Some Properties on Intuitionistic Fuzzy Metric Space

Jong Seo Park*, **Young Chel Kwun**** **and Jin Han Park*****

*Department of Mathematic Education, Chinju National University of Education,
Jinju 660-756, South Korea
**Department of Mathematics, Dong-A University,
Pusan 604-714, South Korea
***Division of Mathematical Sciences, Pukyong National University,
Pusan 608-737, South Korea

Abstract

We define some terminologies on intuitionistic fuzzy metric space and prove that the topology generated by any intuitionistic fuzzy metric space is metrizable. Also, we show that if the intuitionistic fuzzy metric space is complete, then the generated topology is completely metrizable, a Baire space, and that an intuitionistic fuzzy metric space is precompact if and only if every sequence has a Cauchy subsequence.

Key Words: intuitionistic fuzzy metric space, metrizable, complete, compact, precompact, Baire space.

1. Introduction

Many authors studied an appropriate and consistent notion of a fuzzy metric space introduced by Kramosil and Michalek[6]. Also, they investigated this problem and several different notions of a fuzzy metric space have been defined and studied. George and Veeramani([3],[4]) studied an interesting concept of fuzzy metric space, and proved that every fuzzy metric space generates a Hausdorff first countable topology. Also, Gregori and Romaguera[2] proved that the topology generated by any fuzzy metric space is metrizable, and show that if the fuzzy metric space is complete, then the generated topology is completely metrizable.

Recently, Park et.al.[8] defined the intuitionistic fuzzy metric space, and Park etc([8],[9],[11],[12],[13],[14]) defined some notions and studied the several properties on intuitionistic fuzzy metric space.

In this paper, we define some terminologies on intuitionistic fuzzy metric space and prove that the topology generated by any intuitionistic fuzzy metric space is metrizable. Also, we show that if the intuitionistic fuzzy metric space is complete, then the generated topology is completely metrizable, a Baire space, and that an intuitionistic fuzzy metric space is precompact if and only if every sequence has a Cauchy subsequence.

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**Corresponding Author: Young Chel Kwun, yckwun@dau.ac.kr

2. Preliminaries

Throughout this paper, **N** denote the set of all positive integers. Now, we begin with some definitions, properties in intuitionistic fuzzy metric space as following:

Let us recall(see [15]) that a continuous $t-$norm is a operation $* : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (a)$*$ is commutative and associative, (b)$*$ is continuous, (c)$a * 1 = a$ for all $a \in [0,1]$, (d)$a * b \le c * d$ whenever $a \le c$ and $b \le d$ $(a,b,c,d \in [0,1])$. Also, a continuous $t-$conorm is a operation $\diamond : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (a)$\diamond$ is commutative and associative, (b)$\diamond$ is continuous, (c)$a \diamond 0 = a$ for all $a \in [0,1]$, (d)$a \diamond b \ge c \diamond d$ whenever $a \le c$ and $b \le d$ $(a,b,c,d \in [0,1])$.

Furthermore, from ([7]), the following conditions are satisfied: (a)For any $r_1, r_2 \in (0,1)$ with $r_1 > r_2$, there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \ge r_2$ and $r_4 \diamond r_2 \le r_1$. (b)For any $r_5 \in (0,1)$, there exist $r_6, r_7 \in (0,1)$ such that $r_6 * r_6 \ge r_5$ and $r_7 \diamond r_7 \le r_5$.

Definition 2.1. ([8])The $5-$tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous $t-$norm, $\diamond$ is a continuous $t-$conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that

(a)$M(x,y,t) > 0$,

(b)$M(x,y,t) = 1 \Longleftrightarrow x = y$,

(c)$M(x,y,t) = M(y,x,t)$,
(d)$M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,
(e)$M(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous,
(f)$N(x,y,t) > 0$,
(g)$N(x,y,t) = 0 \iff x = y$,
(h)$N(x,y,t) = N(y,x,t)$,
(i)$N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$,
(j)$N(x,y,\cdot) : (0,\infty) \to (0,1]$ is continuous.

Note that (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.2. ([14])Let (X,d) be a metric space. Denote $a * b = ab$ and $a \diamond b = \max\{a,b\}$ for all $a,b \in [0,1]$ and let M_d, N_d be functions defined on $X^2 \times (0,\infty)$ by

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}, \quad N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}. \quad (1)$$

Then $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric space. We call this M_d, N_d as the standard intuitionistic fuzzy metric induced by the metric d.

Definition 2.3. ([14]) Let X be an intuitionistic fuzzy metric space and let $0 < r < 1$, $t > 0$ and $x \in X$. The set

$$B(x,r,t) = \{y \in X; M(x,y,t) > 1-r, N(x,y,t) < r\}$$

is called the open ball with center $x \in X$ and radius r.

Let X be an intuitionistic fuzzy metric space. Define

$$\begin{aligned} \tau &= \{A \subset X; x \in A \iff \text{there exist } r,t > 0, \\ &\qquad 0 < r < 1 \text{ such that } B(x,r,t) \subset A\}, \end{aligned}$$

then τ is a topology on X. Clearly, this topology is Hausdorff and first countable([3]).

Theorem 2.4. Every intuitionistic fuzzy metric space is Hausdorff space.

Proof. Let X be the given intuitionistic fuzzy metric space and let x,y be two distinct points of X. Then $0 < M(x,y,t) < 1$, $0 < N(x,y,t) < 1$. If $M(x,y,t) = r$, $N(x,y,t) = 1-r$ for some r, $0 < r < 1$, then for each r_0, $0 < r_0 < 1$, there exist r_1 from the above result([7]), such that $r_1 * r_1 \geq r_0$, $r_1 \diamond r_1 \leq r_0$. Now, consider the open balls $B(x, 1-r_1, \frac{t}{2})$ and $B(y, 1-r_1, \frac{t}{2})$. Then we will show that $B(x,1-r_1,\frac{t}{2}) \cap B(y,1-r_1,\frac{t}{2}) = \phi$. If there exist $z \in B(x,1-r_1,\frac{t}{2}) \cap B(y,1-r_1,\frac{t}{2})$, then

$$\begin{aligned} r &= M(x,y,t) \\ &\geq M(x,z,\frac{t}{2}) * M(z,y,\frac{t}{2}) \\ &\geq r_1 * r_1 \geq r_0 > r, \\ 1-r &= N(x,y,t) \\ &\leq N(x,z,\frac{t}{2}) \diamond N(z,y,\frac{t}{2}) \\ &\leq (1-r_1) \diamond (1-r_1) \\ &\leq (1-r_0) \diamond (1-r_0) < 1-r \end{aligned}$$

which is a contradiction. Therefore X is Hausdorff space. □

Lemma 2.5. ([5])A T_1 topological space (X,τ) is metrizable if and only if it has an uniformly with a countable base.

Lemma 2.6. ([3])Let X be an intuitionistic fuzzy metric space. Then τ is a Hausdorff topology and for each $x \in X$, $\{B(x,\frac{1}{n},\frac{1}{n}); n \in \mathbf{N}\}$ is a neighborhood base at x for the topology τ.

3. Main Results

Theorem 3.1. Let X be an intuitionistic fuzzy metric space. Then X is a metrizable topological space.

Proof. For each $n \in \mathbf{N}$, define

$$\begin{aligned} U_n &= \{(x,y) \in X \times X; M(x,y,\frac{1}{n}) > 1-\frac{1}{n}, \\ &\qquad N(x,y,\frac{1}{n}) < \frac{1}{n}\}. \end{aligned}$$

We shall prove that $\{U_n; n \in \mathbf{N}\}$ is a base for uniformity U on X. For each $n \in \mathbf{N}$, $\{(x,x); x \in X\} \subseteq U_n$, also, since

$$\begin{aligned} M(x,y,\tfrac{1}{n}) > M(x,y,\tfrac{1}{n+1}) > 1-\tfrac{1}{n+1} > 1-\tfrac{1}{n}, \\ N(x,y,\tfrac{1}{n}) < N(x,y,\tfrac{1}{n+1}) < \tfrac{1}{n+1} < \tfrac{1}{n} \end{aligned}$$

and $M(x,y,\frac{1}{n}) = M(y,x,\frac{1}{n})$, $N(x,y,\frac{1}{n}) = N(y,x,\frac{1}{n})$, hence $U_{n+1} \subseteq U_n$ and $U_n = U_n^{-1}$.

On the other hand, for each that $n \in \mathbf{N}$, there is, by the continuity of $*$ and $\diamond$, an $m \in \mathbf{N}$ such that $m \geq 2n$ and

$$(1-\frac{1}{m}) * (1-\frac{1}{m}) > 1-\frac{1}{n}, \quad \frac{1}{m} \diamond \frac{1}{m} < \frac{1}{n}.$$

Let $(x,y),(y,z) \in U_m$. Then

$$\begin{aligned} M(x,z,\frac{1}{n}) &\geq M(x,z,\frac{2}{m}) \\ &\geq M(x,y,\frac{1}{m}) * M(y,z,\frac{1}{m}) \\ &\geq (1-\frac{1}{m}) * (1-\frac{1}{m}) > 1-\frac{1}{n}, \\ N(x,z,\frac{1}{n}) &\leq N(x,z,\frac{2}{m}) \\ &\leq N(x,y,\frac{1}{m}) \diamond N(y,z,\frac{1}{m}) \\ &\leq \frac{1}{m} \diamond \frac{1}{m} < \frac{1}{n}. \end{aligned}$$

Therefore $(x,z) \in U_n$ and $U_m \circ U_n \subseteq U_n$. Thus $\{U_n; n \in \mathbf{N}\}$ is a base for a uniformity U on X. Since for each $x \in X$ and $n \in N$,

$$\begin{aligned} &U_n(x) \\ &= \{y \in X; M(x,y,\frac{1}{n}) > 1-\frac{1}{n},\ N(x,y,\frac{1}{n}) < \frac{1}{n}\} \\ &= B(x,\frac{1}{n},\frac{1}{n}), \end{aligned}$$

we deduce that the topology induced by U coincides with τ. By Lemma 2.5, X is metrizable topological space. □

Theorem 3.2. Every separable intuitionistic fuzzy metric space is second countable space.

Proof. Let X be the given separable intuitionistic fuzzy metric space and let $A = \{a_n; n \in N\}$ be a countable dense subset of X. Consider $B = \{B(a_j, \frac{1}{k}, \frac{1}{k}); j,k \in \mathbf{N}\}$. Then B is countable. Now, we prove that B is a basis for the family of all open sets in X. Let G be an arbitrary open set in X and $x \in G$, then there exists $r,t > 0$, $0 < r < 1$ such that $B(x,r,t) \subset G$. Since $r \in (0,1)$, there exist $s \in (0,1)$ from the above result([7]), such that $(1-s)*(1-s) > 1-r$, $s \diamond s < r$. Choosing $m \in \mathbf{N}$ such that $\frac{1}{m} < \min\{s, \frac{t}{2}\}$, since A is dense in X, there exists $a_j \in A$ such that $a_j \in B(x, \frac{1}{m}, \frac{1}{m})$.

Now, if $y \in B(x, \frac{1}{m}, \frac{1}{m})$, then

$$\begin{aligned} M(x,y,t) &\geq M(x,a_j,\frac{t}{2}) * M(a_j,y,\frac{t}{2}) \\ &\geq M(x,a_j,\frac{1}{m}) * M(a_j,y,\frac{1}{m}) \\ &\geq (1-\frac{1}{m}) * (1-\frac{1}{m}) \\ &\geq (1-s)*(1-s) > 1-r, \\ N(x,y,t) &\leq N(x,a_j,\frac{t}{2}) \diamond N(a_j,y,\frac{t}{2}) \\ &\leq N(x,a_j,\frac{1}{m}) \diamond N(a_j,y,\frac{1}{m}) \\ &\leq \frac{1}{m} \diamond \frac{1}{m} \leq s \diamond s < r. \end{aligned}$$

Thus, $y \in B(x,r,t)$ and hence B is a basis for a topology τ. By Theorem 3.1, X is a separable metrizable space. Hence X is second countable space([1]). □

Definition 3.3. Let X be an intuitionistic fuzzy metric space.

(a) A sequence $\{x_n\} \subset X$ is convergent to x in X if $\lim_{n\to\infty} M(x_n,x,t) = 1$, $\lim_{n\to\infty} N(x_n,x,t) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\} \subset X$ is called Cauchy sequence if for each r, $0 < r < 1$ and $t > 0$, there exists $n_0 \in \mathbf{N}$ such that $M(x_m,x_n,t) > 1-r$, $N(x_m,x_n,t) < r$ for all $m,n \geq n_0$.

(c) X is complete if every Cauchy sequence is convergent.

(d) We say that M,N are complete intuitionistic fuzzy metric on X if X is a complete intuitionistic fuzzy metric space.

(e)X is a Baire space if the intersection of a countable number of dense open sets is dense in X

(f) X is called precompact if for each $t > 0$ and r with $0 < r < 1$, there is a finite subset A of X such that $X = \cup_{a\in A} B(a,r,t)$.

In this case, M,N are precompact intuitionistic fuzzy metric on X.

Theorem 3.4. Let X be a complete intuitionistic fuzzy metric space. Then X is completely metrizable space.

Proof. Let X be the given intuitionistic fuzzy metric space. From Theorem 3.1, since $\{U_n; n \in \mathbf{N}\}$ is a base for a uniformity U on X, where

$$\begin{aligned} U_n &= \{(x,y) \in X \times X; M(x,y,\frac{1}{n}) > 1-\frac{1}{n}, \\ &\qquad N(x,y,\frac{1}{n}) < \frac{1}{n}\} \end{aligned}$$

for every $n \in \mathbf{N}$. Then there exists M,N on X whose induced uniformity coincides with U. For given a Cauchy sequence $\{x_n\}_{n\in N}$ in X, fixed r,t with $0 < r < 1$ and $t > 0$. Choosing $k \in \mathbf{N}$ such that $\frac{1}{k} \leq \min\{t,r\}$. Then there exist $n_0 \in \mathbf{N}$ such that $(x_m,x_n) \in U_k$ for every $m,n \geq n_0$. That is, for each $m,n \geq n_0$,

$$M(x_m,x_n,t) \geq M(x_m,x_n,\frac{1}{k}) > 1-\frac{1}{k} \geq 1-r,$$
$$N(x_m,x_n,t) \leq N(x_m,x_n,\frac{1}{k}) < \frac{1}{k} \leq r.$$

Hence $\{x_n\}_{n\in\mathbf{N}}$ is a Cauchy sequence in complete intuitionistic fuzzy metric space X. So, $\{x_n\}$ converges to some point in X. Hence M,N are complete intuitionistic fuzzy metric on X. Therefore X is completely metrizable. □

Theorem 3.5. Let X be a complete intuitionistic fuzzy metric space. Then every complete space X is a Baire space.

Proof. Let X be given complete intuitionistic fuzzy metric space and$B_0 (\neq \phi)$ be open set. Also, let $D_1, D_2, \cdots$ be dense open sets in X. Then $B_0 \cap D_1 \neq \phi$ from $\overline{D}_1 = X$. Let $x_1 \in B_0 \cap D_1$, there exists $t_1 > 0, 0 < r_1 < 1$ such that $B(x_1,r_1,t_1) \subset B_0 \cap D_1$. Choosing $r_1' < r_1$ and $t_1' = \min\{t_1, 1\}$ such that

$$B(x_1,r_1',t_1') \subset B_0 \cap D_1.$$

Let $B_1 = B(x_1,r_1',t_1')$. Then $B_1 \cap D_2 \neq \phi$ from $\overline{D}_2 = X$. Let $x_2 \in B_1 \cap D_2$, there exists $t_2 > 0, 0 < r_2 < 1$ such that $B(x_2,r_2,t_2) \subset B_1 \cap D_2$. Choosing $r_2' < r_2$ and $t_2' = \min\{t_2, \frac{1}{2}\}$ such that

$$B(x_2,r_2',t_2') \subset B_1 \cap D_2.$$

Let $B_2 = B(x_2, r'_2, t'_2)$. Then by inductively methods, we can find $x_n \in B_{n-1} \cap D_n$. Hence there exists $t_n > 0$, $0 < r_n < \frac{1}{n}$ such that

$$B(x_n, r_n, t_n) \subset B_{n-1} \cap D_n.$$

Choosing $r'_n < r_n$ and $t'_n = \min\{t_n, \frac{1}{n}\}$ such that

$$B(x_n, r'_n, t'_n) \subset B_{n-1} \cap D_n.$$

Let $B_n = B(x_n, r'_n, t'_n)$. Now, we prove that $\{x_n\}$ is a Cauchy sequence. For given $t > 0$, $\epsilon > 0$, choosing $n_0 \in N$ such that $\frac{1}{n_0} < t$, $\frac{1}{n_0} < \epsilon$. Then for $m \geq n \geq n_0$,

$$M(x_m, x_n, t) \geq M(x_m, x_n, \frac{1}{n}) \geq 1 - \frac{1}{n} > 1 - \epsilon,$$
$$N(x_m, x_n, t) \leq N(x_m, x_n, \frac{1}{n}) \leq \frac{1}{n} < \epsilon.$$

Therefore $\{x_n\}$ is a Cauchy sequence by Definition 3.3. Since X is complete, $x_n \to x$ in X. But $x_k \in B(x_n, r'_n, t'_n)$ for all $k \geq n$ and $B(x_n, r'_n, t'_n)$ is a closed set. Hence $x \in B(x_n, r'_n, t'_n) \subset B_{n-1} \cap D_n$ for all $n \in \mathbf{N}$. Therefore $B_0 \cap (\cap_{n=1}^{\infty} D_n) \neq \phi$. Hence $\cap_{n=1}^{\infty} D_n$ is dense in X. Thus from Definition 3.3(e), X is Baire space. □

Theorem 3.6. An intuitionistic fuzzy metric space X is precompact if and only if every sequence in X has a Cauchy subsequence.

Proof. Suppose that X is a precompact intuitionistic fuzzy metric space. Let $\{x_n\}_{n \in N}$ be a sequence in X. For each $m \in N$, there is a finite subset A_m of X such that $X = \cup_{a \in A_m} B(a, \frac{1}{m}, \frac{1}{m})$. Hence, for $m = 1$, there exist an $a_1 \in A_1$ and $\{x_{1(n)}\}_{n \in \mathbf{N}} \subset \{x_n\}_{n \in \mathbf{N}}$ such that $x_{1(n)} \in B(a_1, 1, 1)$ for every $n \in \mathbf{N}$. Also, for $m = 2$, there exist an $a_2 \in A_2$ and $\{x_{2(n)}\}_{n \in \mathbf{N}} \subset \{x_{1(n)}\}_{n \in \mathbf{N}}$ such that $x_{2(n)} \in B(a_2, \frac{1}{2}, \frac{1}{2})$. By inductively methods, for $m \in \mathbf{N}$, $m > 1$, there exist an $a_m \in A_m$ and $\{x_{m(n)}\}_{n \in \mathbf{N}} \subset \{x_{m-1(n)}\}_{n \in \mathbf{N}}$ such that $x_{m(n)} \in B(a_m, \frac{1}{m}, \frac{1}{m})$ for every $n \in \mathbf{N}$.

Now, consider $\{x_{n(n)}\}_{n \in \mathbf{N}} \subset \{x_n\}_{n \in \mathbf{N}}$. Given $t > 0$ and r with $0 < r < 1$, there exist $n_0 \in \mathbf{N}$ such that $(1 - \frac{1}{n_0}) * (1 - \frac{1}{n_0}) > 1 - r$, $\frac{1}{n_0} \diamond \frac{1}{n_0} < r$ and $\frac{2}{n_0} < t$. Therefore

$$\begin{aligned} & M(x_{k(k)}, x_{m(n)}, t) \\ & > M(x_{k(k)}, x_{m(n)}, \frac{2}{n_0}) \\ & \geq M(x_{k(k)}, a_{n_0}, \frac{1}{n_0}) * M(a_{n_0}, x_{m(n)}, \frac{1}{n_0}) \\ & \geq (1 - \frac{1}{n_0}) * (1 - \frac{1}{n_0}) > 1 - r, \end{aligned}$$

$$\begin{aligned} & N(x_{k(k)}, x_{m(n)}, t) \\ & < N(x_{k(k)}, x_{m(n)}, \frac{2}{n_0}) \\ & \leq N(x_{k(k)}, a_{n_0}, \frac{1}{n_0}) \diamond N(a_{n_0}, x_{m(n)}, \frac{1}{n_0}) \\ & \leq \frac{1}{n_0} \diamond \frac{1}{n_0} < r. \end{aligned}$$

Hence $\{x_{n(n)}\}_{n \in \mathbf{N}}$ is a Cauchy sequence in X.

Conversely, suppose that X is a nonprecompact intuitionistic fuzzy metric space. Then there exist $t > 0$ and r with $0 < r < 1$, such that for each finite subset A of X, $X = \cup_{a \in A} B(a, r, t)$. Fix $x_1 \in X$, there exist $x_2 \in X - B(x_1, r, t)$. Moreover, there exist $x_3 \in X - \cup_{k=1}^{2} B(x_k, r, t)$. By inductively methods, we construct a sequence $\{x_n\}_{n \in \mathbf{N}}$ of distinct points in X such that $x_{n+1} \notin \cup_{k=1}^{n} B(x_k, r, t)$ for every $n \in \mathbf{N}$. Hence $\{x_n\}_{n \in \mathbf{N}}$ has no Cauchy sequence. Therefore if $\{x_n\}_{n \in \mathbf{N}}$ has Cauchy subsequence in X, then X is precompact intuitionistic fuzzy metric space. □

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Jong Seo Park

Professor of Chinju National University of Education
Research Area: Fuzzy mathematics, Fuzzy fixed point theory, Fuzzy differential equation
E-mail : parkjs@cue.ac.kr

Young Chel Kwun

Professor of Dong-A University
Research Area: Fuzzy mathematics, Fuzzy differential equations
E-mail : yckwun@dau.ac.kr

Jin Han Park

Professor of Pukyong National University
Research Area: Fuzzy mathematics, Fuzzy topology
E-mail : jihpark@pknu.ac.kr