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The Role of "Personal Knowledge" in Solid Geometry among Primary School Mathematics Teachers[1]

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Teachers' *personal knowledge* (PK) is an element in their pedagogic-practical knowledge. This study exposes the PK of primary school mathematics teachers regarding solid geometry through reflection. Students are exposed to solid geometry on various levels, from kindergarten age and above. Previous studies attested to the fact that students encounter difficulties—strong dislike and fear engendered by geometry. A good number of teachers have strong dislike to solid geometry, as well. Therefore, those engaged in teaching the subject must address the problem and try to overcome these difficulties. In this paper we have introduced the reflective process among teachers in primary school, including application of Van-Hiele's theory to solid geometry.

Keywords: personal knowledge (PK), reflection, Van-Hiele's theory, solid geometry
MESC Classification: B52, B59, G42, G49
MSC2010 Classification: 97G40

THEORETICAL BACKGROUND

A. "Personal Knowledge" and Reflection

"Personal knowledge" (PK) is an element in the practical-pedagogic knowledge teachers embodies and employs. It is defined as a combination of beliefs, opinions and viewpoints, theories, interpretation of experience and personal philosophies that teachers hold about teaching, the curriculum and the educational material they are teaching (Shulman, 1987; Schön, 1983; Elbaz, 1983; Connelly & Diens, 1982). Exposure of the personal knowledge of primary school mathematics teachers in teaching solid geometry was carried out through reflection.

Reflective thinking is contemplation about thinking and actions. It brings teachers to

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introspection and reflection on their opinions, thoughts, beliefs and feelings (Allen, 1997; Zeichner 1994, 1996; Clandinin & Connelly, 1996; Ross *et al.*, 1993; Louden, 1992). Connelly & Cladinin (1991) explain that "practical personal knowledge" is knowledge tied to the person him/herself - that is to say, teachers construct their personal knowledge in a long-term dialectic process. The concept "personal practical knowledge" means what teachers think about their teaching in three situations: prior to acting, while acting, after acting.

Schön (1983, 1987) designated these three aspects "reflection in action" and "reflection on action." Teachers are labeled "the reflective practical teacher" and are described as structuring different situations in their class while enhancing, improving and reexamining their own personal theories on the educational environment in which they operate. Reflective processes constitute an integral part of teachers' cumulative daily experience and become a basic element in their personal practical knowledge, philosophies and theories as teachers – one's "personal knowledge" (Calderhead, 1989). These processes tie the world of teaching within the personal context of teachers, including outlook, opinions, beliefs and positions vis-a-vis the human and physical content surrounding them, in contrast with the public knowledge of teaching, that is, the product of thought and research in the social context of teaching.

Zeichner (1996) defines reflection as knowledge of self through introspection. He claims that one should define practical reflective teaching as the autonomy of teachers to set targets, examine actions and learn from a position of autonomy. Thus, teachers must be critical of content and objectives, and concurrently develop such critical faculties among the students, as well. Calderhead (1989) emphasizes the importance of encouraging "practical reflective teaching" as part of nurturing professionalism in teaching.

Allen (1997) and Zeichner (1994, 1996) maintain that reflective thinking carries the potential for renewal and creation of professional practical knowledge needed by teachers. Teachers with an abstract style of thinking will develop by adding to their theoretical knowledge, and challenge it with their held views through reflection. Teachers with a concrete style of thinking will attempt to uncover special needs of their students and search for the theoretical knowledge they need to fill. Thus, reflective thinking will bring about internalization of knowledge by reconstruction and reexamination of processes prior, during and after the teaching experience.

Ross *et al.* (1993) stress the importance of reflective thinking in mutual clarification and reexamination of various teaching situations - exercise of judgment, dilemmas, quandaries and decision-making processes viewed through the use of retrospective analytical resection. Integration of these kinds of thinking creates what is defined as teachers' "personal practical philosophy" anchored in personal unique theories (PK).

B. Solid Geometry and Van-Hiele's theory

Solid geometry is a chapter in geometry taught at various stages during the school years, beginning in kindergarten and up to matriculation in high school. The subject is taught in a circular manner at various levels of difficulty. Instructional programs from kindergarten and up to the 12th-grade should enable all students to apply spatial vision, reasoning and geometric modeling in order to solve problems (NCTM, 2000).

Spatial vision is defined as one of people's important skills and it has an impact on our daily life. Spatial vision is considered one of the two primary elements in spatial ability (the second being spatial orientation). Spatial vision constitutes the ability to conceptualize the appearance of objects, movement or changes in their attributes, usually by the shape and exposition of objects. A large number of findings from previous studies support the correlation between spatial vision to mathematics and other sciences. Development of spatial vision is based on exposure and experimentation by the learner focusing on geometric relationships between shapes, their direction, orientation and the prospective of objects in space (NCTM, 1989, 2000). According to the theory of multiple intelligences conceived by Gardner (1983), spatial abilities are viewed as one of people's intelligences. Spatial intelligence has many factors, spread over a wide variety of abilities – cognitive, motor, analytical and behavioral. The term encompasses a vast entirety of non-verbal cognitive competences which help individuals to absorb and decode visual representations creating the image of space. Researchers (Halpern, 2005; Linn & Petersen, 1985; Sorby, Wysocki & Baartmans, 2002) maintain that spatial abilities are essential for all areas of life and greatly influence students' achievements in science, geometry, as well as in many applied subjects. The education system focuses on learning mathematical knowledge and dedicates only a limited room for developing students' spatial abilities. Teachers' prevalent assumption is that these skills are self-acquired during the stages of growth, experience and maturity. Moreover, some researchers (Ben-Chaim, Lappan & Houang, 1989; Caplan, Crawford, Hyde & Richardson, 1997; Colom, Contreras, Botella & Santacreu, 2002) argue that there are gender differences in spatial visualization ability and that these differences affect students' achievements in subjects requiring good spatial ability. These beliefs and stereotypes relating to spatial ability give rise to the mistaken assumption that no help can be provided to students, so that they improve their achievements and reduce gender differences. Spatial vision can be developed by building and manipulating first concrete and then mental representations of shapes, relationships, and transformations. Teachers should plan instruction so that students can explore the relationships of different attributes or change one characteristics of a shape while preserving others.

The professional literature contains a number of reports on enhancement of spatial vision through training and learning (Fuys & Tischler, 1988; Ben-Haim, Lappan & houang, 1989). They survey studies in this area that testify to difficulties encountered by students, characterized by strong dislike and fear. They mention that teachers as well have expressed difficulties and strong dislike.

It has become commonplace to raise Van-Hiele's theory of geometry among those engaged in the teaching of geometry (Gutierres, Jaime & Fortuny, 1991; Mason, 1997; Knight, 2006, Halat & Sahin, 2008). Van-Hiele's theory is usually raised in relation to plane geometry. This theory, developed by the Dutch mathematical educators, Dina and Pierre Van-Hiele, tried to provide an explanation for the fact that many students have difficulties in mastering cognitive processes. According to Van-Hiele's theory, command of mathematics (and geometry in particular), develops in a hierarchic order on four levels: visual level, descriptive level, informal deduction level and formal deduction and rigor level. Moreover, each student can be on different levels of mastery at the same time (Fuys & Tischler, 1988; Gutierres, Jaime & Fortuny, 1991; Van-Hiele, 1999). Therefore, those engaged in teaching geometry must relate to the various levels of thinking among students in the class, striving to overcome the students' difficulties in learning basic geometric concepts as well as to advance students in their levels of thinking.

Van-Hiele's four-level theory is suitable for teaching solid geometry, as well. Van-Hiele themselves employed a cube and other solids as an opening theme for geometry lessons in primary school (NCTM, 1992, 2000). Thus, if one adopts Van-Hiele's theory for solid geometry, then one should advance students along various levels of thinking vis-à-vis 3-dimensional concepts, reducing difficulties and dislike that accompany this learning subject. Expansion of the two-dimensional world into a three-dimensional world can reinforce the learners' spatial abilities.

This paper presents exposure to PK in solid geometry among primary school mathematics teachers through a reflective process that includes application of the Van-Hiele's theory to solid geometry.

Research Population

The research population consisted of 18 primary school mathematics teachers with at least a 5-yeas teaching experience. Teachers attended a course of seven meetings (total of 28 hours; the meetings took place once every two weeks and lasted 4 hours each) for teaching solid geometry in primary school. Within the framework of the course, the meetings were devoted to Van-Hiele's theory (Appendix 1) and various activities in solid geometry.

Research Tools

1. Questionnaires

A. An open-ended written reflective questionnaire – "Questionnaire A". Teachers were requested to analyze their relationship, attitude and feelings towards solid geometry.

The questions were:

1. What are your feelings about solid geometry?
2. What is your opinion about solid geometry?
3. Try to explain your attitude toward solid geometry?

B. An open-ended written reflective questionnaire – "Questionnaire B". Teachers were requested to evaluate themselves, on which level of thinking they are according to Van-Hiele's theory, and to substantiate their answer. The questionnaires were built according to Kinach (1998).

The questions were:

1. Evaluate your own level of thinking according to Van-Hiele's levels of thinking in solid geometry.
2. Try to explain and substantiate how you arrived to this evaluation level.

2. Pool of questions

Teachers were requested to apply, while engaging in reflective thinking, the transfer of Van-Hiele's theory from its original context in plane geometry, to the study of solids. This educational assignment in application of Van-Hiele's theory was presented to teachers after experience and exposure to various activities in solid geometry. They were requested to create problems for pinpointing various levels of thinking in solid geometry - that is, to construct a questionnaire (check-up/test) presenting the first three levels of thinking regarding solid geometry (the three levels suitable for primary school).

Data Analysis and Processing

The data were processed by qualitative methods: Assertions were collected and categories were formulated in accordance with the professional literature (ETIC) and the phenomena under study (EMIC) and frequency of assertions were calculated according to the categories. Analysis of the content was validated by two independent experts in pre-service teacher education, according to a process of expert judgment (Görn, 1977; Creswell, 1998; 2003). They examined a random sample of categorizations on all three questionnaires: a 90% agreement was found.

Research Process

Stage A: Meeting no. 1: Filling-in reflective Questionnaire A. Exposure to Van-Hiele's theory of levels of thinking in plane geometry (Appendix 1).

Stage B: Meeting nos. 2–3: Adaptation and adoption of this theory to solid geometry. In this framework the participants received examples that represented each level of thinking and they, themselves, analyzed additional examples. The intervention was conducted and carried out by an expert in solid geometry.

Stage C: Meetings no.4–5: Filling-in reflective Questionnaire B. Experience and varied activities involving three-dimensional objects, relating to various levels of thinking about solid geometry, including building of objects according to their attributes.

Stage D: Meeting nos. 6–7: Creation of a "bank of questions" designed to pinpoint levels of thinking in solid geometry. Filling-in reflective Questionnaire B.

FINDINGS

1. Reflective Questionnaire A

Regarding feelings and attitudes towards solid geometry (teachers indicated 62 assertions). The categories for sorting the frequency of assertions were established according to the phenomena under study (EMIC).

44% of the teachers' assertions expressed affection, love, interest, a good feeling, self-confidence and a sense of challenge.

56% of the teachers' assertions expressed lack of confidence, fear, lack of mastery of theorems and proofs as well as difficulty with language. Some of the typical responses stated:

> "...My feeling about solid geometry is that of an ominous deficiency, lack of confidence."

> "My feelings about solid geometry are mixed. On the one hand this is one of the most interesting subjects due to the building of the solids. The pupils demonstrate great interest already at the stage of building. Hence, the atmosphere in class is good and I feel it is agreeable teaching the pupils"

10% of the participants' assertions cited the need for special qualifications in mastering solid geometry, such as: command of spatial vision, competence in drawing developments of solids:

> "I don't feel entirely confident because I have a problem with spatial visualization. I frequently have to build or draw and I find it difficult, particularly when I am teaching the topic of object development. I have to cut and attach and then answer the pupils' questions if they ask me."

All teachers cited the course's contribution in enhancing their knowledge of the subject matter, raising their level of thinking, and improving their relationship and attitudes towards solid geometry. One of the typical responses stated:

> "...The course refreshed my memory in general, and I mastered the content taught within the framework of the enrichment course...."

Another teacher reported that:

> "Actually, the lack of confidence was mitigated during the course, particularly after completing the exercise assignment that included building an object according to its attribute..."

One teacher said that:

> "My attitude towards solid geometry is complex. Everyone needs spatial visualization in order to succeed in this subject."

Another teacher mentioned that:

> "I want to point out that in the in-service training course I have learnt to draw and now I feel good with it."

2. Reflective Questionnaire B

Teachers who attended the course categorized their level of spatial thinking according to the level of thinking in Van-Hiele's theory of plain geometry, to which they were exposed in the intervention program during the course (Appendix 1).

Teachers classified themselves as belonging to a certain level of thinking according to their personal knowledge based on significant definitions.

Responses regarding self-evaluations of the level of thinking were as follows:

- 9 rated themselves at the third level of mastery – informal deduction.
- 6 rated themselves at the final stage of mastery – formal deduction and rigor.
- 3 rated themselves at the second stage of mastery – descriptive.
- No one rated him/herself at the first stage of mastering.

Justification of responses was as follows (teachers indicated 58 assertions):

- 55% of the responses had a negative tone - that is, respondents viewed their level of competence as the outcome of lack of total mastery, faulty memory, difficulties in drawing, partial or limited ability to provide proofs, and so forth.

- 45% of the responses had a positive orientation - that is, respondents viewed their level of competence as the outcome of knowledge and familiarity with objects, the ability to analyze, organize and categorize objects, ability to visualize spatial relationships, use of cognitive faculties to analyze objects, and so forth.

Typical responses stated:

> "...On the recognition level - I have mastery, on the analysis level - I have mastery, on the order level - I have good mastery of the objects taught in the class, on the level of deduction and rigor - I have partial mastery...", and "... In studied solid geometry in the past...it was a long time ago. But I don't remember everything that was taught then."

Another teacher:

> "According to Van-Hiele's levels of thinking, I believe I am on the fourth level—formal deduction and rigor. I am familiar with the fundamental concepts. I know the solids and can classify and categorize them. Moreover, I can explain the attributes relating to the solids. In my opinion, my vast experience in learning the subject brings me to that level. "

Another teacher:

> "According to my assessment, my level of thinking is between level 1 and level 2. In some of the questions I answered based on intuition and on my knowledge, without being able to explain why. In some questions (on levels 1, 2) I answered based on previous information. I believe I am on that level because in some of the level 2 questions and in most of the level 3 questions I needed help from an external source."

3. "Bank of Questions"—Evaluation Questionnaire Pinpointing Levels of Spatial Thinking

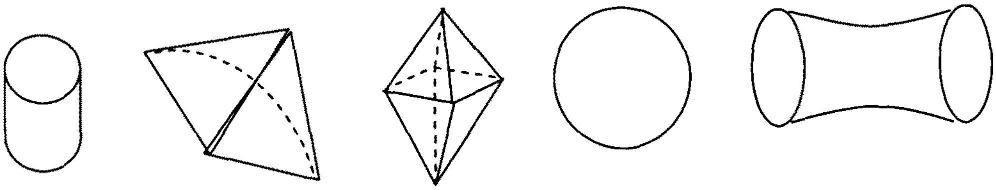
All teachers, without exception, drew up questions that presented the three levels of thinking in solid geometry (the three levels suitable for primary school). In a group workshop format, course participants chose fifteen questions (five for each level of thinking), formulating a questionnaire designed to evaluate mastery of solid geometry.

Choice of questions was based on three criteria: content, question format and composition of the group. The questions chosen represented different shapes in solid geometry (content), questions in the form of diagrams and verbal descriptions (shape), making sure most of the participants are represented. The reason was to form a questionnaire which is interesting, comprehensive, applicable, and analyzing the three levels of thinking according to Van-Hiele's theory in solid geometry.

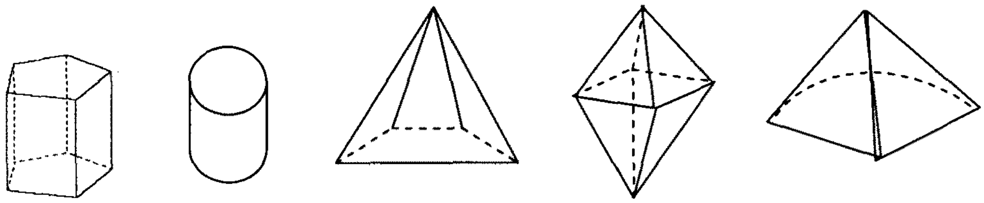
Below is the outcome of the course, namely the "Bank of Questions," which identifies the 3 levels of thinking in solid geometry according to Van-Hiele's theory:

Van-Hiele Level 1:

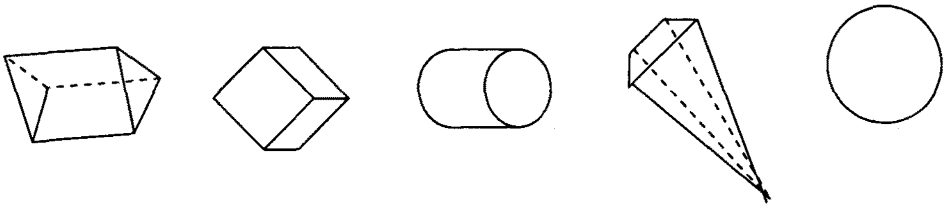
1. Circle the polyhedron among the following objects



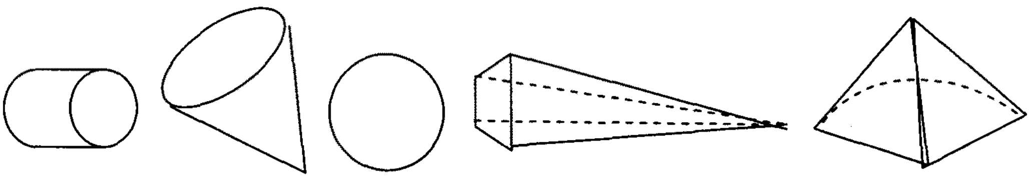
2. Circle the **prism** among the following objects



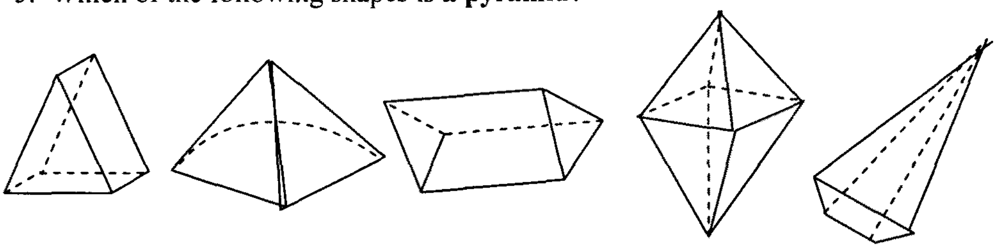
3. Among the following objects, circle the one shaped like a **cylinder**



4. Circle the object shaped like a **cone**



5. Which of the following shapes is **a pyramid**?



Van-Hiele Level 2:

1. Which of the following statements is not true for a prism?

   a. The prism has two congruent and parallel bases.
   b. The lateral surface of the prism is constructed of rectangles or parallelograms.
   c. All the faces of the prism are rectangles or parallelograms.
   d. The prism is a 3-dimensional object.
   e. Statements a-d are false.

2. Which of the following statements is true for every polyhedron?

   a. In a polyhedron all faces are congruent.
   b. All polyhedrons are constructed of pairs of parallel faces.
   c. A polyhedron does not have curved surfaces.
   d. Each vertex in a polyhedron intersects a number of other faces.
   e. Statements a-d are true.

3. Which of the following statements is true for every cylinder?

   a. The bases of a cylinder are circular.
   b. The bases of the cylinder are congruent and parallel squares
   c. The bases of the cylinder are constructed of regular polygons.
   d. The bases of the cylinder are pentagons.
   e. The bases of the cylinder are triangles.

4. Which of the following statements is true for every prism?

   a. The lateral surface of the prism is constructed entirely of triangles.
   b. The prism had two parallel bases.
   c. The base of the prism is rectangular.
   d. The lateral surface of the prism is constructed of regular polygons.
   e. Four faces intersect every vertex of the prism.

5. A cube has

   a. 12 faces
   b. 4 faces
   c. 6 faces
   d. 8 faces
   e. 2 faces

Van-Hiele Level 3:

1. Which of the following statements is true?

   a. If an object is a polyhedron then it is also a prism.
   b. If an object is a prism then it is also a polyhedron.
   c. If an object is not a polyhedron, then it is a prism.
   d. If an object is not a prism, it is not a polyhedron.
   e. If an object is a prism, it is not a polyhedron.

2. Which of the following statements is true?

   a. If an object has 8 vertices it must be a rectangular parallelepiped.
   b. If an object has 8 vertices it must be a cube.
   c. If an object has 8 vertices it must be a pyramid.
   d. If an object has 8 vertices it must be a regular polyhedron.
   e. Statements a-d are false.

3. Which of the following statements is always true?

   a. If an object has two bases then if must be a rectangular parallelepiped.
   b. If an object has two bases then it must be a polygon.
   c. If an object has two bases then it must be a cylinder.
   d. If an object has two bases then it must be a regular polyhedron.
   e. Statements a-d are false.

4. Which of the following statements is true?

   a. If an object is a regular polyhedron, it is also a cube.
   b. If an object is a cube, it is also a regular polyhedron.
   c. If an object is not a regular polyhedron, it is a cube.
   d. If an object is not a cube, it is a regular polyhedron.
   e. Statements a-d are true.

5. All pyramids have in common:

   a. All pyramids have a triangular-shaped base.
   b. The lateral surface is constructed of triangles.
   c. The base is quadrangle and the lateral surface is constructed of triangles.
   d. All faces of the pyramid are triangles.
   e. All statements a-d are false.

DISCUSSION AND CONCLUSIONS

Analysis of findings illustrates that the reflective process accompanying the intervention program enabled teachers to examine their own levels of thinking and their own attitudes and relationship to solid geometry, although the size of the population sample was small.

Findings from the reflective Questionnaire in the first meeting indicate that many of teachers' assertions expressed lack of confidence and fear as well as lack of mastery of theorems and proofs in solid geometry. This may explain the corresponding apprehension and lack of confidence among students, who are influenced by their teachers' attitude (Ben-Haim *et al.*, 1989). Expansion of the world of plane geometry to stereometrics (solid geometry) can enhance the spatial ability of students (Van-Hiele 1999; NCTM, 2000).

Introduction of teachers to Van-Hiele's theory in the context of solid geometry including experimentation, resulted in a cognitive tool that, tied-in to the personal practical philosophy of the participants, enabled them to create reflective situations for contemplation about thinking. Moreover, it helped them to connect theory and practice, and strengthened their knowledge base about mathematics teaching and learning.

Findings from the reflective Questionnaire B show that teachers were able to categorize their levels of thinking in solid geometry by the levels of thinking set forth in Van-Hiele's theory in plain geometry. Teachers sought to explain the rationale behind the rating they gave themselves. There may be a causal relationship between the fact that 56% of teachers' assertions expressed lack of confidence in reflective Questionnaire A and that 55% of the justifications for the levels of mastery in reflective Questionnaire B were stated in negative terms. They perceived subject content through the prism of their own evaluations of their status within the teaching context. The contents constituted a basis for reflection by the participants regarding their knowledge, attitudes, difficulties and thoughts (Allen, 1997; Zeichner, 1994; 1996).

The teachers responded by writing the practical questionnaire designed to create a "bank of questions" for evaluating or pinpointing levels of thinking in solid geometry by levels of thinking set forth by Van-Hiele. The researchers explain this activity as utilization of the outcome of reflective teaching, in that it allows mutual and repeated concept clarification and multiple teaching scenarios. Through a discourse, the participants create conceptualization of knowledge. The process covers raising subjects to self-awareness and, thus, creating cognitive and sometimes also emotional and social essence.

Furthermore, the content analysis indicates that throughout the intervention program the participants cooperated with the researchers - underscored by the candor of their responses to questionnaires. The participants exhibited both openness and a desire to learn, to cope and to improve. Thus, one can assume that integrating exposure of teachers' personal knowledge into mastering contents can be an excellent way of in-service training, within the framework of continued professional development, bearing in mind the limitation of a small sample.

All teachers who participated in the study's intervention program succeeded in formulating solid geometry questions in line with the levels determined by Van-Hiele. In addition, all teachers reported that they managed to overcome a lack of confidence regarding the subject and that as a result of formulating a "bank of questions" they would be able to teach these contents in their classrooms.

Consequently, in view of the methodological limitations, we would recommend adopting this approach in coping with additional geometric material.

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APPENDIX 1

Levels of Thinking in Plane Geometry

Van-Hiele's theory of Levels developed by two Dutch mathematical educators in the late 1950's explains the fact that many students encounter difficulties in cognitive processes on the high levels, particularly when they must cope with providing proofs. According to this theory, development in the study of geometry progresses in a hierarchical order along four levels of mastery, where partial mastery at a particular level is necessary, but not sufficient for understanding on a higher level, and students cannot function on a particular level if they have not achieved mastery on previous levels.

The four levels presented by Van-Hiele are:

1 Visual level: At this elementary stage, students are able to identify geometric shapes and differentiate between them. Each of the concepts or shapes is viewed as a whole-as it appears. Students are able to differentiate between similar shapes and are able to assign name to them. At this stage students are unable to specify the attributes of each shape.
2. Descriptive level: At this stage students are able to analyze the attributes of the shapes, but do not have the ability to assign attributes to a particular element (*i.e.*, a cube) to the attributes of a group to which it belongs (*i.e.*, a quadrilateral).
3. Informal deduction level: At this stage students identify a hierarchical order of inclusion among groups of shapes by their attributes and definitions. However, they cannot prove arguments involving the attributes of geometric shapes.
4. Formal deduction and rigor level: At this stage students know the purpose of basic terms, axioms, definitions, statements and proofs and the connections among them. They can use suppositions to prove statements and understand the significance of essential and satisfying conditions. At this stage students are able to provide reasons and arguments at various stages of the proof. In addition, they understand the importance of precision in their proofs, and the context from the general to the particular and even the necessity of providing a proof from each category.

Clarification

Students can function on various levels of mastery set forth by Van-Hiele at the same time, depending on the level of understanding reached regarding the particular geometric concept under study.

Many studies conducted during the past three decades found that there is a gap between the students' level of ability and understanding and the level of teaching of geometry teachers. In many cases, teachers were found to be teaching pm a higher level than the ability of the students to understand.