, r)$.

Definition 2.4 ([9]). Let $(X, \mathcal{M}_X)$ be an r-FMS and (Y, σ) a fuzzy topological space. Then $f : X \to Y$ is said to be *fuzzy r-minimal continuous* if for every fuzzy r-open set A in Y, $f^{-1}(A)$ is fuzzy r-minimal open in X.

Theorem 2.5 ([9]). Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). Then we have the following:

(1) f is fuzzy r-minimal continuous.

(2) $f^{-1}(B)$ is a fuzzy r-minimal closed set for each fuzzy r-closed set B in Y.

(3) $f(mC(A, r)) \subseteq cl(f(A), r)$ for $A \in I^X$.

(4) $mC(f^{-1}(B), r) \subseteq f^{-1}(cl(B, r))$ for $B \in I^Y$.

(5) $f^{-1}(int(B, r)) \subseteq mI(f^{-1}(B), r)$ for $B \in I^Y$.

Then $(1) \Leftrightarrow (2) \Rightarrow (3) \Leftrightarrow (4) \Leftrightarrow (5)$.

3. Fuzzy weakly r-minimal continuous function

Definition 3.1. Let $(X, \mathcal{M}_X)$ be an r-FMS and (Y, σ) a fuzzy topological space. Then $f : X \to Y$ is said to be *fuzzy weakly r-minimal continuous* if for each fuzzy point x_α and for each fuzzy r-open set V with $f(x_\alpha) \in V$, there exists a fuzzy r-minimal open set U such that $x_\alpha \in U$ and $f(U) \subseteq cl(V, r)$.

Remark 3.2. Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). Then every fuzzy r-minimal continuous mapping f is clearly fuzzy weakly r-minimal continuous but the converse is not always true as shown in the next example.

Example 3.3. Let $X = I$ and let us consider two fuzzy sets A, B defined as

$$A(x) = \frac{1}{2}x, \;\; x \in I;$$

$$B(x) = -\frac{1}{2}(x - 1), \;\; x \in I.$$

Consider a fuzzy family

$$\mathcal{M}_X(U) = \begin{cases} \frac{1}{2}, & \text{if } U = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{2}{3}, & \text{if } U = A, B, \\ 0, & \text{otherwise}, \end{cases}$$

and a fuzzy topology

$$\sigma(U) = \begin{cases} 1, & \text{if } U = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, A, B, \\ \frac{1}{3}, & \text{if } U = A \cap B, A \cup B, \\ 0, & \text{otherwise}. \end{cases}$$

Then the identity function $f : (X, \mathcal{M}_X) \to (X, \sigma)$ is fuzzy weakly $\frac{1}{3}$-minimal continuous but not fuzzy $\frac{1}{3}$-minimal continuous.

Theorem 3.4. Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). Then the following statements are equivalent:

(1) f is fuzzy weakly r-minimal continuous.

(2) $f^{-1}(B) \subseteq mI(f^{-1}(cl(B, r)), r)$ for each fuzzy r-open set B of Y.

(3) $mC(f^{-1}(int(F, r)), r) \subseteq f^{-1}(F)$ for each fuzzy r-closed set F in Y.

(4) $mC(f^{-1}(int(cl(B, r), r)), r) \subseteq f^{-1}(cl(B, r))$ for each $B \in I^Y$.

(5) $f^{-1}(int(B, r)) \subseteq mI(f^{-1}(cl(int(B, r), r)), r)$ for each $B \in I^Y$.

(6) $mC(f^{-1}(V), r) \subseteq f^{-1}(cl(V, r))$ for each fuzzy r-open set V in Y.

Proof. (1) $\Rightarrow$ (2) Let B be a fuzzy r-open set in Y. Since f is fuzzy weakly r-minimal continuous, for each $x_\alpha \in f^{-1}(B)$, there exists a fuzzy r-minimal open set U_{x_α} of x_α such that $f(U_{x_\alpha}) \subseteq cl(B)$. Now we can say for each $x_\alpha \in f^{-1}(B)$, there exists a fuzzy r-minimal open set U_{x_α} such that

$$x_\alpha \in U_{x_\alpha} \subseteq f^{-1}(f(U_{x_\alpha})) \subseteq f^{-1}(cl(B, r)).$$

This implies $x_\alpha \in mI(f^{-1}(cl(B, r)), r)$. Hence $f^{-1}(B) \subseteq mI(f^{-1}(cl(B, r)), r)$.

(2) $\Rightarrow$ (1) Let x_α be a fuzzy point in X and V a fuzzy r-open open set containing $f(x_\alpha)$. Then since $x_\alpha \in$

$f^{-1}(V) \subseteq mI(f^{-1}(cl(V,r)),r)$, there exists a fuzzy r-minimal open set U containing x_α such that $x_\alpha \in U \subseteq f^{-1}(cl(V,r))$. This implies $f(U) \subseteq f(f^{-1}(cl(V,r))) \subseteq cl(V,r)$. Hence f is fuzzy weakly r-minimal continuous.

(1) ⇒ (3) Let F be any fuzzy r-closed set of Y. Then $\tilde{\mathbf{1}} - F$ is a fuzzy r-open set in Y, from Theorem 2.2 and Theorem 2.4, it follows

$$\begin{aligned} f^{-1}(\tilde{\mathbf{1}} - F) &\subseteq mI(f^{-1}(cl(\tilde{\mathbf{1}} - F, r)), r) \\ &= mI(f^{-1}(\tilde{\mathbf{1}} - int(F,r)), r) \\ &= mI(\tilde{\mathbf{1}} - f^{-1}(int(F,r)), r) \\ &= \tilde{\mathbf{1}} - mC(f^{-1}(int(F,r)), r). \end{aligned}$$

Hence we have $mC(f^{-1}(int(F,r)),r) \subseteq f^{-1}(F)$.

(3) ⇒ (4) Let B be any fuzzy set in Y. Since $cl(B,r)$ is a fuzzy r-closed set in Y, by (3),

$$mC(f^{-1}(int(cl(B,r),r)),r) \subseteq f^{-1}(cl(B,r)).$$

(4) ⇒ (5) For $B \in I^Y$, $f^{-1}(int(B,r)) = \tilde{\mathbf{1}} - f^{-1}(cl(\tilde{\mathbf{1}} - B,r)) \subseteq \tilde{\mathbf{1}} - mC(f^{-1}(int(cl(\tilde{\mathbf{1}} - B,r),r)),r) = mI(f^{-1}(cl(int(B,r),r)),r)$.

Thus $f^{-1}(int(B,r)) \subseteq mI(f^{-1}(cl(int(B,r),r)),r)$.

(5) ⇒ (6) Let V be any fuzzy r-open set of Y. Then by (5),

$$\begin{aligned} \tilde{\mathbf{1}} - f^{-1}(cl(V,r)) &= f^{-1}(int(\tilde{\mathbf{1}} - V, r)) \\ &\subseteq mI(f^{-1}(cl(int(\tilde{\mathbf{1}} - V,r),r)),r) \\ &= mI(\tilde{\mathbf{1}} - f^{-1}(int(cl(V,r),r)),r) \\ &= \tilde{\mathbf{1}} - mC(f^{-1}(int(cl(V,r),r)),r) \\ &\subseteq \tilde{\mathbf{1}} - mC(f^{-1}(V),r). \end{aligned}$$

Hence we have $mC(f^{-1}(V),r) \subseteq f^{-1}(cl(V,r))$.

(6) ⇒ (1) Let V be a fuzzy r-open set containing $f(x_\alpha)$. By (6),

$$\begin{aligned} f^{-1}(V) &\subseteq f^{-1}(int(cl(V,r),r)) \\ &= \tilde{\mathbf{1}} - f^{-1}(cl(\tilde{\mathbf{1}} - cl(V,r),r)) \\ &\subseteq \tilde{\mathbf{1}} - mC(f^{-1}(\tilde{\mathbf{1}} - cl(V,r)),r) \\ &= mI(f^{-1}(cl(V,r)),r). \end{aligned}$$

It implies $x_\alpha \in mI(f^{-1}(cl(V,r)),r)$. Thus there exists a fuzzy r-minimal open set U such that $M_x \in U \subseteq f^{-1}(cl(V,r))$. Hence $f(U) \subseteq cl(V,r)$.

□

Theorem 3.5. Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y,σ). Then the following statements are equivalent:

(1) f is fuzzy weakly r-minimal continuous.

(2) $mC(f^{-1}(int(cl(G,r),r)),r) \subseteq f^{-1}(cl(G,r))$ for each fuzzy r-open set G in Y.

(3) $mC(f^{-1}(int(cl(V,r),r)),r) \subseteq f^{-1}(cl(V,r))$ for each fuzzy r-preopen set V in Y.

(4) $mC(f^{-1}(int(K,r)),r) \subseteq f^{-1}(K)$ for each fuzzy r-regular closed set K in Y.

(5) $mC(f^{-1}(int(cl(G,r),r)),r) \subseteq f^{-1}(cl(G,r))$ for each fuzzy r-β-open set G in Y.

(6) $mC(f^{-1}(int(cl(G,r),r)),r) \subseteq f^{-1}(cl(G,r))$ for each fuzzy r-semiopen set G in Y.

Proof. (1) ⇒ (2) Let G be a fuzzy r-open set of Y; then by Theorem 3.4 (3), we have $mC(f^{-1}(int(cl(G,r),r)),r) \subseteq f^{-1}(cl(G,r))$.

(2) ⇒ (3) Let V be a fuzzy r-preopen of Y. Then $V \subseteq int(cl(V,r),r)$. Set $A = int(cl(V,r),r)$. Then since A is a fuzzy r-open set, from (2), it follows

$$mC(f^{-1}(int(cl(A,r),r)),r) \subseteq f^{-1}(cl(A,r)).$$

Since $cl(A,r) = cl(V,r)$, we have

$$mC(f^{-1}(int(cl(V,r),r)),r) \subseteq f^{-1}(cl(V,r)).$$

(3) ⇒ (4) Let K be a fuzzy r-regular closed set of Y. Then since $int(K,r)$ is an a fuzzy r-preopen set, by (3), $mC(f^{-1}(int(cl(int(K,r),r),r)),r) \subseteq f^{-1}(cl(int(K,r),r))$. Since K is fuzzy r-regular closed and $int(K,r) = int(cl(int(K,r),r),r)$, we have

$$mC(f^{-1}(int(K,r)),r) \subseteq f^{-1}(K,r).$$

(4) ⇒ (5) Let G be a fuzzy r-β-open set. Then $G \subseteq cl(int(cl(G,r),r),r)$ and $cl(G,r) = cl(int(cl(G,r),r),r)$. So $cl(G)$ is a fuzzy r-regular closed set. Hence by (4), we have

$$mC(f^{-1}(int(cl(G,r),r)),r) \subseteq f^{-1}(cl(G,r)).$$

(5) ⇒ (6) It is obvious.

(6) ⇒ (1) Let V be a fuzzy r-open set; then since V is a fuzzy r-semiopen set, by (6) and $V \subseteq int(cl(V,r),r)$, we have $mC(f^{-1}(V),r) \subseteq mC(f^{-1}(int(cl(V,r),r)),r) \subseteq f^{-1}(cl(V,r))$. Hence, by Theorem 3.4 (6), f is fuzzy weakly r-minimal continuous. □

Definition 3.6. Let $f : X \to Y$ be a mapping between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y,σ). Then f is to be *fuzzy co-r-minimal open* if for every fuzzy r-minimal open set A in X, $f(A)$ is fuzzy r-open in Y.

Theorem 3.7. Let $f : X \to Y$ be a function between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological topological space (Y,σ). Then the following are equivalent:

(1) f is fuzzy co-r-minimal open.

(2) $f(mI(A,r)) \subseteq int(f(A),r)$ for $A \in I^X$.

(3) $mI(f^{-1}(B),r) \subseteq f^{-1}(int(B,r))$ for $B \in I^Y$.

Proof. (1) $\Rightarrow$ (2) For $A \in I^X$, $f(mI(A,r)) = f(\cup\{B \in I^X : B \subseteq A, B$ is fuzzy r-minimal open$\})$
$= \cup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B)$ is fuzzy r-open$\}$
$\subseteq \cup\{U \in I^Y : U \subseteq f(A), U$ is fuzzy r-open$\}$
$= int(f(A), r)$.
Hence $f(mI(A,r)) \subseteq int(f(A), r)$.

(2) $\Rightarrow$ (1) For every fuzzy r-minimal open set A in X, $A = mI(A,r)$ and by (2), $f(A) = f(mI(A,r)) \subseteq int(f(A), r)$. This implies $f(A)$ is fuzzy r-open, and hence f is fuzzy co-r-minimal open.

(2) $\Rightarrow$ (3) For $B \in I^Y$, from (2) it follows that

$$f(mI(f^{-1}(B), r)) \subseteq int(f(f^{-1}(B)), r) \subseteq int(B, r).$$

Hence we get (3).
Similarly, we get (3) $\Rightarrow$ (2). $\square$

Definition 3.8 ([5]). Let $(X, \mathcal{M}_X)$ be an r-FMS and $\mathcal{A} = \{A_i \in I^X : i \in J\}$. $\mathcal{A}$ is called a *fuzzy r-minimal cover* if $\cup\{A_i : i \in J\} = \tilde{\mathbf{1}}$. It is a *fuzzy r-minimal open cover* if each A_i is a fuzzy r-minimal open set. A subcover of a fuzzy r-minimal cover $\mathcal{A}$ is a subfamily of it which also is a fuzzy r-minimal cover. A fuzzy set A in X is said to be *fuzzy r-minimal compact* (resp., *almost fuzzy r-minimal compact, nearly fuzzy r-minimal compact*) if for every fuzzy r-minimal open cover $\mathcal{A} = \{A_i \in I^X : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{j\in J_0} A_j$ (resp., $A \subseteq \cup_{j\in J_0} mC(A_j, r)$, $A \subseteq \cup_{j\in J_0} mI(mC(A_j, r), r)$).

Definition 3.9 ([4]). Let (X, τ) be a fuzzy topological space. A fuzzy set A in X is said to be *r-fuzzy compact* (resp., *r-fuzzy almost compact, r-fuzzy nearly compact*) if for every fuzzy r-open cover $\mathcal{A} = \{A_i \in I^X : \tau(A_i) \geq r, i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{j\in J_0} A_i$ (resp., $A \subseteq \cup_{j\in J_0} cl(A_j, r)$, $A \subseteq \cup_{j\in J_0} int(cl(A_j, r), r)$).

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy family $\mathcal{M}$ is said to have the property $(\mathcal{U})$ [11] if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) \geq \wedge \mathcal{M}(A_i).$$

Theorem 3.10 ([11]). Let $(X, \mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$. Then
(1) $mI(A,r) = A$ if and only if $A \in \mathcal{M}_r$ for $A \in I^X$.
(2) $mC(A,r) = A$ if and only if $A^c \in \mathcal{M}_r$ for $A \in I^X$.

Theorem 3.11. Let $f : X \to Y$ be a fuzzy weakly r-minimal continuous between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). If A is a fuzzy r-minimal compact set and if $\mathcal{M}_X$ has the property $(\mathcal{U})$, then $f(A)$ is r-fuzzy almost compact.

Proof. Let $\mathcal{B} = \{B_i \in I^Y : i \in J\}$ be a fuzzy r-open cover of $f(A)$ in Y. Then by the property $(\mathcal{U})$, $\{mI(f^{-1}(cl(B_i, r)), r) : B_i \in \mathcal{B}$ for $i \in J\}$ is a fuzzy r-minimal open cover of A in X. Since A is fuzzy r-minimal compact, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{j\in J_0} mI(f^{-1}(cl(B_j, r)), r) \subseteq \cup_{j\in J_0} f^{-1}(cl(B_j, r))$. Hence $f(A) \subseteq \cup_{j\in J_0} cl(B_j, r)$. $\square$

Theorem 3.12. Let $f : X \to Y$ be a fuzzy weakly r-minimal continuous and fuzzy co-r-minimal open mapping between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). If A is an almost fuzzy r-minimal compact set and if $\mathcal{M}_X$ has the property $(\mathcal{U})$, then $f(A)$ is r-fuzzy almost compact

Proof. Let $\mathcal{B} = \{B_i \in I^Y : i \in J\}$ be a fuzzy r-open cover of $f(A)$ in Y. Then by the property $(\mathcal{U})$, $\{mI(f^{-1}(cl(B_i, r)), r) : B_i \in \mathcal{B}$ for $i \in J\}$ is a fuzzy r-minimal open cover of A in X. Since A is an almost fuzzy r-minimal compact set, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{j\in J_0} mC(mI(f^{-1}(cl(B_j, r)), r), r)$. Since $int(cl(B_j, r), r)$ is fuzzy r-open in in Y, from Theorem 3.4 (2) and Theorem 3.7, it follows
$\cup_{j\in J_0} mC(mI(f^{-1}(cl(B_j, r)), r), r)$
$\subseteq \cup_{j\in J_0} mC(f^{-1}(int(cl(B_j, r), r)), r)$
$\subseteq \cup_{j\in J_0} f^{-1}(cl(B_j, r))$.
Hence $f(A) \subseteq \cup_{j\in J_0} cl(B_j, r)$. $\square$

Theorem 3.13. Let $f : X \to Y$ be a fuzzy weakly r-minimal continuous and fuzzy co-r-minimal open mapping between an r-FMS $(X, \mathcal{M}_X)$ and a fuzzy topological space (Y, σ). If A is a nearly fuzzy r-minimal compact set and if $\mathcal{M}_X$ has the property $(\mathcal{U})$, then $f(A)$ is r-fuzzy nearly compact.

Proof. Let $\mathcal{B} = \{B_i \in I^Y : i \in J\}$ be a fuzzy r-open cover of $f(A)$ in Y. Then $\{mI(f^{-1}(cl(B_i, r)), r) : B_i \in \mathcal{B}$ for $i \in J\}$ is a fuzzy r-minimal open cover of A in X. By definition of nearly fuzzy r-minimal compactness, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{j\in J_0} mI(mC(mI(f^{-1}(cl(B_j, r)), r), r), r)$. Since $int(cl(B_j, r), r)$ is fuzzy r-open, from Theorem 3.4 (2) and Theorem 3.7, it follows
$\cup_{j\in J_0} mI(mC(mI(f^{-1}(cl(B_j, r)), r), r), r)$
$\subseteq \cup_{j\in J_0} mI(mC(f^{-1}(int(cl(B_j, r), r)), r), r)$
$\subseteq \cup_{j\in J_0} mI(f^{-1}(cl(B_j, r)), r)$
$\subseteq \cup_{j\in J_0} f^{-1}(int(cl(B_j, r), r))$.
This implies $f(A) \subseteq \cup_{j\in J_0} int(cl(B_j, r), r)$, and hence $f(A)$ is r-fuzzy nearly compact. $\square$

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