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애매 bi-군과 퍼지 bi-함수의 성질에 관한 연구

On some properties of vague bi-groups and fuzzy bi-functions

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Abstract

M. Demirci[Vague groups, J. math. Anal. Appl. vol.230, pp. 142-156, 1999] studied the vague group operation on a crisp set as a fuzzy function and estabished the vague group structure on a crisp set. In this paper we consider bi-groups which are studied by A.A.A. Agboola and L.S. Akinola. And we also will define vague bi-groups and fuzzy bi-functions and we investigate some basic operations on the vague bi-group and fuzzy bi-functions.

Key Words : bi-groups, vague groups, fuzzy equality, vague binary operation, vague bi-groups, fuzzy bi-functions.

1. Introduction

Fuzzy sets proposed by Zadeh in 1968([8]) and fuzzy settings of various algebraic concepts were studied by several authors. Many authors have worked to present the fuzzy setting of various algebraic concepts based on the papers [1,5,6,7].

To get a more general extension, Demirchi [5,7] defined the concept of vague group based on fuzzy equalities and fuzzy functions. He also established the vague group structure on a crisp set.

The concept of fuzzy equality and fuzzy function given in [5,6,7] provides a good tool for fuzzifying the group operation on a crisp set.

Let X, Y be crisp sets. A mapping $E_X: X\times X \to [0,1]$ is called a fuzzy equality on X if and only if the following conditions are satisfied:

(i) $E_X(x,y)=1$ if and only if $x=y$, $\forall x,y\in X$,
(ii) $E_X(x,y)=E_X(y,x)$, $\forall x,y\in X$, and
(iii) $\min[E_X(x,y), E_X(y,z)] \le E_X(x,z)$, $x,y,z\in X$.

The real number $E_X(x,y)$ is called the degree of x and y in X.

Let E_X and E_Y be two fuzzy equalities an X and Y, respectively. $f: X\to Y$ is called fuzzy function with respect to E_X and E_Y if and only if the membership function $\mu_f: X\times Y\to[0,1]$ of f satisfies the following conditions:

(i) $\forall x\in X$, $\exists y\in Y$ such that $\mu_f(x,y)>0$, and
(ii) $\min[\mu_f(x,z), \mu_f(y,w), E_X(x,y)] \le E_Y(z,w)$, $\forall x,y\in X$, $\forall w,z\in Y$.

A fuzzy function f is called a strong fuzzy function if and only if it satisfies the following additionally condition:

$$\forall x\in X, \exists y\in Y \text{ such that } \mu_f(x,y)=1.$$

In this paper we consider bi-groups which are defined by A.A.A. Agboola and L.S. Akinola [2] and W.B. Vasabtha Kadasamy [3]. And we also will define vague bi-groups and fuzzy bi-functions and we investigate some basic operations on vague bi-groups and fuzzy bi-functions.

2. Vague bigroups.

In this section, we consider vague binary operations, vague closed under the operations, vague semigroups, vague groups, fuzzy functions, etc.

Definition 2.1 ([6]) (1) A strong fuzzy function $f: X\times X\to X$ with respect to a fuzzy equality $E_{X\times X}$ on $X\times X$ and fuzzy equality E_X on X is said to be a vague binary operation with respect to $E_{X\times X}$ and E_X.
(2) A vague binary operation f with repsect to $E_{X\times X}$ and E_X is said to be transitive of first order if it satisfies the following condition:

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$$\min[\mu_f(a,b,c), E_X(c,d)] \le \mu_f(a,b,d), \ \forall a,b,c \in X.$$

(3) A vague binary operation f on X with respect to $E_{X\times X}$ and E_X is said to be transitive of the second order if it satisfies the following condition:

$$\min[\mu_f(a,b,c), E_X(b,d)] \le \mu_f(a,d,c), \ \forall a,b,c \in X.$$

(4) A vague binary operation f on X with respect to $E_{X\times X}$ and E_X is said to be transitive of the third order if it satisfies the following condition:

$$\min[\mu_f(a,b,c), E_X(a,d)] \le \mu_f(d,b,c), \ \forall a,b,c \in X.$$

(5) Let f be a vague binary operation on X. A crisp subset B of X is said to be vague closed under f if it satisfies the following condition:

$$\mu_f(a,b,c) = 1 \Rightarrow c \in B, \text{ for } \forall a,b \in B, \forall c \in X.$$

Definition 2.2 ([6]) Let $\circ$ be a vague binary operation on X with respect to a fuzzy equality $E_{X\times X}$ on $X\times X$ and E_X on X. Then

(1) $(X,\circ)$ is called a vague semigroup if the membership function $\mu_\circ : X\times X\times X \to [0,1]$ of $\circ$ satisfies the following condition:

$$\min[\mu_\circ(b,c,d), \mu_\circ(a,d,m), \mu_\circ(a,b,q), \mu_\circ(q,c,w)] \le E_X(m,w), \qquad \forall a,b,c,d,q,m,w \in X.$$

(2) A vague semigroup $(X,\circ)$ is called a vague monoid if and only if it satisfies the following condition:

$\exists e \in X$ such that

$$\min[\mu_\circ(e,a,a), \mu_\circ(a,e,a)] = 1, \ \forall a \in X.$$

(3) A vague monoid $(X,\circ)$ is called a vague group if it satisfies the following condition:

$\forall a \in X, \exists a^{-1} \in X$ such that

$$\min[\mu_\circ(a^{-1},a,e), \mu_\circ(a,a^{-1},e)] = 1.$$

(4) A vague group $(X,\circ)$ is said to be abelian (commutative) if $\circ$ satisfies the following condition:

$$\min[\mu_\circ(a,b,m), \mu_\circ(b,a,w)] \le E_X(m,w), \ \forall a,b,m,w \in X.$$

Proposition 2.3 ([6]) Let $\circ$ be a X with respect to $E_{X\times X}$ and E_X if $(X,\circ)$ is a semigroup and $\circ$ is transitive of the second and the third order, then $(X,\circ)$ is a vague group.

Now, we will define vague bi-group and investigate cancellative law of vague bi-group, and some properties of them.

Definition 2.4 (1) A set $(X,\oplus,\odot)$ with two vague binary operation $\oplus$ and $\odot$ is called a vague bi-group if there exist two proper subsets X_1 and X_2 of X such that

(i) $X = X_1 \cup X_2$,
(ii) $(X_1,\oplus)$ is a vague group,
(iii) $(X_2,\odot)$ is a vague group.

(2) A vague bigroup $(X,\oplus,\odot)$ is said to be abelian if $(X_1,\oplus)$ and $(X_2,\odot)$ are vague abelian.

Theorem 2.5 (Cancellation law) Let $(X,\oplus,\odot)$ be vague bi-group with respect to $E_{X\times X}$ on $X\times X$ and E_X on X. Then we have

(1) $\min[\mu_\oplus(a,b,u), \mu_\oplus(a,c,u)] \le E_{X_1}(b,c)$, for $\forall a,b,c,u \in X_1$

and

$$\min[\mu_\odot(a',b',u'), \mu_\odot(a',c',u')] \le E_{X_2}(b',c'), \ \forall a',b',c',u' \in X_2.$$

(2) $\min[\mu_\oplus(b_1,a_1,u_1), \mu_\oplus(c_1,a_1,u_1)] \le E_{X_1}(b_1,c_1)$, for $\forall a_1,b_1,c_1,u_1 \in X_1$

and

$$\min[\mu_\odot(b_2,a_2,u_2), \mu_\odot(c_2,a_2,u_2)] \le E_{X_2}(b_2,c_2), \ \forall a_2,b_2,c_2,u_2 \in X_2.$$

Proposition 2.6 Let $(X,\oplus,\odot)$ be a vague bi-group with respect to $E_{X\times X}$ on $X\times X$ and E_X on X with $X = X_1 \cup X_2$. Then we have

(1) The identity of $(X_1,\oplus)$ and $(X_2,\odot)$ is unique.

(2) Each element of $(X,\oplus,\odot)$ has a unique inverse element in X. That is, each element of $(X_1,\oplus)$ and $(X_2,\odot)$ have a unique inverse element in X.

(3) For all $a = \begin{cases} a_1 & \text{if } a \in X_1 \\ a_2 & \text{if } a \in X_2 \end{cases}$, we have

$$\oplus^{-1}(\oplus^{-1}a_1) = a_1 \text{ and } \odot^{-1}(\odot^{-1}a_2) = a_2.$$

Proof. (1) Suppose that e_1 and f_1 are identities in $(X_1,\oplus)$ and that e_2 and f_2 are identities in $(X_2,\odot)$. Then we have

$$\mu_\oplus(e_1,a_1,a_1) = \mu_\oplus(a_1,e_1,a_1) = 1$$

and

$$\mu_\odot(e_2,a_2,a_2) = \mu_\odot(a_2,e_2,a_2) = 1.$$

Also, we have

$$\mu_\oplus(f_1,a_1,a_1) = \mu_\oplus(a_1,f_1,a_1) = 1$$

and

$$\mu_\odot(f_2,a_2,a_2) = \mu_\odot(a_2,f_2,a_2) = 1.$$

Thus, by cancellation law, we see that

$$\min[\mu_{\oplus}(a_1,e_1,a_1),\mu_{\oplus}(a_1,f_1,a_1)] \le E_{X_1}(e_1,f_1)$$

and

$$\min[\mu_{\odot}(a_2,e_2,a_2),\mu_{\odot}(a_2,e_2,a_2)] \le E_{X_2}(e_2,f_2).$$

Then, we obtain that

$$E_2(e_2,f_2)=1 \text{ and } E_1(e_1,f_1)=1.$$

So we have $e_2=f_2$ and $e_1=f_1$ and hence the identity of $(X_1,\oplus)$ and $(X_2,\odot)$ is unique.

(2) Let $e=\begin{cases} e_1 \text{ if } e\in X_1 \\ e_2 \text{ if } e\in X_2 \end{cases}$ be identities in $X=X_1\cup X_2$. Suppose that $b=\begin{cases} b_1 \text{ if } b\in X_1 \\ b_2 \text{ if } b\in X_2 \end{cases}$ and $c=\begin{cases} c_1 \text{ if } c\in X_1 \\ c_2 \text{ if } c\in X_2 \end{cases}$ are inverse of $a=\begin{cases} a_1 \text{ if } a\in X_1 \\ a_2 \text{ if } a\in X_2 \end{cases}$. Then we have

$$\mu_{\oplus}(a_1,b_1,e_1)=\mu_{\oplus}(b_1,a_1,e_1)=1$$

and

$$\mu_{\odot}(a_2,b_2,e_2)=\mu_{\odot}(b_2,a_2,e_2)=1.$$

Since $c=\begin{cases} c_1 \text{ if } c\in X_1 \\ c_2 \text{ if } c\in X_2 \end{cases}$ is inverse of $a=\begin{cases} a_1 \text{ if } a\in X_1 \\ a_2 \text{ if } a\in X_2 \end{cases}$, we also see that

$$\mu_{\oplus}(a_1,c_1,e_1)=\mu_{\oplus}(c_1,a_1,e_1)=1$$

and

$$\mu_{\odot}(a_2,c_2,e_2)=\mu_{\odot}(c_2,a_2,e_2)=1.$$

Thus, by cancellation law, we see that

$$\min[\mu_{\oplus}(a_1,b_1,e_1),\mu_{\oplus}(a_1,c_1,e_1)] \le E_{X_1}(b_1,c_1)$$

and

$$\min[\mu_{\odot}(a_2,b_2,e_2),\mu_{\odot}(a_2,c_2,e_2)] \le E_{X_2}(b_2,c_2).$$

Then, we obtain that

$$E_1(b_1,c_1)=1 \text{ and } E_2(b_2,c_2)=1.$$

So we have $b=c$ and hence the identity of $(X,\oplus,\odot)$ is unique.

(3) Let $e=\begin{cases} e_1 \text{ if } e\in X_1 \\ e_2 \text{ if } e\in X_2 \end{cases}$ be identities in $X=X_1\cup X_2$. Suppose that $a=\begin{cases} a_1 \text{ if } a\in X_1 \\ a_2 \text{ if } a\in X_2 \end{cases}$ has a unique inverse $a^*=\begin{cases} \oplus^{-1}(a_1) \text{ if } a\in X_1 \\ \odot^{-1}(a_2) \text{ if } a\in X_2 \end{cases}$. Then we note that we note that

$$\mu_{\oplus}(\oplus^{-1}(a_1),a_1,e_1)=1=\mu_{\oplus}(\oplus^{-1}(a_1),\oplus^{-1}\oplus^{-1}(a_1),e_1)$$

and

$$\mu_{\bullet}(\odot^{-1}(a_2),a_2,e_2)=1=\mu_{\bullet}(\odot^{-1}(a_2),\odot^{-1}\odot^{-1}(a_2),e_2).$$

Thus, by cancellation law, we have

$$\min[\mu_{\oplus}(\oplus^{-1}(a_1),a_1,e_1),\mu_{+}(\oplus^{-1}(a_1),\oplus^{-1}\oplus^{-1}(a_1),e_1)] \le E_{X_1}(a_1,\oplus^{-1}\oplus^{-1}(a_1))$$

and

$$\min[\mu_{\odot}(\odot^{-1}(a_2),a_2,e_2),\mu_{\bullet}(\odot^{-1}(a_2),\odot^{-1}\odot^{-1}(a_2),e_2)] \le E_{X_2}(a_2,\odot^{-1}\odot^{-1}(a_2)).$$

Then, we obtain that

$$E_1(a_1,\oplus^{-1}\oplus^{-1}(a_1))=1 \text{ and } E_2(a_2,\odot^{-1}\odot^{-1}(a_2))=1.$$

So we have $\oplus^{-1}\oplus^{-1}(a_1)=a_1$ and $\odot^{-1}\odot^{-1}(a_2)=a_2$.

From Proposition 2.3, we can obtain the following proposition.

Proposition 2.7 If $(X_1,\oplus)$ is a semigroup and $\oplus$ is a transtive of the second and third order, and if $(X_1,\odot)$ is a semigroup and $\odot$ is a transtive of the second and third order, then $(X,\oplus,\odot)$ is a vague bigroup.

3. Fuzzy bi-functions.

In this section, we define fuzzy bi-functions and investigate some characterizations of them.

Definition 3.1 Let X be crisp sets. If there exist two proper subsets X_1 and X_2 such that

(i) $X_1\cup X_2=X$,

(ii) X_1 is closed under $\oplus$,

(iii) X_2 is closed under $\odot$,

then X is called a crisp bi-set.

Example 3.2 Let $\mathbb{Q},\mathbb{Q}^+,\mathbb{Q}^-$ be the set of rational numbers, the set of nonnegative rational integers, the set of negative integers, respectively. Then $(\mathbb{Q},+,\bullet)$ is a crisp bi-set. In fact, $\mathbb{Q}=\mathbb{Q}^+\cup\mathbb{Q}^-$ and $\mathbb{Q}^+$ is closed under the usual addition $+$ and $\mathbb{Q}^-$ is closed under the usual multiplication $\bullet$.

Definition 3.3 Let X be a crisp biset with $X=X_1\cup X_2$. A mapping $E_X: X\times X\to[0,1]$ is called a fuzzy bi-equality on X if it satisfies the following conditions:

(i) $E_{X_1}(x_1,y_1)=1$ and $E_{X_2}(x_2,y_2)=1$ if and only if $x_1=y_1$ and $x_2=y_2$, where $x=\begin{cases} x_1 \text{ if } x\in X_1 \\ x_2 \text{ if } x\in X_2 \end{cases}$ and $y=\begin{cases} y_1 \text{ if } y\in Y_1 \\ y_2 \text{ if } y\in Y_2 \end{cases}$,

(ii) $E_{X_1}(x_1,y_1)=E_{X_1}(y_1,x_1)$ and $E_{X_2}(x_2,y_2)=E_{X_2}(y_2,x_2)$,

(iii) $\min[E_{X_1}(x_1,y_1),E_{X_1}(y_1,z_1))] \le E_{X_1}(x_1,z_1)$

and

$$\min[E_{X_2}(x_2,y_2),E_{X_2}(y_2,z_2))] \le E_{X_2}(x_2,z_2)$$

where $x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases}$, $y=\begin{cases} y_1 & \text{if } y \in Y_1 \\ y_2 & \text{if } y \in Y_2 \end{cases}$, and $z=\begin{cases} z_1 & \text{if } z \in Z_1 \\ z_2 & \text{if } z \in Z_2 \end{cases}$.

Now, we consider the following notation:

$$X \otimes Y \equiv (X_1 \times Y_1) \cup (X_2 \times Y_2)$$

where $X_i \times Y_i$ is the Cartesian product of X_i and Y_i for $i=1,2$.

Definition 3.4 Let X and Y be crisp bi-sets with $X=X_1 \cup X_2$ and $Y=Y_1 \cup Y_2$, respectively and let E_X and E_Y be three fuzzy bi-equalities on X and Y, respectively.

(1) $f: X \to Y$ is called a fuzzy bi-function with respect to E_X and E_Y if it satisfies the following conditions:

(I) there exist two fuzzy functions f_1 and f_2 of f such that $f=f_1 \cup f_2$,

(II) the membership function $\mu_f: X \otimes Y \to [0,1]$ of f satisfies the following conditions:

(i) for all $x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases}$, $\exists\, y=\begin{cases} y_1 & \text{if } y \in Y_1 \\ y_2 & \text{if } y \in Y_2 \end{cases}$ such that $\mu_{f_1}(x_1,y_1)>0$ and $\mu_{f_2}(x_2,y_2)>0$,

(ii) $\min[\mu_{f_1}(x_1,z_1),\mu_{f_1}(y_1,w_1),E_{X_1}(x_1,y_1)] \le E_{Y_1}(z_1,w_1)$

and

$$\min[\mu_{f_2}(x_2,z_2),\mu_{f_2}(y_2,w_2),E_{X_2}(x_2,y_2)] \le E_{Y_2}(z_2,w_2)$$

where $x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases}$, $y=\begin{cases} y_1 & \text{if } y \in X_1 \\ y_2 & \text{if } y \in X_2 \end{cases}$, $w=\begin{cases} w_1 & \text{if } w \in Y_1 \\ w_2 & \text{if } w \in Y_2 \end{cases}$ and $z=\begin{cases} z_1 & \text{if } z \in Y_1 \\ z_2 & \text{if } z \in Y_2 \end{cases}$.

(2) A fuzzy bi-function f is called a strongy fuzzy bifunction if it satisfies the following additional condition:

(iii) for all $x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases}$, $\exists\, y=\begin{cases} y_1 & \text{if } y \in Y_1 \\ y_2 & \text{if } y \in Y_2 \end{cases}$ such that $\mu_{f_1}(x_1,y_1)=1$ and $\mu_{f_2}(x_2,y_2)=1$.

Definition 3.5 Let X and Y be crisp bisets and let $f=f_1 \cup f_2$ be a fuzzy bi-function with respect to E_X on X and E_Y on Y.

(1) A fuzzy bi-function f is said to be surjective if and only if

$\forall\ y=\begin{cases} y_1 & \text{if } y \in Y_1 \\ y_2 & \text{if } y \in Y_2 \end{cases}$, $\exists\ x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases}$ such that

$$\mu_{f_1}(x_1,y_1)>0 \text{ and } \mu_{f_2}(x_2,y_2)>0.$$

(2) A fuzzy bi-function f is said to be strong surjective if and only if

$\forall\ y=\begin{cases} y_1 & \text{if } y \in Y_1 \\ y_2 & \text{if } y \in Y_2 \end{cases}$, $\exists\ x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases}$ such that

$$\mu_{f_1}(x_1,y_1)=1 \text{ and } \mu_{f_2}(x_2,y_2)=1.$$

(3) A fuzzy bi-function f is said to be injective if and only if

$$\forall\ x=\begin{cases} x_1 & \text{if } x \in X_1 \\ x_2 & \text{if } x \in X_2 \end{cases},\ y=\begin{cases} y_1 & \text{if } y \in X_1 \\ y_2 & \text{if } y \in X_2 \end{cases},$$

$$\forall\ w=\begin{cases} w_1 & \text{if } w \in Y_1 \\ w_2 & \text{if } w \in Y_2 \end{cases},\ z=\begin{cases} z_1 & \text{if } z \in Y_1 \\ z_2 & \text{if } z \in Y_2 \end{cases},$$

$$\min[\mu_{f_1}(x_1,z_1),\mu_{f_1}(y_1,w_1),E_{Y_1}(z_1,w_1)] \le E_{X_1}(z_1,y_1)$$

and

$$\min[\mu_{f_2}(x_2,z_2),\mu_{f_2}(y_2,w_2),E_{X_2}(x_2,y_2)] \le E_{Y_2}(z_2,w_2).$$

(4) A fuzzy bi-function f is said to be bijective if and only if it is surjective and injective.

(5) A fuzzy bi-function f is said to be strong bijective if and only if it is strong surjective and injective.

Definition 3.6 Let $X=X_1 \cup X_2$ be a crisp bi-set. The fuzzy bi-relation $U=U_1 \cup U_2$ on $X \otimes X$ defined by

$$\mu_{U_1}(x_1,y_1)=\begin{cases} 0 & \text{if } x_1=y_1 \\ 1 & \text{if } x_1 \ne y_1 \end{cases},\quad (x_1,y_1) \in X_1 \times X_1$$

and

$$\mu_{U_2}(x_2,y_2)=\begin{cases} 0 & \text{if } x_2=y_2 \\ 1 & \text{if } x_2 \ne y_2 \end{cases},\quad (x_2,y_2) \in X_2 \times X_2$$

is called a unit fuzzy bi-function on $X=X_1 \cup X_2$ and is denoted by $U_X=U_{X_1} \cup U_{X_2}$.

Definition 3.7 Let $X=X_1 \cup X_2$, $Y=Y_1 \cup Y_2$, and $Z=Z_1 \cup Z_2$ be crisp bi-sets and let $R=R_1 \cup R_2$, $K=K_1 \cup K_2$, and $S=S_1 \cup S_2$ be fuzzy bi-relations on $X \otimes Y$, $Y \otimes X$, and $Y \otimes Z$, respectively.

(1) The sup-min composition $R \circ S$ of R and S on $X \otimes Z$ is defined by a fuzzy bi-relation with the membership function

$$\mu_{R \circ S}=\mu_{R_1 \circ S_1} \cup \mu_{R_2 \circ S_2}: X \otimes Z \to I$$

given by

$$\mu_{R_1 \circ S_1}(x_1,z_1) = \sup_{y_1 \in Y_1}[\min[\mu_{R_1}(x_1,y_1),\mu_{S_1}(y_1,z_1)]]$$

for $i=1,2$.

(2) The fuzzy bi-relation K is said to be the inverse of the fuzzy bi-relation R if and only if

$$\mu_K(y,x)=\mu_R(x,y),\ \forall (x,y) \in X\otimes Y$$

where $\mu_K(\cdot,\cdot)=\mu_{K_1}(\cdot,\cdot)\cup\mu_{K_2}(\cdot,\cdot)$.

We note that $\mu_K(y,x)=\mu_R(x,y),\ \forall (x,y) \in X\otimes Y$ if and only if

$$\mu_{K_1}(y,x)=\mu_{R_1}(x,y),\ \forall (x,y) \in X_1\otimes Y_1$$

and

$$\mu_{K_2}(y,x)=\mu_{R_2}(x,y),\ \forall (x,y) \in X_2\otimes Y_2.$$

From the above definition, we can obatin the following two propositions.

Proposition 3.8 Let $X=X_1\cup X_2, Y=Y_1\cup Y_2$, and $Z=Z_1\cup Z_2$ be crisp bi-sets and $f=f_1\cup f_2: X\to Y$ be a fuzzy bi-function on $X\otimes Y$ and $g=g_1\cup g_2: Y\to Z$ be fuzzy bi-function $Y\otimes Z$ on with respect to $E_X=E_{X_1}\cup E_{X_2}$ on X and $E_Y=E_{Y_1}\cup E_{Y_2}$ on Y, and $E_Y=E_{Y_1}\cup E_{Y_2}$ on Y and $E_Z=E_{Z_1}\cup E_{Z_2}$ on Z, respectively. Then the sup-min composition $g\circ f$ of f and g is a fuzzy bi-function on $X\otimes Z$ with respect to $E_X=E_{X_1}\cup E_{X_2}$ on X and $E_Z=E_{Z_1}\cup E_{Z_2}$ on Z.

Proof. Let $\mu_{g\circ f}: X\otimes Z\to[0,1]$ be the membership function of $g\circ f$. Then we have for each $x=\begin{cases}x_1 & \text{if } x\in X_1\\ x_2 & \text{if } x\in X_2\end{cases}$ and $z=\begin{cases}z_1 & \text{if } z\in Z_1\\ z_2 & \text{if } z\in Z_2\end{cases}$,

$$\begin{aligned}\mu_{g\circ f}(x,z) &= \sup_{y\in Y}[\min(\mu_g(y,z),\mu_f(x,y))]\\ &= \sup_{y\in Y}[\min[\min(\mu_{g_1}(y_1,z_1),\mu_{g_2}(y_2,z_2)),\\ &\qquad \min(\mu_{f_1}(x_1,y_1),\mu_{f_2}(x_2,y_2))]]\\ &= \sup_{y_1\in Y_1}[\min(\mu_{g_1}(y_1,z_1),\mu_{f_1}(x_1,y_1))]\vee\\ &\quad \sup_{y_2\in Y_2}[\min(\mu_{g_2}(y_2,z_2),\mu_{f_2}(x_2,y_2))]\\ &= \mu_{g_1\circ f_1}(x_1,z_1)\vee\mu_{g_2\circ f_2}(x_2,z_2).\end{aligned}$$

That is, $g\circ f=(g_1\circ f_1)\cup(g_2\circ f_2)$.

Therefore, $g\circ f$ satisfies the condition (I) of Definition 3.4. Since $g_1\circ f_1$ and $g_2\circ f_2$ are fuzzy functions,

$$\min[\mu_{g_1\circ f_1}(x_1,z_1),\mu_{g_1\circ f_1}(y_1,w_1),E_{X_1}(x_1,y_1)] \le E_{Z_1}(z_1,w_1)$$

and

$$\min[\mu_{g_2\circ f_2}(x_2,z_2),\mu_{g_2\circ f_2}(y_2,w_2),E_{X_2}(x_2,y_2)] \le E_{Z_2}(z_2,w_2).$$

Thus,

$$\min[\mu_{g\circ f}(x,z),\mu_{g\circ f}(y,w),E_X(x,y)] \le E_Z(z,w).$$

That is, $g\circ f$ satisfies the condition (II) of Definition 3.4. Therefore, $g\circ f$ is a fuzzy bi-function.

Proposition 3.9 Let $X=X_1\cup X_2$ and $Y=Y_1\cup Y_2$ be crisp bi-sets and $f=f_1\cup f_2: X\to Y$ be a fuzzy bi-function on $X\otimes Y$ with respect to $E_X=E_{X_1}\cup E_{X_2}$ on X and $E_Y=E_{Y_1}\cup E_{Y_2}$ on Y. If f is bijective, then we have the inverse bi-relation $f^{-1}=f_1^{-1}\cup f_2^{-1}$ of $f=f_1\cup f_2$ is a fuzzy bi-function on $Y\otimes X$.

Proof. The proof is clear!

Proposition 3.10 Let $X=X_1\cup X_2, Y=Y_1\cup Y_2$, and $Z=Z_1\cup Z_2$ be crisp bi-sets and $f=f_1\cup f_2: X\to Y$ on $X\otimes Y$ and $g=g_1\cup g_2: Y\to Z$ be fuzzy bi-functions on $Y\otimes Z$ with respect to $E_X=E_{X_1}\cup E_{X_2}$ on X and $E_Y=E_{Y_1}\cup E_{Y_2}$ on Y, and $E_Y=E_{Y_1}\cup E_{Y_2}$ on Y and $E_Z=E_{Z_1}\cup E_{Z_2}$ on Z, respectively. If f and g are bijective, then

$$(g\circ f)^{-1}=f^{-1}\circ g^{-1}.$$

Proof. The proof is clear!

References

[1] 김태균, 박달원, 박홍경, 임석훈, 장이채, *대수학 입문과 퍼지 응용*, 교우사, 2003.

[2] A.A.A. Agboola and L.S. Akinola, “On Bicoset of a Bivector space”, Preprint.

[3] W.B. Vasantha Kandasamy, “Bivector spaces”,” *U. Sci. Phy. Sci.* vol. 11, pp.186-190.

[4] W.B. Vasantha Kadasamy, “Bialgebaric structures and smarandache bialgebraic structures”, *American Research Press Rehoboth*, 2003.

[5] M. Demirci, “Vague groups”, *J. of Math. Anal. and Appl.* vol.230, pp.142-156, 1999.

[6] S. Sezer, “Vague groups and generalized vague subgroups on the basis of $([0,1],\le,\wedge)$”, *Information Sciences,* vol.174, pp.123-142, 2003.

[7] M. Demirci, “The generalized associative law in vague groups and its applications II”, *Information Sciences*, vol.176, pp.1488-1530, 2006.

[8] L.A. Zadeh, "Fuzzy sets", *Inform. Control,* vol.8, pp.338-353, 1968.

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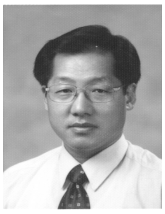

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