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A Study on an Adaptive Robust Fuzzy Controller with GAs for Path Tracking of a Wheeled Mobile Robot

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Abstract

This paper proposes an adaptive robust fuzzy control scheme for path tracking of a wheeled mobile robot with uncertainties. The robot dynamics including the actuator dynamics is considered in this work. The presented controller is composed of a fuzzy basis function network (FBFN) to approximate an unknown nonlinear function of the robot complete dynamics, an adaptive robust input to overcome the uncertainties, and a stabilizing control input. Genetic algorithms are employed to optimize the fuzzy rules of FBFN. The stability and the convergence of the tracking errors are guaranteed using the Lyapunov stability theory. When the controller is designed, the different parameters for two actuator models in the dynamic equation are taken into account. The proposed control scheme does not require the accurate parameter values for the actuator parameters as well as the robot parameters. The validity and robustness of the proposed control scheme are demonstrated through computer simulations.

Key Words : Fuzzy basis function network, adaptive robust control, genetic algorithms, robot dynamics, actuator dynamics, uncertainty.

1. Introduction

In the past, many research results on the path tracking control problem for a wheeled mobile robot have been proposed. "*Perfect velocity tracking*" was put forward in Kanayama et al. [1] to solve this problem for the kinematic model. In Fierro and Lewis [2], a dynamic controller was presented to integrate into the kinematic controller. However, the controller assumed that the dynamic parameters have to be completely known. This requirement cannot be carried out in practical situations where it is very difficult to completely obtain the exact parameters of the model. Das and Kar [3] proposed an adaptive fuzzy controller to approximate a nonlinear function involving the robot dynamics so that no knowledge of the robot parameters could be required. In Das and Kar [3], although the actuator dynamics was taken into account, the parameters for the actuators were still required and the same parameters for the right and left actuator models were also used.

The fuzzy basis function network with a powerful competence for uniformly approximating any nonlinear function over compact input space has been suggested by many researchers, as shown in Das and Kar [3], Wang and Mendel [4], and Wang [7]. Although the previous robot controllers have showed good performance in many simulations and experiments, few literatures on the robustness of the controller against parameter variations and disturbances have been discussed. Shin et al. [5] and Kim et al. [6] presented a robust adaptive controller for robots and showed the robustness to uncertainties. Depending on the number of inputs and linguistic degrees of control variable, a lot of logic rules are produced. In many case they are potential redundant rules that not only have no effect on fuzzy inference but also make the system response worse. Therefore, it is necessary to optimize those fuzzy rules. Probabilistic optimization methods, such as GAs have been proven suitable for selecting those fuzzy rules as presented in Pishkenari et al. [8], Herrera et al. [9], and Lekova et al. [10].

In this paper, we establish a new control scheme so that a wheeled mobile robot can track the desired reference path asymptotically against uncertainties. The scheme is based on the structure of FBFN and employs the adaptive and genetic algorithm techniques. The actuator dynamics for the two wheels of a wheeled mobile robot are included in the robot dynamic model, and the accurate parameters for the actuator parameters as well as the robot parameters are not required in the proposed controller.

2. A Wheeled Mobile Robot System

2.1 Kinematics and dynamics of a wheeled mobile robot

The pose of a wheeled mobile robot in the global reference coordinate frame $\{O, X, Y\}$ is completely specified by the generalized coordinates $q = [x_c \quad y_c \quad \theta]^T$, where x_c and y_c are the coordinates of the point C of the center of mass (COM) with respect to the global reference coordinate frame and θ is the orientation of the local frame $\{C, X_c, Y_c\}$ attached on the robot platform measured from X axis of the

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global reference coordinate frame. A wheeled mobile robot and the coordinate frames are shown in Fig. 1.

A nonholonomic mobile robot system having a n dimensional configuration space $\mathbf{C}$ with n generalized configuration variables $(q_1, q_2, ..., q_n)$ and subject to m constraints can be described by

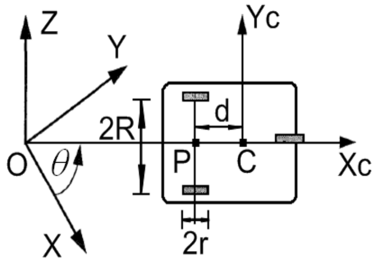

Fig. 1 A nonholonomic wheeled mobile robot

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + \tau_d = B(q)\tau - A(q)\lambda \quad (1)$$

where $M(q) \in \Re^{n\times n}$ is a symmetric positive definite inertia matrix, $V_m(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal and coriolis matrix, $\tau_d \in \Re^{n\times 1}$ denotes bounded unknown disturbances including unmodeled dynamics, $B(q) \in \Re^{n\times r}$ is the input transformation matrix, $\tau \in \Re^{r\times 1}$ is the input torque vector, $A(q) \in \Re^{n\times m}$ is the matrix associated with the constraints, and $\lambda \in \Re^{m\times 1}$ is the vector of constraint forces.

The complete equations of motion of a wheeled mobile robot can be rewritten as

$$\dot{q} = S(q)v(t) \quad (2)$$

$$\bar{M}\dot{v} + \bar{V}_m v + \bar{\tau}_d = \bar{B}\tau \quad (3)$$

Wher $\bar{M} = S^T MS$, $\bar{V}_m = S^T(M\dot{S} + V_m S)$, $\bar{\tau}_d = S^T \tau_d$, $\bar{B} = S^T B$, and the wheel torque vector $\tau = [\tau_r \quad \tau_l]^T$. The actual velocity vector v is written by $v = [v_l \quad w_a]$, and v_l and w_a are the linear velocity of the point P along the robot axis and angular velocity, respectively.

The matrix $S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}$ can be easily obtained from the following nonholonomic constraint equation, which is $-\dot{x}_c \sin\theta + \dot{y}_c \cos\theta - d\dot{\theta} = 0$ under the pure rolling and non-slipping condition of a mobile robot with two wheels.

Property 1: $\bar{M}$ is uniformly positive definite, as shown in Fierro and Lewis [2] and Kim et al. [6].

Property 2: $\left(\dot{\bar{M}} - 2\bar{V}_m\right)$ is skew symmetric, as shown in Fierro and Lewis [2], and Kim et al. [6].

Property 3: There exist unknown positive constants $\bar{M}_{\max}$, $\bar{V}_{\max}$, $\bar{\tau}_{d\max}$, and $\bar{B}_{\max}$ such that $\|\bar{M}(q)\| \le \bar{M}_{\max}$, $\|\bar{V}_m(q,\dot{q})\| \le \bar{V}_{\max}\|\dot{q}\|$, $\|\bar{\tau}_d\| \le \bar{\tau}_{d\max}$, and $\|\bar{B}\| \le \bar{B}_{\max}$, as found in Kim et al. [6].

2.2 Dynamic model of a wheeled mobile robot including actuator dynamics

The actuator dynamics with the same parameters for two DC motors was presented in Das and Kar [3]. In this paper, it is considered that two DC motors have the different parameters. The dynamics equation of a wheeled mobile robot including the actuator dynamics can be written as

$$\bar{M}\dot{v} + \bar{V}_m v + \bar{B}D_v v + \bar{\tau}_d = Hu \quad (4)$$

where $H = \bar{B}D_u$, $\bar{B} = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ R & -R \end{bmatrix}$,

$D_u = diag\left(\frac{N_r K_{Tr}}{R_{ar}}, \frac{N_l K_{Tl}}{R_{al}}\right)$, $X = \frac{1}{r}\begin{bmatrix} 1 & R \\ 1 & -R \end{bmatrix}$,

$D_v = diag\left(\frac{N_r^2 K_{Tr} K_{br}}{R_{ar}}, \frac{N_l^2 K_{Tl} K_{bl}}{R_{al}}\right) \cdot X$, and $diag(a_1, a_2)$ represents a 2×2 diagonal matrix of the diagonal elements a_1 and a_2. N_r and N_l are gear ratios of the right and left motors, respectively. K_{Tr} and K_{Tl} are the motor torque constants, R_{ar} and R_{al} are the electric resistances, K_{br} and K_{bl} are the counter electromotive force coefficients of left and right motors, respectively. The length constants R and r in two wheels are shown in Fig. 1. The vector $u = [u_r \quad u_l]^T$ is the actuator input voltage vector and used as the control input instead of the wheel torque vector τ. In this robot model (4), the motor inductances are neglected.

3. Design of An Adaptive Robust Fuzzy Controller with GAs

3.1 Controller Design

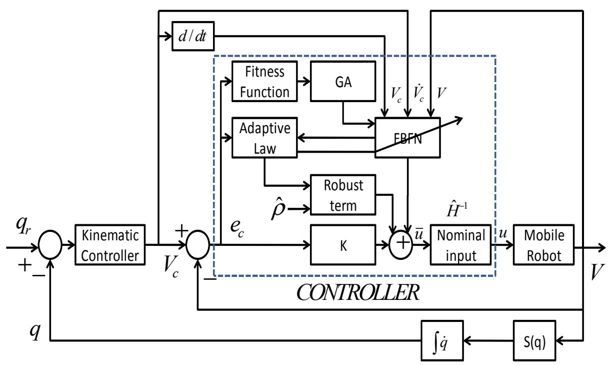

Fig. 2 Controller scheme.

Fig.2 shows the structure of the whole system. The output of the kinematic controller is employed as the input of the dynamic controller.

If we assume "*perfect velocity tracking*" for the kinematic model (2), then the kinematic model is asymptotically stable. The desired reference trajectory $q_r(t)$ and the actual posture $q(t)$ of the mobile robot are expressed by $q_r(t)=[x_r(t) \quad y_r(t) \quad \theta_r(t)]^T$ and $q(t)=[x(t) \quad y(t) \quad \theta(t)]^T$, respectively. The tracking errors are obtained in the basis of a frame fixed on the mobile robot as

$$E_p = T_e(q_r - q) = T_e E_q,\ E_q = q_r - q$$

$$E_p = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (5)$$

The velocity control input vector v_c that achieves the asymptotic tracking for the kinematic model is

$$v_c = \begin{bmatrix} v_{lc} \\ w_{ac} \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 + k_1 e_1 \\ w_r + k_2 v_r e_2 + k_3 v_r \sin e_3 \end{bmatrix} \quad (6)$$

where v_r and w_r are the reference linear velocity and angular velocity of the mobile robot, the control gains $k_1 > 0,\ k_2 > 0,$ and $k_3 > 0$ are design parameters, as shown in Kanayama et al [1]. This velocity control input (6) can guarantee that E_p converges to zero asymptotically as time goes to infinity when d is zero.

Once the auxiliary velocity control input vector $v_c(t) \in \Re^{n-m}$ is obtained for the asymptotically stable kinematic steering system, we need to design an actuator input voltage $u(t)$ to guarantee the robust tracking despite unknown robot and actuator dynamic parameters and external disturbances.

The auxiliary velocity tracking error is denoted as

$$e_c = v_c - v \quad (7)$$

Differentiating (7) with respect to time and substituting it into (4), the robot dynamics can be expressed as

$$\bar{M}\dot{e}_c = -\bar{V}_m e_c + \bar{M}\dot{v}_c + \bar{V}_m v_c + \bar{B}D_v v - Hu + \bar{\tau}_d \quad (8)$$

where the nonlinear function containing the dynamic parameters of the robot and actuator parameters is

$$f(x) = \bar{M}\dot{v}_c + \bar{V}_m v_c + \bar{B}D_v v \quad (9)$$

The above nonlinear function (9) can be approximated using a following FBFN as referred to Wang and Mendel [4].

$$f(x) = \begin{bmatrix} \sum_{j=1}^{N_1} p_{vj}(x)\theta_{vj} \\ \sum_{j=1}^{N_2} p_{wj}(x)\theta_{wj} \end{bmatrix} + \begin{bmatrix} \varepsilon_v(x) \\ \varepsilon_w(x) \end{bmatrix} = R(v_c, \dot{v}_c, v)\Phi + \varepsilon(x) \quad (10)$$

$$p_{vj_1}(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^{j_1}}(x_i)}{\sum_{j_1=1}^{N_1} \prod_{i=1}^{n} \mu_{A_i^{j_1}}(x_i)},\ p_{wj_2}(x) = \frac{\prod_{i=1}^{n} \mu_{\bar{A}_i^{j_2}}(x_i)}{\sum_{j_2=1}^{N_2} \prod_{i=1}^{n} \mu_{\bar{A}_i^{j_2}}(x_i)},$$

$$j_1 = 1,,2,...,N_1,\ j_2 = 1,,2,...,N_2 \quad (11)$$

where $x = (v_c, \dot{v}_c, v)$ is the input variable vector of fuzzy basis functions, $p_{vj}(x)$ and $p_{wj}(x)$ are called the fuzzy basis functions which correspond to fuzzy IF-THEN rules, θ_{vj} and θ_{wj} are free parameters, $R(v_c, \dot{v}_c, v) \in \Re^{2\times[2*\max(N_1,N_2)]}$ is called a fuzzy basis function matrix, $\Phi = [\theta_v^T \quad \theta_w^T]^T \in \Re^{2*\max(N_1,N_2)}$ is a desired parameter vector, which is an unknown constant vector to be determined to closely approximate the nonlinear function. $\varepsilon(x)$ is the approximation error vector. $\mu_{A_i^j}(x)$ and $\mu_{\bar{A}_i^j}(x)$ represent the Gaussian membership functions, defined by

$$\mu_{A_i^j}(x_i) = a_i^j \exp\left[-\frac{1}{2}\left(\frac{x_i - \bar{x}_i^j}{\sigma_i^j}\right)^2\right],$$

where $a_i^j, \bar{x}_i^j$ and σ_i^j are real-valued parameters with $0 < a_i^j \le 1$.

Property 4 : From Property 3 and the structure of the FBFN (10)~(11), there exist unknown positive constants $\bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3$ and $\bar{\theta}_4$ such that

$$\|\varepsilon(x)\| \le \bar{\theta}_1 + \bar{\theta}_2\|\dot{v}_c\| + \bar{\theta}_3\|\dot{q}\|\|v_c\| + \bar{\theta}_4\|v\| = \rho_e. \quad (12)$$

3.2 Central ideal for optimization of fuzzy rules by GAs

GAs are the adaptive stochastic methods of searching in a population of abstract representations (chromosomes) of candidate solutions based on the evaluation of fitness function to rate potential solutions in terms of their fitness. GAs proceed to initialize a population of randomly generated solutions and meliorate it by applying genetic operations inspired by evolutionary biology such as selection, crossover, mutation. A new generation of solutions with higher fitness is produced to replace its parent.

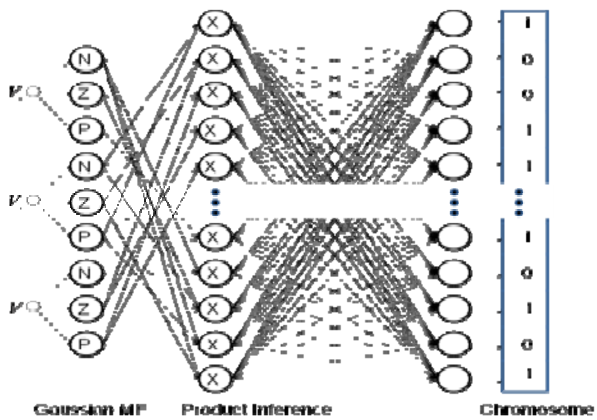

Fig. 3 FBFN structure with GAs.

Each chromosome is represented by a bit string, and each logic if-then rule derived from the FBFN is denoted by a gene. Gene “1” of the chromosome represents that the rule is selected, while gene “0” infers the contrary. For instance, for three input variables in the FBFN, and three membership functions for each variable, there are totally 27 rules are possibly produced. Hence, a chromosome with 27 binary values is needed to present the selected rules in the FBFN. The parallel structure of a FBFN with GAs is shown in Fig. 3.

In the reproduction process, we use two basic genetic operators: mutation and crossover. The crossover operation allows two chromosomes to exchange their information randomly so that new generation with good genes can be produced. The mutation operation randomly alters the genes to create diversity in the population. After these procedures, a new solution represented for selected fuzzy rules is applied to the FBFN to control the system. Fig. 4 describes the flowchart of GAs in the proposed controller.

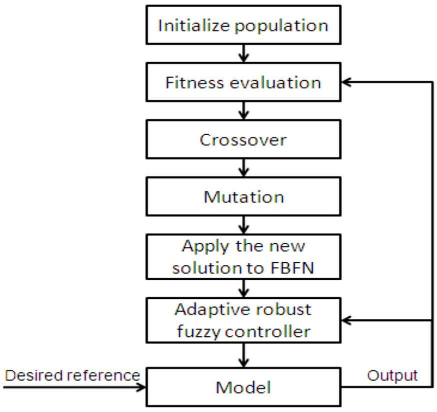

Fig. 4 Flowchart of GAs in the controller.

During control process, the FBFN is modified on-line after each sampling time by the procedure of selecting solutions according to the definition of the fitness to minimize the dynamic tracking error. The sampling time of GAs tuning is chosen as 0.2 second so that the fuzzy rules do not change so rapidly, which might become the potential source of disturbance to the system.

The fitness function of GAs can be simply defined as :

$$f_{fitness} = \frac{1}{(v_c - v)^2}$$

The goal is to make the tracking error e_c as small as possible, yet still guarantee the stability of the system; hence improve the kinematic tracking performance.

3.3 Controller structure

It is considered that the input matrix H including the actuator parameters is unknown. Hence the control input is chosen as follows,

$$u = \hat{H}^{-1}\bar{u} \tag{13}$$

where $\hat{H}$ is the guessed nominal parameter matrix of H and $\bar{u}$ is an adaptive fuzzy control input.

Substituting (13) into (8), the closed-loop error dynamics for e_c is

$$\bar{M}\dot{e}_c = -\bar{V}_m e_c + f(x) + (I - H\hat{H}^{-1})\bar{u} - \bar{u} + \bar{\tau}_d \tag{14}$$

Assumption 1: There exist a positive constant C_0 such that

$$\left\| I - H\hat{H}^{-1} \right\| \le C_0 < 1 \tag{15}$$

Theorem 1: Under Assumption 1, if the following control and adaptation laws (16)~(19) are applied to the wheeled mobile robot system (2) and (4), then e_c converges to zero asymptotically when $v_r > 0$. Therefore, in the case that d is zero, the tracking errors e_1, e_2, and e_3 converge to zero asymptotically by the velocity control (6). Thus, the actual posture $q(t)$ converges to the desired reference trajectory $q_r(t)$ asymptotically as time t goes to infinity.

$$u = \hat{H}^{-1}\bar{u} = \hat{H}^{-1}\left(\hat{f} + u_s\right), \quad \hat{f}(x) = R(v_c, \dot{v}_c, v)\hat{\Phi}, \tag{16}$$

$$u_s = \hat{\rho}\frac{e_c}{\|e_c\|} + Ke_c, \quad \hat{\rho} = \hat{\Theta}^T\psi \tag{17}$$

$$\dot{\hat{\Phi}} = \Gamma_\phi R^T e_c, \qquad \dot{\hat{\Theta}} = \Gamma_\theta \psi \|e_c\| \in \Re^6 \tag{18}$$

$$\psi = \left[1, \|\dot{v}_c\|, \|\dot{q}\|\|v_c\|, \|v\|, \|R\hat{\Phi}\|, \|e_c\|\right]^T \tag{19}$$

where $\hat{f}(x)$ is an estimate of the nonlinear robot function $f(x)$ estimated by using a FBFN, and u_s is the adaptive robust control input and stabilizing control term, $\hat{\Phi}$ is an estimate updated by (18). The dimension of the estimate $\hat{\Phi}$ is determined by the fuzzy basis functions and fuzzy rules used in the FBFN. $\hat{\Theta}$ is an estimate of the real norm-bounded parameter vector Θ and updated by (18). The initial values $\hat{\Theta}(0)$ of the estimate vector $\hat{\Theta}(t)$ have to be set as positive constants. ψ is the bounding function obtained in the stability proof. The gains K, Γ_ϕ and Γ_θ are positive constant diagonal gain matrices.

3.4 Proof of stability

Consider a Lyapunov function candidate:

$$V = V_1 + V_2 + V_3 \tag{20}$$

where $V_1 = \frac{1}{2}(e_1^2 + e_2^2) + \frac{1}{k_2}(1 - \cos e_3)$,

$V_2 = \frac{1}{2}e_c^T\bar{M}e_c + \frac{1}{2}\tilde{\Phi}^T\Gamma_\phi^{-1}\tilde{\Phi}$, and $V_3 = \frac{(1 - C_0)}{2}\tilde{\Theta}^T\Gamma_\theta^{-1}\tilde{\Theta}$.

The time derivative of V_1 in (20) is as follows by (6).

$$\dot{V}_1 = e_1\dot{e}_1 + e_2\dot{e}_2 + \frac{1}{k_2}\dot{e}_3\sin e_3 = -k_1e_1^2 - \frac{k_3}{k_2}v_r\sin^2 e_3. \tag{21}$$

After differentiating V_2 with respect to time, substituting it into the error dynamics (14), adopting Property 2, and applying it to (16)~(18), we obtain

$$\dot{V}_2 = e_c^T \varepsilon - e_c^T u_s + e_c^T (I - H\hat{H}^{-1})(R\hat{\Phi} + u_s) + e_c^T \overline{\tau}_d \quad (22)$$

where $\tilde{\Phi} = \hat{\Phi} - \Phi$.

From Property 3, Property 4 and Assumption 1, the boundedness of (22) can be obtained as

$$\dot{V}_2 \le \overline{\rho}_e \|e_c\| + C_o \|e_c\| \|R\hat{\Phi}\| + C_o \|e_c\| \|u_s\| - e_c^T u_s \quad (23)$$

where $\overline{\rho}_e = \rho_e + \overline{\tau}_{d\max}$.

From the definition of u_s in (17), it can be inferred that

$$\|u_s\| \le \|\hat{\rho}\| + \|K\| \|e_c\| \le \hat{\rho} + K_m \|e_c\| \quad (24)$$

where the initial values $\hat{\Theta}(0)$ are positive constant values and thus $\hat{\rho}$ is also positive. Here, $\|K\| \le K_m$, where K_m is a positive constant.

Thus we can obtain the following inequality.

$$\begin{aligned} \dot{V}_2 &\le \overline{\rho}_e \|e_c\| + C_o \|e_c\| \|R\hat{\Phi}\| + C_o K_m \|e_c\|^2 \\ &+ (C_o - 1)\hat{\rho} \|e_c\| - e_c^T K e_c \end{aligned} \quad (25)$$

Differentiating V_3 with respect time, we obtain

$$\begin{aligned} \dot{V}_2 + \dot{V}_3 &\le -e_c^T K e_c - \hat{\rho}(1 - C_o)\|e_c\| + \rho(1 - C_0)\|e_c\| \\ &+ (1 - C_o)\tilde{\Theta}^T \Gamma_\theta^{-1} \dot{\hat{\Theta}} \end{aligned} \quad (26)$$

where $\rho = \frac{1}{(1 - C_0)} \left[\overline{\rho}_e + C_o \|R\hat{\Phi}\| + C_o K_m \|e_c\| \right]$.

Since $\tilde{\rho} = \hat{\rho} - \rho = \hat{\Theta}^T \psi - \Theta^T \psi = \tilde{\Theta}^T \psi$, then

$$\dot{V}_2 + \dot{V}_3 \le -e_c^T K e_c - \tilde{\Theta}^T \psi (1 - C_o)\|e_c\| + (1 - C_0)\tilde{\Theta}^T \Gamma_\theta^{-1} \dot{\hat{\Theta}} . \quad (27)$$

By choosing the adaptation law (18) and using (21) and (27), we can conclude that

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \le -k_1 e_1^2 - \frac{k_3 v_r \sin^2 e_3}{k_2} - e_c^T K e_c < 0 . \quad (28)$$

It can be easily found that the velocity tracking error vector e_c converges to zero asymptotically when $v_r > 0$. In the case that d is zero, the tracking errors e_1, e_2, and e_3 converge to zero asymptotically by the velocity control (6). Thus, the actual posture $q(t)$ converges to the reference trajectory $q_r(t)$ asymptotically as time t goes to infinity.

Remark 1: The presented GA in this work is used for the optimization and reduction of the fuzzy membership functions shown in equations (10), (11) and Fig. 3. According to the proposed GA, the dimension of the matrix $R(v_c, \dot{v}_c, v)$ composed of the fuzzy membership functions is determined. The fuzzy membership functions used in this FBFN are all Gaussian functions and all bounded. The matrix $R(v_c, \dot{v}_c, v)$ is bounded irrespective of the number of the fuzzy membership functions and fuzzy rules selected by the presented GA.

Therefore, the stability of the closed-loop control system is not dependent on the selection of the fuzzy rules. In other words, the action of GA does not affect the stability of the proposed adaptive robust fuzzy controller with GAs. In conclusion, the stability of the closed-loop control system including the presented GA can be guaranteed.

4. Simulation Results

The simulation for path tracking control of the wheeled mobile robot illustrated in Fig. 1 is performed to verify the proposed control scheme. In the presence of uncertainties such as parameter variations and disturbance, conventional PID, ARFC(Adaptive Robust Fuzzy Controller) and ARFC-GA(Adaptive Robust Fuzzy Controller with GAs) have been conducted and compared in order to show the effectiveness of the proposed controller.

The desired reference trajectory is set as following:

$$x_r(t) = \frac{t}{25},\ y_r(t) = \sin(\frac{2\pi t}{50}),\ \text{and}\ \theta_r(t) = \tan^{-1}\left(\frac{\dot{y}_r}{\dot{x}_r}\right).$$

The initial posture of the reference trajectory is $q_r(0) = [0 \quad 0 \quad 1.2626]^T$ and the initial actual posture of the robot is chosen as $q(0) = [-0.3 \quad 0 \quad \pi/6]^T$.

In the simulation, the actuator and robot parameters are defined as $N_r = 100$, $N_l = 80$, $K_{br} = 0.0274$, $K_{bl} = 0.025$, $K_{tr} = 0.0274(Nm/A)$, $K_{tl} = 0.025(Nm/A)$, $R_{ar} = 4\Omega$, $R_{ar} = 3\Omega$, $m = 10kg$, $I = 5kg \cdot m^2$, $R = 0.2(m)$, $r = 0.05(m)$, and $d = 0.01(m)$.

The knowledge of the actuator and robot parameters are set to 70% of the real values. The disturbance is unknown in the control system and generated in this simulation by $\tau_d = 5\sin(2t)$. The parameter variations are considered during the total control process. In this simulation, it is used that the mass and moment of inertia of the robot vary as follows.

1) $0(\text{sec}) \le t < 20(\text{sec})$, $m = 10(kg)$, $I = 5(kgm^2)$;
2) $20(\text{sec}) \le t < 30(\text{sec})$, $m = 20(kg)$, $I = 10(kgm^2)$;
3) $30(\text{sec}) \le t < 40(\text{sec})$, $m = 25(kg)$, $I = 15(kgm^2)$;
4) $40(\text{sec}) \le t \le 50(\text{sec})$, $m = 30(kg)$, $I = 20(kgm^2)$.

In the presented FBFN, for each input variable, three Gaussian basis functions are defined as following:

$$\mu_{A_i^j}(x_i) = \exp\left[-\frac{1}{2}\left(\frac{x_i - \overline{x}_i^j}{\sigma_i^j}\right)^2\right]$$

where $\overline{x}_i^j = -0.5, 0, 0.5$ and $\sigma_i^j = 0.2$

Therefore, there are totally 27 rules are used in the FBFN.

For the GAs, we execute our experiments with the following parameters: population size: 100, crossover rate: 0.8, mutation rate: 0.01.

The gains used in the controller are chosen as $k_1 = 0.25$,

$k_2 = 50,\ k_3 = 10,\ K = 50I_2,\ \Gamma_\phi = 5000I_{10}$, and $\Gamma_\theta = 0.00005I_7$

$\Gamma_\theta = 0.00005I_7$ where $I_n \in \Re^{n\times n}$ is an identity matrix.

Table 1 shows the performance comparison of conventional PID algorithm, adaptive robust fuzzy and adaptive robust fuzzy with GAs.

Fig. 5, 6 and 7 show the tracking performance for the proposed path tracking control of the mobile robot: (a) Path tracking of the robot $(q(t) \to q_r(t))$; (b) Control input voltages $(u = [u_r \quad u_l]^T)$; (c) Tracking errors of the robot posture $(E_q = q_r - q)$; (d) Tracking of linear and angular velocities $(v \to v_c)$ when applying to the conventional PID controller, ARFC, and ARFC-GA, respectively. It can be seen from the implementation that the PID controller cannot make the system converge asymptotically, because it cannot overcome the uncertainties and disturbances. The big errors appear when the mobile robot changes direction rapidly. On the contrary, the ARFC makes the system stable asymptotically. The tracking errors converge to zero as time goes to infinity.

Table 1. Performance comparison.

Controller	Velocity tracking RMS error		Position tracking RMS error(mm)
	v_{Ic} (m/sec)	w_{Ic} (rad/sec)	
PID	0.0385	0.0159	98.9
ARFC	0.0131	0.0034	29.0
ARFC-GA	0.0073	0.0028	23.1

The proposed ARFC-GA takes an advantage of the evolution characteristics of GAs to select the optimized fuzzy rules online from the product inference process of a fuzzy basis function network. Therefore, the tracking errors are reduced. The biggest aim for using GA in this controller is the optimization and reduction of the fuzzy membership functions through the given genetic scheme from the proposed ARFC.

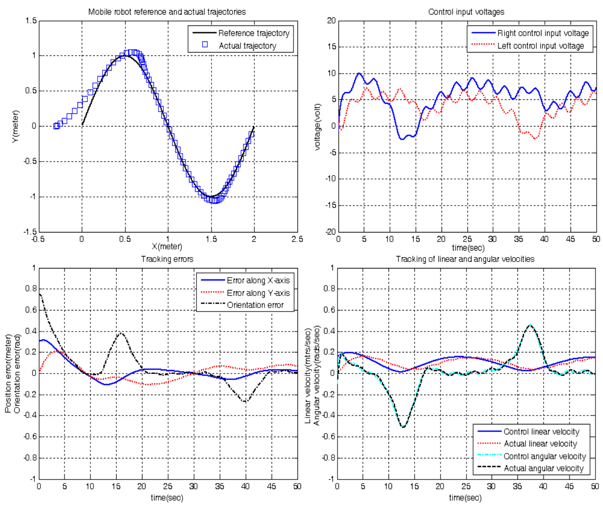

Fig. 5 System performance of PID controller.

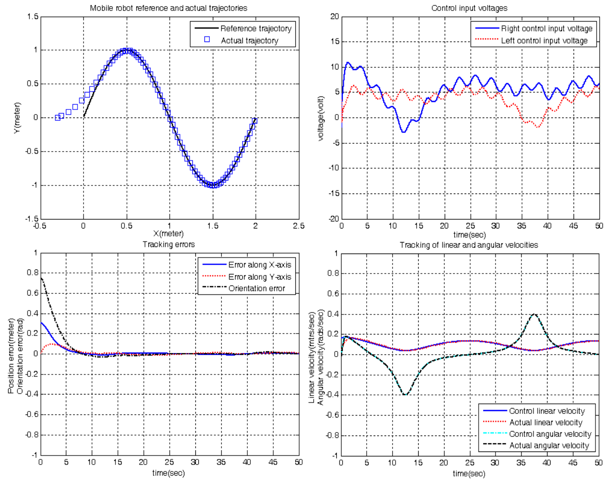

Fig. 6 System performance with ARFC.

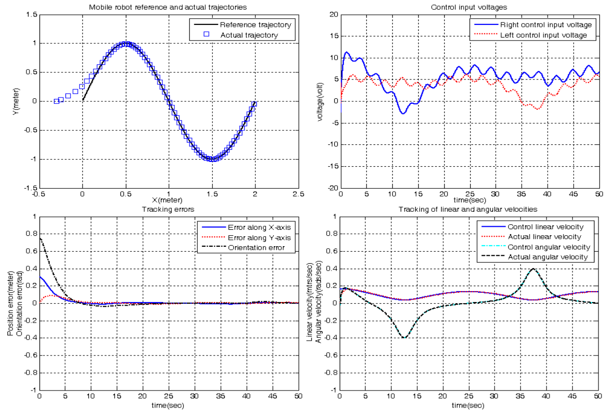

Fig. 7 System performance with ARFC-GA.

It can be seen from the simulation results that conventional PID cannot overcome disturbances and parameter variations, while the proposed controller give the best performance over all. ARFC-GA is valid and robust against parameter variations and disturbances.

5. Conclusions

In this paper, we have developed an adaptive robust fuzzy controller with genetic algorithms for a wheeled mobile robot that asymptotically tracks a desired reference path in the presence of uncertainties. The robot dynamics including the actuator dynamics is considered in this work. The stability and the convergence of the tracking errors have been guaranteed based on the Lyapunov function theory. The different parameters for two actuator models in the dynamic equation have been considered in the proposed controller. The proposed control scheme does not require the accurate parameter values for the robot and actuator parameters. The effectiveness and robustness of the proposed controller has been shown through the simulation.

In addition to this work, a study on control schemes considering the inductances of the actuators in the actuator dynamics is left to a further research.

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