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Pricing the Storage of Inbound Containers with a Discrete Probability Distribution of Retrieval Times

Ki Young Kim
Department of Systems and Management Engineering, Dongseo University,
San69-1, Churye-2dong, Sasang-ku, Pusan, 617-716, Korea
Tel: 82-51-320-1637, Fax: 82-51-320-1751, E-mail: kiykim@gdsu.dongseo.ac.kr

Kap Hwan Kim†
Department of Industrial Engineering, Pusan National University,
30 Changjeon-dong, Kumjeong-ku, Pusan, 609-735, Korea
Tel: 82-51-510-2419, Fax: 82-51-512-7603, E-mail: kapkim@pusan.ac.kr

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Abstract. This paper discusses a method for pricing the storage of inbound containers in a container yard. The pricing structure is characterized by a free-time-limit and a storage price for the storage time that extends beyond the free-time-limit. A cost model is developed from the viewpoint of a public terminal operator as well as a private terminal operator. Unlike a previous study on this issue, this study assumes that the retrieval times follow a discrete probability distribution, which is more realistic than the previous study. A solution procedure is suggested and illustrated by using numerical examples.

Keywords: Container Terminal, Pricing, Space Allocation, Inbound Containers

1. INTRODUCTION

There are four different types of operations in container terminals: discharging inbound containers from vessels; loading outbound containers onto vessels; receiving outbound containers from road trucks or trains; and delivering inbound containers to road trucks or trains for hinterland transportation. Figure 1 shows a typical yard configuration and container flows in a container terminal. This figure illustrates a typical layout of indirect-transfer systems with yard cranes. In a typical storage yard of a container terminal, there are 10~50 blocks. Each block consists of 30~40 yard-bays. Each bay usually has 6~10 stacks (rows) of between four and eight tiers. The handling operations in the container terminal include three types of operations: vessel operations associated with container-ships, receiving/delivery operations for road trucks, and container handling and storage operations in the yard. The following focuses on the discharging and delivery operations that are directly related to the issues in this paper.

During the discharging operation, inbound containers in a vessel are unloaded from the vessel and stacked in the yard. Quay cranes (QCs) transfer containers from the ship to a yard truck. Then, the yard truck delivers the inbound (import/discharged) container to a yard crane (YC) that picks it up and stacks it into a position in the yard. When a road truck arrives at the gate, the location of the corresponding inbound container is communicated to the road truck. When the road truck arrives at the transfer point, the arrival of the truck is communicated to a YC. Before the YC picks up the target container from the stack, it first undertakes gantry travel to the parking position of the road truck. If there are other containers on top of the container requested by the road truck, the YC has to first relocate all the containers that are atop the target container to other stacks; this is called the "rehandling operation" and is a major factor that lowers the productivity of the YC during the delivery operation. When all the containers are relocated, the YC transfers the target container to the truck. Figure 2 illustrates various terminologies for a storage block in a yard.

It is very important to efficiently utilize the storage space in container terminals. The efficiency of the usage of storage space significantly influences the profitability of a terminal as well as the productivity of the handling operation in the terminal. When containers are stored in the yard for a long time, there is likely to be high congestion in the yard, which results in low productivity in the handling operation.

† : Corresponding Author

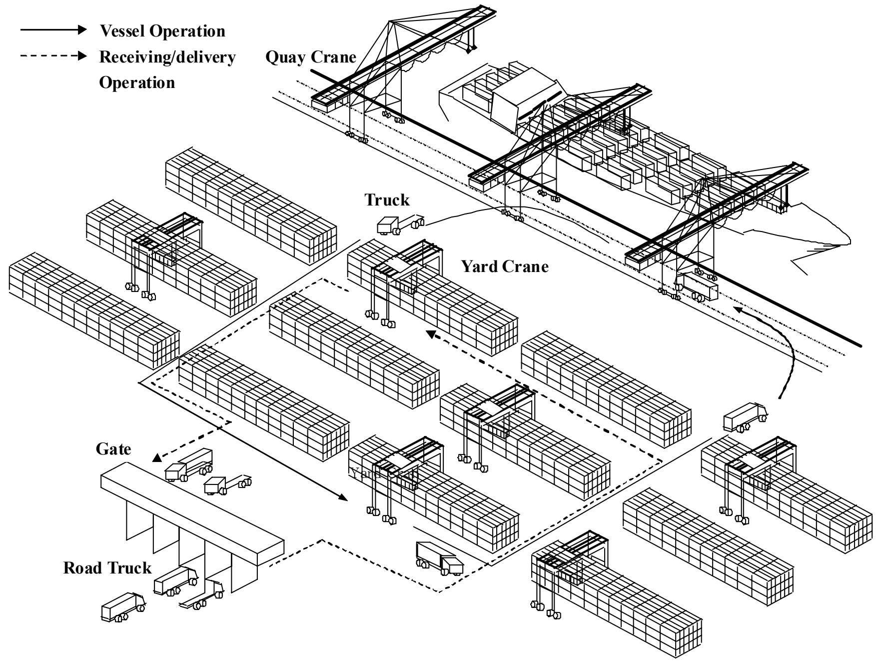

Figure 1. An illustration of the layout of a container terminal.

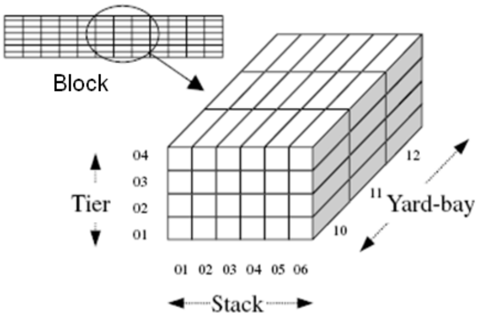

Figure 2. A storage block.

Because the storage charge restrains shippers from storing containers for a long time, terminal operators usually receive a storage charge-which is higher than the storage fees at outside warehouses-for storing a container beyond a pre-specified duration (called the free-time-limit). The price schedule for the storage of containers is expressed by the free-time-limit and the daily storage price that is applied to the daily storage of a container beyond the free-time-limit. Thus, when the price schedule of the storage charge is suggested by a terminal operator, shippers determine how long to store their containers in the container yard so as to minimize their total cost.

Figure 3 shows different flows of inbound containers. In Figure 3(a), a container that is unloaded from a vessel in a port container terminal (PCT) is directly shipped to the customer. Case (b) shows that an unloaded container is temporarily stacked at the container yard (CY) and then transported to the customer. In case (c), an unloaded container is directly shipped to an outside container depot, which is called an off-dock container yard (ODCT), and stacked during a certain period of time and then delivered to the customer. Case (d) shows that an unloaded container is stored at CY in PCT during a certain period of time, and then transported to an ODCY and stacked for another period of time before it is delivered to the customer.

The flow of an inbound container depends on its delivery due date, and on the price structures for storage and transportation in both CY and ODCY. If the free-time-limit increases and the daily storage price decreeases, shippers will keep their containers for longer. As a result, PCT will earn less revenue per container but keep a larger number of containers in the yard, which implies a higher cost of congestion in its CY. If the free-time-limit decreases and the daily storage price increases, shippers will move their containers earlier to an ODCY. As a result, PCT will earn more revenue per container but keep a smaller number of containers in the yard, which implies a lower cost of congestion in its CY. This study suggests a method to determine the free-time-limit and the daily storage price from the viewpoint of a terminal operator.

Until recently, the operational aspects of container terminals have not received much attention in the academic literature. Taleb-Ibrahimi *et al*. (1993) analyzed the space-allocation problem in container terminals with either a constant or cyclic space requirement. Kozan (2000) suggested a network model for deciding container flows and types of handling equipment so that the handling and traveling times of containers in multimodal

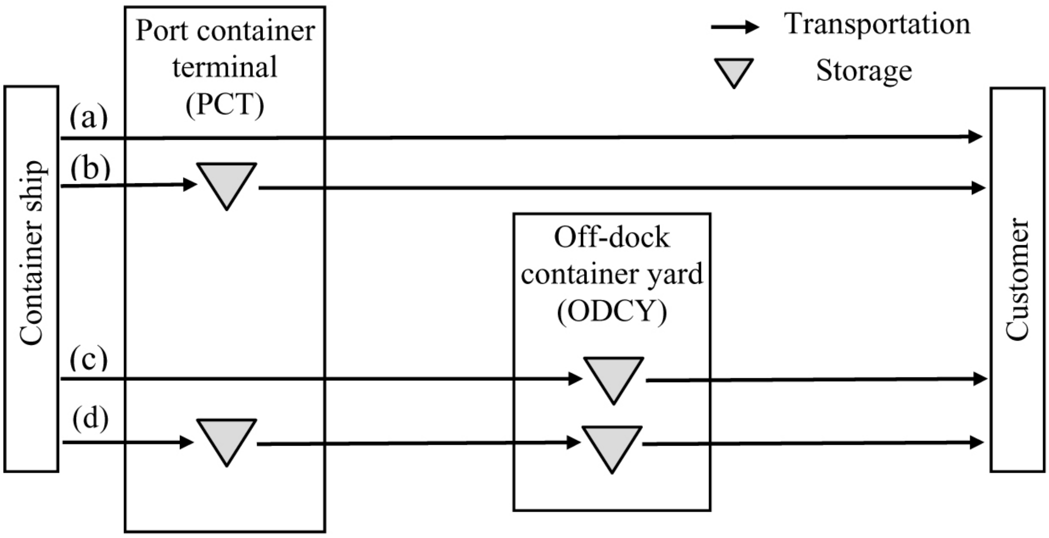

Figure 3. Alternatives for the storage of inbound containers.

container terminals are minimized. Castilho and Daganzo (1993) addressed the stacking problem of inbound containers in port container terminals. Cao and Uebe (1995) suggested a transportation model for assigning the available yard space to space requirements in container terminals. Kim *et al*. (2000) suggested a method for determining the storage locations of individual outbound containers in container terminals. Kim (1997) proposed a formula for estimating the number of rehandles in inbound container yards. Kim and Kim (1999) addressed the space allocation problem for inbound containers. Kim and Kim (2002) proposed a method for determining the optimal size of the storage space and yard cranes for inbound container yards.

The following three papers are directly related to the pricing issue in this paper. Castilho and Daganzo (1991) addressed a pricing problem for temporary storage facilities at ports. They analyzed shipper behaviors for different price schedules when the shippers seek to maximize their external cost savings-resulting from the use of temporary storage space instead of warehouses at a distance-minus the sum of the moving expenses and holding costs. Holguin-Veras and Jara-Diaz (1999) proposed a method for determining the space allocation and prices for priority systems in CYs. They analyzed three different pricing rules: welfare maximization, welfare maximization subject to a breakeven constraint, and profit maximization. Kim and Kim (2007) proposed a method of determining the free-time-limit and the storage price per unit time for inbound containers. They assumed that the time when customers want to pick up their container after discharging follows an exponential distribution and that the free-time-limit can be any positive continuous value. However, in reality, most terminals propose a discrete-valued, free-time-limit (in the unit

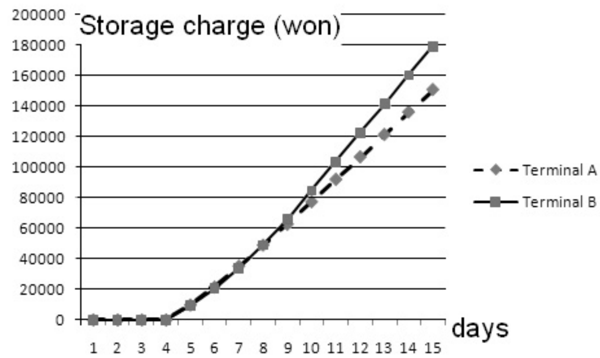

Figure 4. Price schedules of two real container terminals (A and B) in Busan.

of days). That is, the free-time-limit is given as 4, 5, ⋯, or 10 days. Then, terminal operators collect data on the distribution of the retrieval time of inbound containers at the 1^{st}, 2^{nd}, 3^{rd}, ⋯, 10^{th} day after the departure of the associated vessel. Unlike the study of Kim and Kim (2007), the present study assumes that the probability that a customer wants to pick up his/her container after it is discharged is given by a discrete probability distribution and that the free-time-limit is also specified in the unit of days.

This paper assumes a storage charge that is proportional to the length of time by which the storage time exceeds the free-time-limit. Figure 4 illustrates the real price schedules that are being used by two container terminals in Busan. Both terminals provide a free-time-limit of 4 days and terminals A and B charge approximately 14700 and 18900 Korean won per day in excess of that limit, respectively. Both terminals apply a slightly lower daily rate during storage days 5-9. However, in general, the total charge increases linearly beyond the free-time-limit, which is an assumption in the model of this paper.

In this paper, three cost models for optimally determining the pricing structure are developed from differing viewpoints of the terminal operators. First, this paper proposes a model for maximizing the profit of the PCT. Second, a model is suggested that maximizes the profit of the PCT with a constraint on the service level for customers. The last model is for minimizing the total cost of both the PCT and customers.

The following section describes the structure of the price schedule that is assumed in this paper and the response of customers to the suggested price schedule. Section 3 proposes a method for deriving the optimal schedule when the terminal operator maximizes his/her own profit. Section 4 addresses the case where the terminal operator maximizes his/her own profit under the condition that the customer service level is guaranteed to be at a pre-specified level. Section 5 discusses the case where the terminal operator is a public agency and thus, the objective is to minimize the total cost of the terminal and customers. Finally, a summary and conclusions are provided.

2. PRICE SCHEDULE AND THE BEHAVIOR OF CUSTOMERS

This section proposes mathematical models for determining the price schedule for the storage of inbound containers. The mathematical models in this paper are based on the following assumptions and notation.

• ***Assumptions***

(1) Yard cranes are used in CYs.
(2) CYs are considered only for inbound containers.
(3) The number of inbound containers discharged from vessels per day is deterministic and constant.
(4) The yard space allocated to inbound containers is constant.
(5) Each yard crane serves road trucks based on the FIFO rule.
(6) When a container is rehandled, it is put again into another slot in the same bay.
(7) Containers unloaded from different vessels are not mixed with each other in the same bay.

• ***Notation***

m = Number of inbound containers (TEUs) unloaded from vessels and stacked in the CY per day; this number corresponds to those for (b), (c), and (d) in Figure 3.
γ = Factor that converts TEUs to the number of containers. $\gamma = 0.7$ is assumed in this study.
λ = Average arrival rate of road trucks.
r = Number of stacks in a bay ($r = 6$ is assumed in this study).
g = Total number of ground slots in the CY for inbound containers.
C_c = Variable cost of a YC per unit time.
C_t = Variable cost of a road truck per unit time.
s_o = Daily storage price per TEU in ODCYs that are located outside the PCT.
c_h = Delivery charge for a container from the PCT to ODCYs, inclusive of the handling charge at ODCYs.
t_d = Time required for a YC to transfer a container to a truck. This is the movement time of the spreader for transferring a container to a truck.
t_r = Time to relocate a container that is atop the target container to one of the other stacks. This is the cycle time for the relocation of a container.
t_R = Time to relocate all the containers that are atop the target container that is to be picked up. Note that more than one container may be relocated for picking up the target container.
t_t = Gantry travel time of a YC for picking up an inbound container (refer the second paragraph of the introduction for a detailed explanation of the operation).
$p(i)$ = Probability that a container is delivered to its customer on the i^{th} day after it is discharged; $p(i) > 0$ for $i = 1, 2, \cdots, T$, where T is the latest delivery possible day.
$q(j)$ = Probability that the duration of stay at the CY of a container is j days.
t_s = Maximum value of i in p(i) for which containers are delivered directly from the CY of the PCT to customers. That is, shippers of the containers whose delivery dates are later than t_s will move their containers from the CY to ODCYs and then move again from the ODCYs to their doors.
F = Free-time-limit for inbound containers (days) (a decision variable).
S = Daily storage price of a container (TEU) in the CY of the PCT after the free-time-limit (F) (a decision variable).

When a terminal operator proposes the price schedule of (F, S) to shippers, shippers will choose one of the flow paths in Figure 3 for their containers based on their costs and delivery requirements. Because Case (a) in Figure 3 corresponds to the case where freight in the container had no time to be stored at either the CY of the PCT or an ODCY, it is excluded from the analysis. The number of containers in Case (a) does not depend on the price schedule for storage at the CY of the PCT, while the number of containers in the other cases will be affected by the price schedule.

The cost of a container as a function of the storage ime can be represented by Figure 5(a). Thus, containers with storage time longer than t_s will be moved to an ODCY after staying at the CY until the free-time-limit. The curve for the revenue per container can be drawn as shown in Figure 5(b). When the storage period of a container is shorter than F, the storage charge is zero. However, when the storage period of a container exceeds F, the charge increases linearly until the duration (inclusive of F) reaches t_s. If the storage period of a container is longer than t_s, it is better for the corresponding shipper of the container to move the container to an ODCY at time t_s, which means the terminal does not receive any charge from the shipper beyond t_s days.

Suppose that the probability distribution of the time when inbound containers are delivered to customers is as shown in Figure 6. However, responding to the pricing schedule proposed by the terminal operator, shippers will determine cost-minimizing flow paths, and may relocate their containers to ODCYs before the containers are delivered to their doors. Thus, the probability mass function of the duration-of-stay must be different from that of the delivery time. *q(i)* represents the latter Figure 7. The probability mass function, *q(i)*, can be evaluated by adding the probability, *p(i)*, of all the containers that will be moved to an ODCY to the probability for the end of the free-time-limit. Thus, *q(i)* becomes as follows:

$$q(i)=\begin{cases} \mathrm{p}(i) & \textit{for} \quad 0 \le i < F \\ \mathrm{p}(i)+\sum\limits_{k=t_s+1}^{T}\mathrm{p}(k) & \textit{for} \quad i=F \\ \mathrm{p}(i) & \textit{for} \quad F < i \le t_s \\ 0 & \textit{for} \quad i > t_s. \end{cases} \tag{1}$$

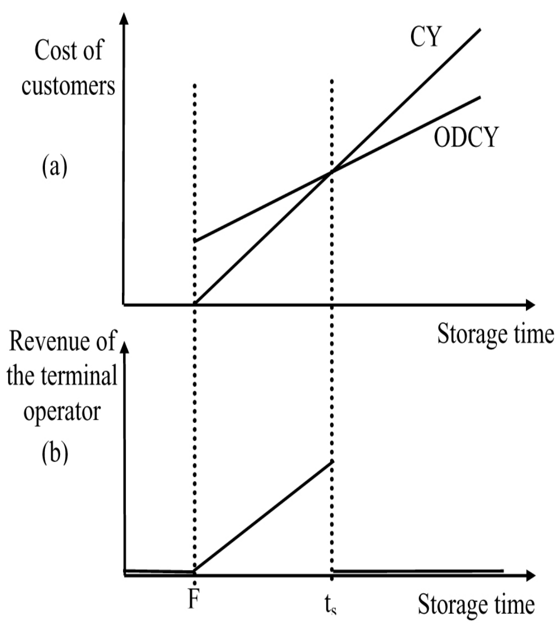

Figure 5. The cost function of a shipper and the revenue of the terminal operator.

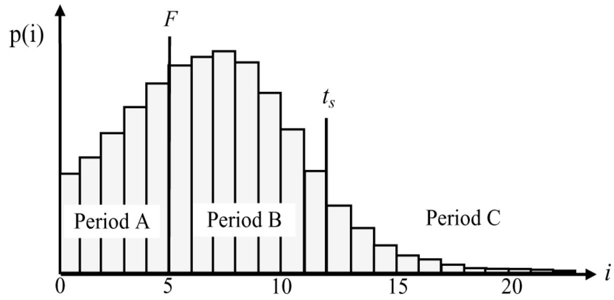

Figure 6. Probability mass function of the time for pickup requests for containers (p(i)).

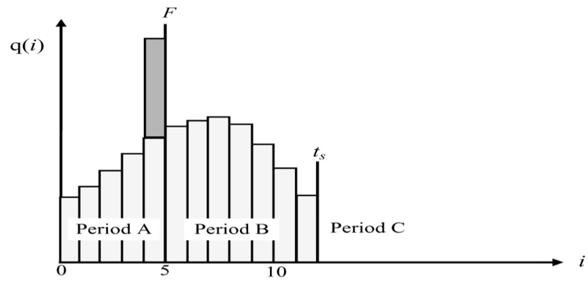

Figure 7. Probability mass function of the actual duration-of-stay of containers ($q(i)$).

The free-time-limit and the storage price can be determined from different perspectives. When the terminal operator is a privately owned company, the objective of the terminal operator will be profit maximization. Some terminal operators will attempt to not only maximize their profit but also provide a certain level of service for their customers. However, when a container terminal is owned by either a public agency or the government, the objective will be to maximize the public utility or minimize the public cost. The following provides a mathematical model for each of the above three cases.

This paper assumed a gamma distribution, Gamma (ϕ, θ), as the probability mass function for *p(i)*. The gamma distribution was discretized for numerical experiments as shown in the Appendix.

3. OPTIMAL PRICE SCHEDULE WHEN THE TERMINAL OPERATOR MAXIMIZES THEIR PROFIT

When the terminal operator suggests a lower value of S and a higher value of F, the duration-of-stay of containers increases; thus, the average height of stacks increases. As the average height of containers increases, the number of rehandles also increases.

The model in this subsection maximizes the expected profit of the terminal operator for the storage of an inbound container, which is the expected revenue for the container (TEU) minus the expected cost of handling by a YC for the container (TEU). In this case, the expected profit per TEU can be expressed as follows:

$$\text{(P1): } Max \quad \mathrm{E}[\mathrm{PF}(F,\ S)] = \sum_{i=F+1}^{t_s} S(i-F)\,\mathrm{p}(i)$$
$$-c_c\gamma\{\mathrm{E}[t_d] + \mathrm{E}[t_R] + \mathrm{E}[t_t]\},$$

where t_s is the maximum value of t such that $S(t-F) \le c_h\gamma + s_o(t-F)$.

From the definition of t_s in (P1), S can be rewritten as follows:

$$S \le [c_h\gamma + s_o(t-F)]/(t-F) \tag{2}$$

The right-hand-side of (2) is a monotonically decreasing function of *t*. Note that t_s can be F+1, F+2, ⋯, *T*. Because the objective function of the problem, P1, is an increasing function with respect to S for a given value of t (or t_s), it can be maximized when (2) holds as an equality. This fact implies that although *S* is a continuous decision variable, the optimal values of *S* and F can be found by calculating the objective function values with respect to all the combinations of all the possible discrete values of *S* and *F*.

Note that the first term of the objective function of P1 represents the expected revenue from the storage charge per container, while the latter is the variable cost for a YC for handling a container in a yard. Considering that $\mathrm{E}[t_d]$ and $\mathrm{E}[t_t]$ do not depend on the decision variables, F and S, the objective function can be rewritten as follows:

$$\text{(P1'): } Max\ \mathrm{E}[\mathrm{PF}(F,\ S)] = \sum_{k=F+1}^{t_s} S(i-F)\,\mathrm{p}(i) - c_c\gamma\,\mathrm{E}[t_R]\,, \tag{3}$$

where t_s is the maximum value of *t* such that $S(t-F) \le c_h\gamma + s_o(t-F)$.

The following describes how to evaluate $\mathrm{E}[t_R]$ for given values of F and S.

The expected number of containers on the yard can be represented as $\sum_{i=1}^{t_s} m\, iq(i)$. Thus, the average height of stacks becomes $\sum_{i=1}^{t_s} m\, iq(i)$ /g. Because we can assume that containers are retrieved uniformly, the maximum (initial) height of stacks becomes:

$$h = 2\sum_{i=1}^{t_s} m\, iq(i)/g \tag{4}$$

Suppose that a randomly chosen container is retrieved from a bay with a maximum (initial) height of *h* and *r* stacks. Kim (1997) proposed a simple formula, $\frac{h-1}{4} + \frac{h+2}{16r}$, to estimate the average number of rehandles per pickup.

Then, the expected rehandling time, $\mathrm{E}[t_R]$, which is the time to relocate all the containers atop the target container that is randomly chosen from a yard-bay, can be estimated by using:

$$\mathrm{E}[t_R] = E[t_r]\left[\frac{h-1}{4} + \frac{h+2}{16r}\right] \tag{5}$$

The experimental data used in this paper are summarized as follows: $\mathrm{E}(t_r)$, m, g, c_c, s_0, and c_h were set to 260 seconds, 2580 TEUs, 4875 ground slots, 100 won per second, 2000 won per TEU-day, and 40000 Korean won per container, respectively. The time values of the input parameters were estimated based on real data collected from PECT, which is one of the largest container terminals in Korea.

First, it was assumed that p(i) follows Gamma (3, 1). Table 1 illustrates the changes in the first term of P1, which is the revenue per TEU, for various values of F and S. Table 1 shows that the revenue decreases as the value of F increases in cases where the value of S is smaller than 11333, while in the other cases, the revenue increases first and decreases later as the value of F increases. Table 1 also shows that for a given value of F, the revenue increases first and decreases later as the value of S increases. Table 2 shows $\mathrm{E}(t_R)$, which is the expected time for rehandling per target container, for various values of F and S. This table shows that the rehandling time of a YC per target container decreases as the value of S increases, while the rehandling time increases as the value of F increases. Table 3 shows the values of the objective function (3) for various F and S. This table shows that the profit decreases as the value of F increases. Also, it shows that the profit increases first and decreases later as the value of S increases in case the value of F is 0, 1, or 2, while in case the value of F is greater than 2, the profit just increases (the loss decreases) as the value of S increases. Thus, the objective function is maximized when the values of S and F become 11333 and 0, respectively. Note that the blank spaces in Tables 1, 2, and Tables 3 represent infeasible cases.

The optimal values of S and F for the problem, P1', can be found by calculating the objective function values with respect to all the combinations of all the possible discrete values of S and F.

Thus, the algorithm for P1' can be described as follows.

(Step 0) F := 0. PF* := 0. F* := 0. S:= 0.

(Step 1) F ← F + 1. t_s := F. If F > T, then stop. Otherwise, go to Step 2.

(Step 2) t_s ← t_s + 1. If t_s > T, go to Step 1. Otherwise, set $S := [c_h\gamma + s_o(t_s - F)]\ /(t_s - F)$. Evaluate E[PF(*F*, *S*)]. If E[PF(*F*, *S*)] > PF*, then PF*← E[PF(*F*, *S*)], *F** ← *F*, t_s* ← t_s, and S* ← S. Go to the beginning of Step 2.

Table 1. The expected revenues for various values of F and S.

S \ F	0	1	2	3	4	5	6
6000	20868						
6666	21532	16377					
7600	21413	17053	11462				
9000	19764	17103	11978	7193			
11333	15695	15901	11969	7452	4171		
16000	8831	12716	10931	7222	4187	2233	
30000	2190	7184	8329	6083	3729	2061	1063

Table 2. The rehandling times per container for various values of F and S (unit: seconds).

S \ F	0	1	2	3	4	5	6
6000	190						
6666	172	190					
7600	142	174	190				
9000	98	150	177	190			
11333	40	114	157	180	190		
16000	0	71	131	165	182	190	
30000	0	31	102	147	172	185	190

Table 3. The profit per container for various values of F and S (unit: won).

S \ F	0	1	2	3	4	5	6
6000	7590						
6666	9500	3100					
7600	11449	4843	-1814				
9000	12918	6616	-409	-6083			
11333	12919	7909	960	-5113	-9105		
16000	8831	7776	1792	-4309	-8555	-11043	
30000	2190	5029	1225	-4201	-8324	-10860	-12213

This is a kind of total enumeration within a feasible region. Hereby, it is assumed that p(i) follows Gamma (4, 2). Figure 8 shows the changes in the optimal S for various values of c_h and s_o. During the numerical experiment for P1, no solution was found with a positive value of F. The optimal value of S increased as the values of c_h and s_o increased. Figure 9 also shows the changes in the optimal S for various values of c_c, the variable cost of a yard crane. For small values of c_c, an increase in the variable costs caused the price of storage space, S, to increase proportionally. However, when the variable cost became large, the optimal value of S did not change.

In order to generate the distribution, p(i), which approximately follows Gamma (ϕ, θ), the values of ϕ and θ were varied discretely from 1 to 4. Sixteen distributions of p(i) were assumed by using all combinations of the values of ϕ and θ. Note that the expectation and the variance can be expressed as $\phi\theta$ and $\phi\theta^2$, respectively. Figures 10 and 11 respectively show the changes in the optimal value of S for various means and variances of p(i). These figures respectively show that the value of S decreased as the values of the mean and variance of p(i) increased. Figures 12 and 13 respectively show the optimal profit for different means and variances of p(i), when F = 0. These figures respectively show that the profit decreased as the values of the mean and variance of p(i) increased. This seems to result from the increase in the handling cost.

4. OPTIMAL PRICE SCHEDULE WHEN A TERMINAL OPERATOR MAXIMIZES THEIR PROFIT SUBJECT TO A CUSTOMER SERVICE CONSTRAINT

In the previous subsection, because the terminal operator does not consider the service level for customers, the number of containers tends to increase to such a level that the turnaround time of road trucks becomes too high. Thus, it is a reasonable assumption that the terminal operator will consider the customer service

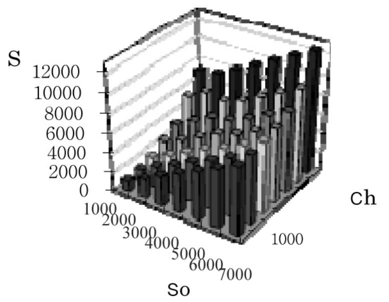

Figure 8. The optimal S for various values of c_h and s_o.

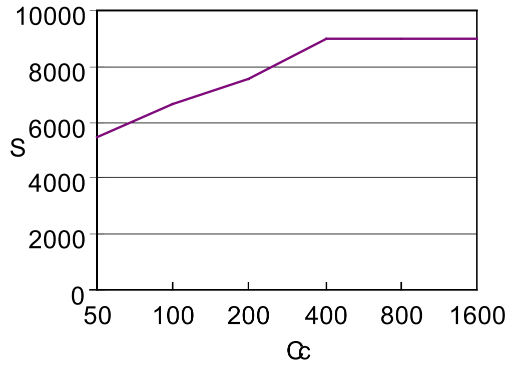

Figure 9. The optimal S for various values of c_c.

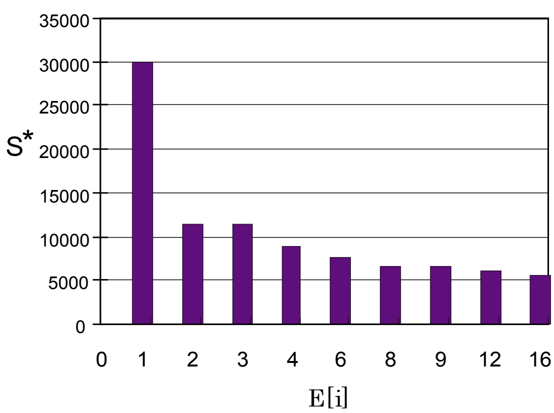

Figure 10. The optimal value of S for various means of p(i).

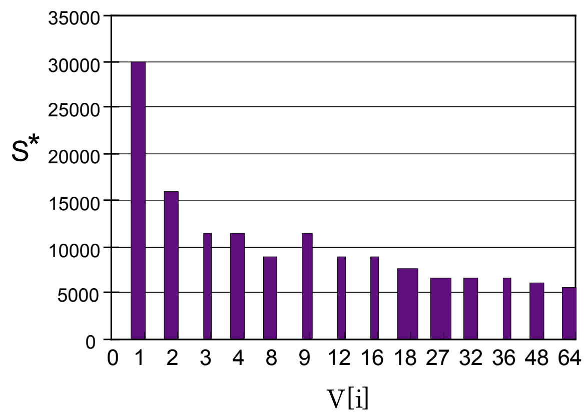

Figure 11. The optimal value of S for various variances of p(i).

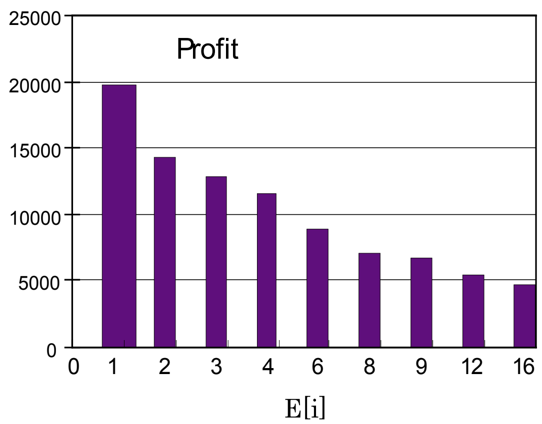

Figure 12. The optimal profit for various means of p(i).

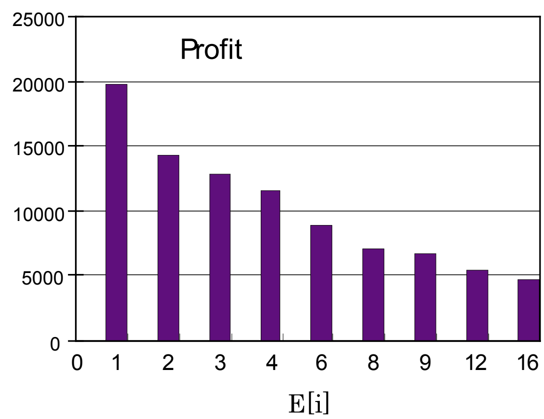

Figure 13. The optimal profit for various variances of p(i).

when deciding the storage price and the free-time-limit. Then, the problem can be formulated as follows:

$$\text{(P2)}: \ Max\ \mathrm{E}[\mathrm{PF}(F, S)] = \sum_{k=F+1}^{t_s} S(i-F)\,\mathrm{p}(i) - c_c \gamma E[t_R] \quad (6)$$

subject to

$$\mathrm{E}(w_t | \ F, S) \ \leq \ w, \quad (7)$$

where t_s is the maximum value of t such that $S(t-F) \leq c_h\gamma + s_o(t-F)$, and w_t and w, respectively, are the waiting time of road trucks and the maximum allowable average waiting time of road trucks.

The following discusses how to evaluate $E[w_t]$. The waiting time of road trucks depends on the inter-arrival time of trucks and the service time needed by YCs. The service time consists of the handling time for a target container (t_d), the travel time between yard-bays (t_t), and the rehandling time (t_R).

The expectation and the variance of t_d can be estimated based on practical data. Also, expressions for the expectation and the variance of t_t were derived in Kim and Kim (2002). However, the expectation and the variance of t_d and t_t do not depend on the decision variables, F and S. Thus, they are considered as constant values in the evaluation of the expected waiting time.

Thus, the only time value that depends on the decision variables, F and S, is t_R. The following discussion is about evaluating the expectation and the variance of the rehandling time. In order to find the distribution of the rehandling time, 30 sets of data were collected by actual observations at two of the largest container terminals in Busan, Korea. Using statistical analysis, it was found that the Gamma distribution fits well with the empirical data on the rehandling time per container. Let the rehandling time per container follow Gamma(α, β).

However, for picking up a container, it may be necessary to rehandle more than one container. Let the probability that *i* containers must be relocated for picking up a container be r(i). Then, r(i) can be calculated as in Table 4. When the value of r(i) increases, the value of E[t_R] in (6) also increases. Detailed explanations on the procedure for calculating r(i) are found in Kim and Kim (2002).

Then, by a derivation in Kim and Kim (2002), the expectation and the variance of the total rehandling time (t_R) can be expressed as follows:

$$E(t_R) = \alpha\beta \sum_{u=0}^{J} u\, r(u) \tag{8}$$

and

$$V(t_R) = (\alpha\beta^2) \sum_{u=0}^{J} r(u)u(1+u\alpha) - \{\alpha\beta \sum_{u=0}^{J} u r(u)\}^2 \tag{9}$$

where J is the maximum number of rehandles possible. From the measured data, we could estimate α as 16.9 and β as 7.3.

Let the arrival process of road trucks follow a Poisson process with an expected arrival rate of λ. From the definition of the cycle time for inbound containers, it follows that the cycle time (seconds) to transfer an inbound container, t_c, can be expressed by $t_d+t_R+t_t$. Because the handling time for a target inbound container, the rehandling time, and the travel time can be assumed to be independent of each other, $E(t_c) = E(t_d)+E(t_R)+E(t_t)$, and $V(t_c) = V(t_d)+V(t_R)+V(t_t)$. $E(t_d)$, $V(t_d)$, $E(t_t)$, and $V(t_t)$ can be estimated directly using the measured data, while $E(t_R)$ and $V(t_R)$ can be respectively calculated by (8) and (9). Then, the transfer operation of inbound containers can be formulated through an M/G/1 queuing model. The expected waiting time of trucks in the system can be evaluated as $E(w_t \mid F, S) = \frac{\rho}{\lambda} + \frac{\lambda^2 V(t_c) + \rho^2}{2(1-\rho)\lambda}$, where $\rho = \lambda/\mu$ and $\mu = 1/\{E(td) + E(tR) + E(t_t)\}$.

Table 4. The probability that the number of rehandles is i (r(i)).

Initial number of containers (*hr*)	r(0)	r(1)	r(2)	r(3)	r(4)
6	1				
7	0.918	0.082			
8	0.857	0.143			
9	0.810	0.190			
10	0.771	0.229			
11	0.740	0.260			
12	0.714	0.272	0.014		
13	0.691	0.266	0.043		
14	0.669	0.262	0.069		
15	0.648	0.261	0.091		
16	0.629	0.260	0.110		
17	0.611	0.261	0.123	0.005	
18	0.594	0.261	0.129	0.016	
19	0.578	0.261	0.129	0.031	
20	0.563	0.261	0.131	0.045	
21	0.549	0.260	0.133	0.058	
22	0.536	0.259	0.135	0.067	0.002
23	0.523	0.258	0.138	0.073	0.008
24	0.511	0.256	0.140	0.077	0.015

The optimal values of S and F for the problem, P2, can be found by calculating the objective values for all combinations of all the possible discrete values of S and F. Thus, the algorithm for P2 can be described as follows.

(Step 0) F: = 0. PF* := 0. F* := 0. S:= 0.

(Step 1) F ← F + 1. $t_s \leftarrow F$. If $F > T$, then stop. Otherwise, go to Step 2.

(Step 2) $t_s \leftarrow t_s + 1$. If $t_s > T$, go to Step 1. Otherwise, $S := [c_h + s_o(t_s - F)]/(t_s - F)$ set

(Step 3) Check if E(w_t| *F*, *S*) > w; then, go to Step 1. Otherwise, evaluate E[PF(*F*, *S*)]. If E[PF(*F*, *S*)] > PF*, then PF*← E[PF(*F*, *S*)], *F** ← *F*, t_s^* ← t_s, and *S** ← *S*. Go to Step 2.

Note that once it is found that E(w_t| *F*, *S*) > w, it is not necessary to further increase t_s because an increase in t_s will also increase the value of E(w_t| *F*, *S*).

The following numerical experiment assumed that λ, E(t_d), Var(t_d), and w were 70 per hour, 79 seconds, 683 seconds, and 90 seconds, respectively. E(t_t) and Var(t_t) were calculated by considering the speed of YCs. The moving speed of YCs was assumed to be 0.3 of one bay-distance per second. It was assumed that p(i) followed Gamma (4, 2). Table 5 shows the changes in the objective function, (6), for various values of F and S. Note that it shows that for a given value of S, the objective function increased first and decreased subsequently as the value of F increased. Note that for a given value of F, the objective function decreased as the value of S increased.

Figures 14 and 15 show the changes in the optimal value of S for different means and variances of p(i). These figures respectively show that the value of S decreased as the values of the mean and the variance of p(i) increased. Figures 16 and 17 show the changes in the optimal revenue for different means and variances of p(i). Figures 18 and 19 show the changes in the rehandling cost for different means and variances of p(i).

Table 5. The objective function, (6), for various values of F and S.

S \ F	0	1	2	3
5500	13223			
6000	11793			
6666	9792	9044		
7600	8190	6967	5713	
9000	4768	5075	4039	
11333	2142	3016	2096	79
16000	544	1205	98	-813
30000	45	488	-1040	-2729

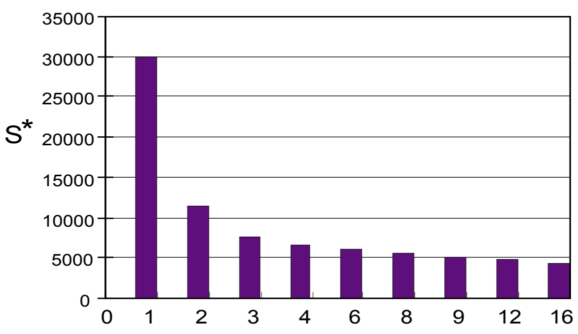

Figure 14. The optimal value of S for various means of p(i).

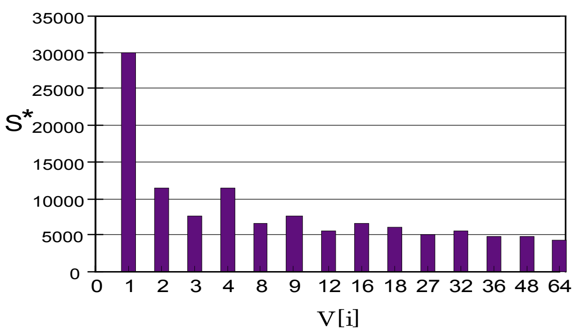

Figure 15. The optimal value of S for various variances of p(i).

5. OPTIMAL PRICE SCHEDULE FOR PUBLIC TERMINAL OPERATORS

Suppose that a container terminal is owned and operated by a public agency whose objective is to either maximize the welfare or minimize the cost of the public. This paper assumes that the objective is to minimize the total cost of both PCT and customers related to the decisions on the free-time-limit and the storage price for inbound containers. The total cost includes the operating cost of the PCT operator and those of customers at both the PCT and ODCYs. The expected total cost per TEU can be formulated as follows.

$$\text{(P3):}\quad E[TC(F,\ t_s)] = (c_c + c_t)\gamma\{E[t_d] + E[t_R] + E[t_t]\} + c_t\gamma E[w_t] + \sum_{k=t_s+1}^{T}[c_h\gamma + s_o(k-F)]\mathrm{p}(k)$$

Because $E[t_d]$ and $E[t_t]$ do not depend on F and t_s, they will be deleted from the model. Then, we have the following objective function:

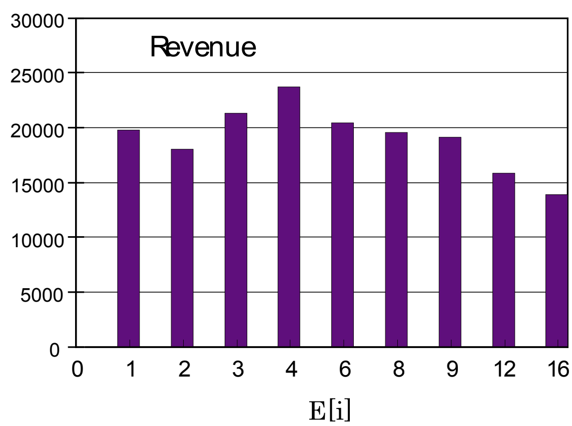

Figure 16. The revenue in optimal solutions for various means of p(i).

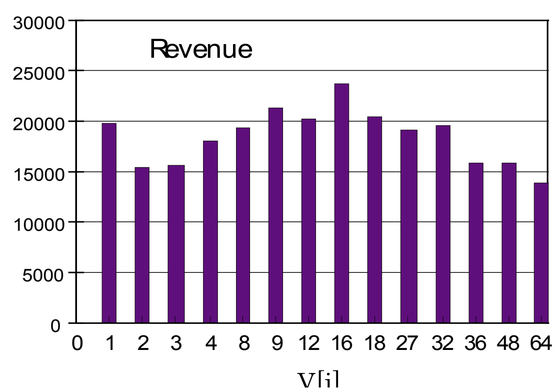

Figure 17. The revenue in optimal solutions for various variances of p(i).

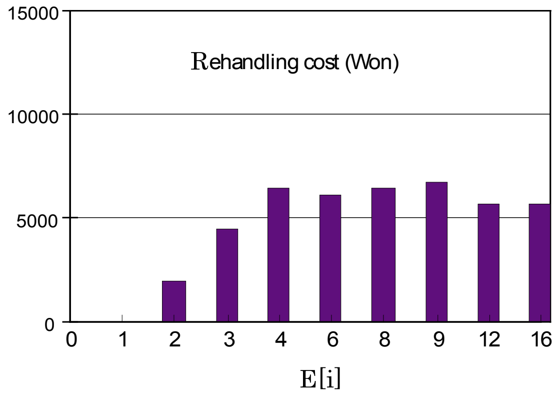

Figure 18. The rehandling cost in optimal solutions for various means of p(i).

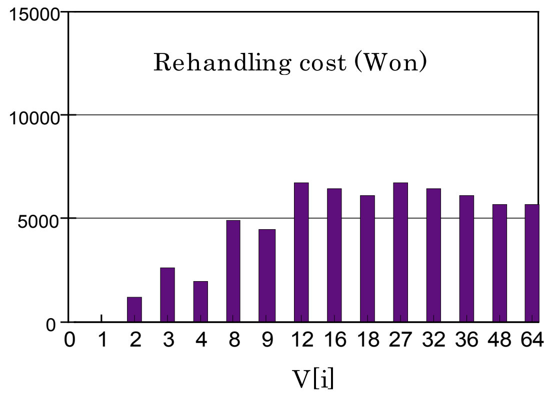

Figure 19. The rehandling cost in optimal solutions for various variances of p(i).

$$Min\ \mathrm{E}[\mathrm{TC}(F,\ t_s)] = (c_c + c_t)\gamma E[t_R] + c_t \gamma E[w_t] + \sum_{k=t_s+1}^{T} [c_h\ \gamma + s_o(k-F)]\mathrm{p}(k)\,. \qquad (10)$$

Note that the first term in (10) represents the cost that results from the relocation of containers by YCs. Because both a YC and a truck must wait for the completion of the relocation operation, the relocation cost is imposed on both the YC and the truck. The second term is the waiting cost of road trucks. The last term represents the cost of containers that are moved to an ODCY before they are delivered to their final destinations.

The value of the objective function, (10), depends only on t_s and F; it does not depend on the value of S. However, by the definition of t_s that t_s is the maximum value of t such that $S(t-F) \le c_h + s_o(t-F)$, the range of S under which the values of the optimal F and t_s are feasible is restricted. Thus, the optimal range of S can be found by:

$$\frac{c_h}{t_s+1-F} + s_o < S \le \frac{c_h}{t_s-F} + s_o \qquad (11)$$

The optimal values of S and F for the problem, P3, can be found by enumerating the objective values for all the combinations of the possible values of t_s and F.

Thus, the algorithm for the problem, P3, can be described as follows.

(Step 0) $F := 0$. TC* := 0. $F^* := 0$.

(Step 1) $F \leftarrow F + 1$. $t_s \leftarrow F$. If $F > T$, go to Step 3.

(Step 2) $t_s \leftarrow t_s + 1$. If $t_s > T$, go to step 1. Otherwise, evaluate E[TC(F, t_s)]. If E[TC (F, t_s)] < TC*, TC* ← E[TC(F, t_s)], F* ← F, and $t_s^* \leftarrow t_s$. Go to the beginning of Step 2.

(Step 3) Calculate the optimal range of S by Eq. (11).

The following numerical experiment set ct to 10 won per second. And, it was assumed that p(i) followed Gamma (4, 2). Figure 20 shows the changes in the values of the objective function, viz., (10), for various values of F and S. The optimal solution could be found at the point, F = 0 and $t_s = 8$. In this case, the range of the value of S was 5111 < S ≤ 5500. Although the optimal value of F was zero in most instances of P3, positive optimal values of F could also be found in some instances of P3. For example, if p(i) followed Gamma (1, 4), the optimal solution could be found at the point, F = 1 and $t_s = 11$. In this case, the range of S became 4545 < S ≤ 4800.

Figure 21 shows the last term of the objective function, (10), for various values of t_s. Note that in Figure 21, V[i] = 16 and V[i] = 32 with the same expectation, E[i] = 8, corresponding to the means and variances of the gamma distributions, Gamma (4, 2) and Gamma (2, 4), respectively. Also, E[i] = 4 and E[i] = 8 with the same variance, V[i] = 16, corresponding to the means and variances of the gamma distributions, Gamma (1, 4) and Gamma (4, 2), respectively. Figure 21 shows that the value of the last term of the objective function, (10), decreased as the value of t_s increased. It also shows that for a given value of t_s, the value of the last term decreased as the mean and the variance of the distribution, p(i), decreased. Figure 22 shows that the rehandling cost increased as the value of t_s increased. It also shows that there was no big difference in the rehandling cost across the different distributions of p(i).

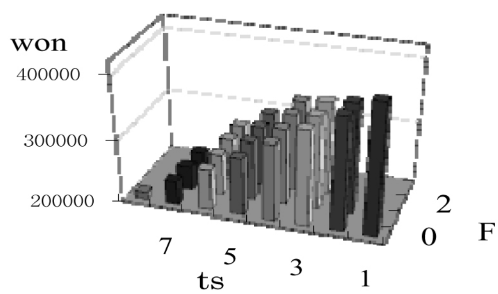

Figure 20. The values of the objective function of (10) for various values of F and t_s.

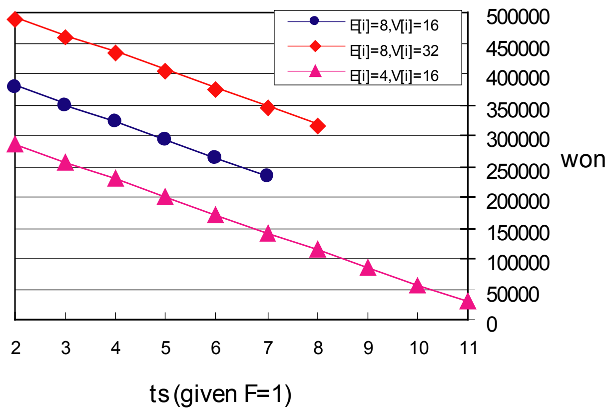

Figure 21. The values of the last term of (10) for various values of t_s.

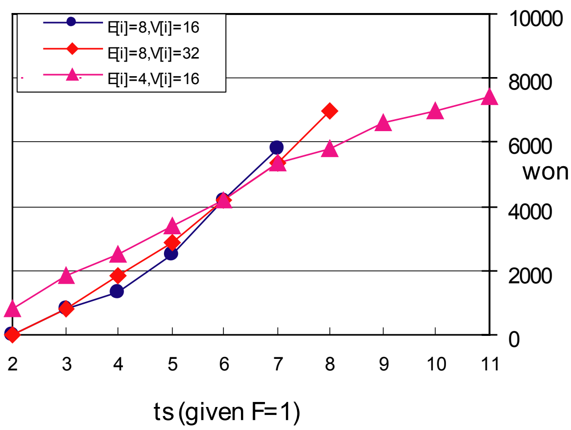

Figure 22. The values of the rehandling cost in PCT for various values of t_s.

6. SUMMARY AND CONCLUSIONS

In this paper, three models were suggested for determining the optimal free-time-limit and the daily storage price for inbound containers. The models can be utilized in private and public port container terminals for increasing their revenue or profit. A solution procedure was suggested for each model. It was found that the optimal solution could be found by a simple enumeration of the limited number of combinations of all possible values of the free-time-limit and the daily storage price.

In order to evaluate the proposed models, numerical experiments were conducted. According to the numerical experiments, most of the optimal solutions indicated that the free-time-limit of the storage was either zero or a value that was much smaller than the values being used in practice. This means that the price schedule with a large, positive free-time-limit is not beneficial to any of the players in the system including the terminal operator and customers. Considering that vessel liners consider the free-time-limit as a benefit that the terminal operator should provide to carriers, it may be necessary for the terminal operators to suggest new price schedules after careful analysis of the benefits and costs of all the related participants in the system.

This study considered only a linear price schedule. Other types of non-linear (but convex) price schedules may also be studied for comparing the non-linear models with the linear model in this study in terms of performance.

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APPENDIX

The gamma distribution (Gamma(ϕ, θ))-which can be adapted to various different shapes and expressed as follows-was used to express p(i). Its density function is:

$$f(t) = \frac{1}{\Gamma(\varphi)\theta^{\varphi}} t^{\varphi-1} e^{-t/\theta}, \text{ where } 0 \le t < \infty. \quad \text{(A-1)}$$

Because p(i) is a discrete probability distribution, it can be evaluated as follows:

$$p(i) = \int_{i-1}^{i} \frac{1}{\Gamma(\varphi)\theta^{\varphi}} t^{\varphi-1} e^{-t/\theta} \mathrm{dt} \quad \text{for i = 1, 2, } \cdots\text{, T.} \quad \text{(A-2)}$$

In the above, T can be defined as the value that satisfies $\sum_{i=1}^{T} \mathrm{p}(i) \cong 1$.