# 소성해석법의 선체 GRILLAGE 설계에의 응용

김기성<sup>1,†</sup>·신승욱<sup>2</sup> 인하대학교 기계공학부 조선해양공학전공<sup>1</sup> 인하대학교 대학원 조선해양공학과<sup>2</sup>

# Application of Plastic Analysis Method to Ship Grillage Design

Ki-Sung Kim<sup>1,†</sup>·Sung-Uk Shin<sup>2</sup> Department of Naval Architecture and Ocean Engineering<sup>1</sup> Department Naval Architecture and Ocean Engineering, Graduate School<sup>2</sup>

## Abstract

A plastic analysis method is commonly used in ship and offshore structural system to utilize the ultimate strength. In this paper, the basic principle of plastic analysis method is applied to ship grillages such as transverse oil-tight bulkheads. The main emphasis is placed on the optimum arrangement of grillage system to give minimum weight. Additional parametric study is carried out to find the effect of various arrangement of grillage system. The above methods are applied to oil-tight bulkhead design, and results are compared with the existing one.

Keywords : Plastic analysis method(소성해석법), Structural optimization(최적구조설계), Oil-tight bulkhead design(유밀격벽설계), Grillage design(grillage 설계)

# 1. Introduction

The basic foundation of the application of plastic analysis and design methods was established for idealized grid structures and introduced by Kim (1982) and Kim and Hong (2004). Further applications of plastic design method to ship grillages are shown in other references (Kim, et al., 1995, 2001, 2004, 2009).

In the present paper, those basic principles are applied to the more realistic ship grillages.

The first part of the chapter deals with the general analysis and design principles for realistic grillages.

The last part concerns the application of the above design principles to transverse oil-tight bulkheads.

# 2. Analysis and Design Procedure

### 2.1 Analysis Method

The basic analysis method applied to the grillage design is the strength design method, which utilizes the concept of plastic collapse in conjunction with a load factor. In the modelling technique applied to the grillage design, the structure is idealized, firstly as closely as possible to the realistic structures and secondly as simple as possible, for the purpose of easy investigation of the effect of the variation of design parameters. The general mathematical optimization procedures are discarded here in the grillage design process simply because the design model is not as complex as for the complex mathematical optimization.

Then the main emphasis on the design procedure is placed on the parametric study of the design variables, by which the designer can recognize the sensitivity of each design variable.

Since the grillages of ship structures generally consist of a small number of heavy beams and a large number of small size stiffeners, the grillages are idealized as three components:

- (a) Grids of heavy members: Girders and webs.
- (b) Secondary small members: Stiffeners.
- (c) Plate.

In the following sub-sections, each component of the grillage design process is examined in detail with particular reference to grillages such as transverse bulkheads.

### 2.2 Plate Design

In the case of plate for grillage structures, plate may be

assumed fixed against rotation at the stiffeners if this is symmetrical loading.

For the plate the plastic moment per unit width is given by  $\sigma_y \cdot t^2/4$ , where t is the plate thickness and  $\sigma_y$  is the material yield stress. By applying the usual plastic analysis method for the strip of plate, the following relationship can be obtained:

$$LF \cdot h = \frac{4\sigma_y}{w} (\frac{t}{s})^2 \tag{1}$$

where LF = Design load factor

h = Pressure head (mm)

 $\sigma_y$  = Material yield stress (N/mm<sup>2</sup>) w = Weight density (N/mm<sup>2</sup>)

t = Plate thickness (mm)

s = Stiffener space (mm).

For a plate with a low aspect ratio, the bending resistance in the orthogonal direction to the strip of plate may be considered by ierroducing a correction factor(e), then the above expression becomes:

$$LF \cdot h = \frac{4\sigma_y}{w \cdot e} (\frac{t}{s})^2 \tag{2}$$

where e = Constant for aspect ratio

Then, Eqn. (2) can be rewritten as follows:

$$t = [LF \cdot h \ge \frac{w \cdot e^2}{4\sigma_y}]^{1/2} \ge s$$
(3)

For design purpose the 'e' value may be taken as unity for plates whose length is equal or greater than four times the width. For smaller aspect ratios a safe approximation is given by a lower bound solution as follows (Broughton, 1962):

$$e = 1.1 - 0.4 \ge s/S \tag{4}$$

where s = Width of plate (mm)S = Length of plate (mm)

To give reasonable correspondence with the present rule requirement, the load factor LF = 1.6 may be taken for plate of bulkheads.

### 2.3 Selection of Beam Sizes

Plastic design methods under point loads at intersections were dealt with by Kim and Hong (2004). One of the important findings for grillage weight optimization under uniform pressure load was that, in a two-directional grillage system, the minimum weight design uses small size beams along the longer span or large size beams along the shorter span, when the plate panel thickness is not considered. In a practical design situation, a grillage consists of a small number of heavy members in one or two directions and large number of small stiffeners. In this case, the transformation of pressure load into intersection point loads may not be appropriate. Therefore, in the grillage design study to be explored in this paper, the lateral pressure load is transformed into line loads along the beams. For a conservative assumption, the pressure load of a full beam space is considered to be applied to each beam.

The beam end conditions can be determined by considering the surrounding structures at boundaries. For example, the end conditions for webs and girders of transverse bulkheads may be considered fixed and the end conditions of beams on a pontoon deck may be considered simple.

For a span of stiffener, it is generally accepted that the end conditions are fixed when plastic analysis method is applied.

For the beam size selection of heavy members of grillages (e.g. webs and girders in transverse bulkheads), the grillage is idealized as in Fig. 1(a). In this figure, the horizontal girders are assumed to take the whole load in the form of line loads due to the stiffener end reactions, and on the vertical webs, only the internal reactions are applied. These internal reactions are represented by the girder line load of one web space. And the sizes of girders and webs depend on these reaction values. Different magnitudes of internal reactions along each girder are chosen for the present design study.

The intensity of line loads varies on each girder depending on the position of the girders, and so do the internal reactions. The plastic collapse modes for girders and webs are shown in Figs. 1 (b) and (c).

The collapse equations for these two main heavy members and stiffeners can be calculated by applying the plastic collapse theorems for each beam as follows.

#### (1) Horizontal Girders

Collapse equations for the beam under constant line load and constant internal reactions are fully described by



(a) Loading and Internal Reactions







Chowdhury (1977). According to his lower bound solution, the collapse equations for different numbers of supports are [see Fig. 1(b)]:

$$M_{pg} = \frac{(n+1)^2}{C} [(1-\alpha) + \frac{\alpha^2}{(n+1)^2}] \cdot W_g \cdot l^2$$

$$[n = odd]$$

$$M_{pg} = \frac{(n+1)^2}{C} [1 + \frac{n \cdot (n+2) \cdot \alpha^2}{(n+1)^2}] \cdot W_g \cdot l^2$$

$$[n = even]$$
(5)

### Where,

 $M_{pg}$  = Fully plastic bending moment of girder (N-mm) n = Number of weds l = Web beam space (mm)  $W_g$  = Design line load on girder (N/mm) C = Constant for boundary conditions [C=8 for simple ends, C=16 for fixed ends]

 $\alpha$ = Non-dimensional internal reaction.

$$= R/(W_g \cdot l)$$

R = Internal reaction at intersection (N)

In these equations, the magnitude of  $M_{pg}$  is a function of  $\alpha$  value, e.g. when with higher support reactions,  $M_{pg}$  value is smaller, or vice versa.

The value for minimum weight design is obvious as a result of the optimization study by Kim (1982), where  $\alpha$  values usually take extremes of 0 or 1 depending on panel aspect ratio. Then the designer has to consider other factor such as structural integrity or rule requirements for the number and sizes of beams.

(2) Vertical Webs

As mentioned above, only loads on a web are intersection reactions.

The loading and collapse mode for this web with four intersection reactions of different magnitudes are shown in Fig. 2.



Fig. 2 Vertical web collapse under point loads

Then the collapse work equation for the web with arbitrary loading can be expressed as follows:

$$2 \ge (\theta + \phi) \ge M_{pw} = \sum_{i=1}^{n} (R_i \ge d_i)$$
(6)

Where.

 $M_{pw}$ = Plastic bending moment of web (N-mm) $R_i$ = Internal reactions at intersection i (N)  $d_i\text{=}$  Plastic deflection value at support point i~(mm)

- $d_m\text{=}$  Deflection value at central hinge point (mm)
- n = Number of intersection points
- a = Distance from left end to hinge point  $\left(mm\right)$
- $\boldsymbol{b}$  = Distance from right end to hinge point (mm)
- $\theta = d_m/a$
- $\phi = d_m/b$

Then, by rearranging the above equation, the plastic moment is obtained as follows:

$$M_{pw} = \frac{\sum_{i=1}^{n} (R_i \ge d_i)}{2 \ge (\theta + \phi)}$$
(7)

The actual collapse condition can be found numerically by finding the hinge position for the maximum plastic bending moment  $(M_{nn})$ .

As was the case with the horizontal girders, the magnitude of  $M_{pw}$  is dependent on the a values, therefore the designer has to choose an appropriate value to get the required  $M_{pw}$  and  $M_{pa}$ .

#### (3) Stiffeners

For the stiffeners, the beam size can be calculated by considering the collapse equations between heavy primary members. As mentioned before, the stiffener end condition is considered fixed. Fig. 3 shows a stiffener under a varying intensity of line load along the beam. The plastic collapse equation of a stiffener in Fig. 3 is as follows:



Fig. 3 Stiffener collapse under trapezoidal load

$$M_{ps} = \frac{a \cdot b}{12 \cdot l^2} [3l^2 \cdot H_c + H_v \cdot (2a^2 + 3a \cdot b + b^2)]$$
(8)

Where,

 $M_{ns}$  = Plastic moment of stiffener (N-mm)

$$\begin{split} l &= \text{Stiffener span } (mm) \\ a &= l \cdot \left[ -r + (r^2 + r + \frac{1}{3})^{1/2} \right] \\ b &= 1 - a \\ r &= H_{\!\!e}/H_v \text{ [see Fig. 3]} \end{split}$$

The above equation is also valid for the constant line load case when  $H_{\!v}=0.$ 

Once the plastic collapse equation is established for heavy primary members and stiffeners, the required beam sizes are found by the relationship of  $Z_p = M_p/\sigma_y$ . The determination of beam dimensions to satisfy the required  $Z_p$  is discussed in the next sub-section entitled 'Beam Section Design'.

# 3. Beam Section Design

For design and optimization studies of stiffened grillages, three types of beam sections are considered, i.e. flat bars or T-bars of rolled section and deep girder sections. The limitations on the plate thickness of beam sections and ratios of thickness to depth are given as beam design data. The minimum plate thickness may be controlled by the availability of plates and robustness as well as production controlled by considering the possibility of local and lateral buckling. The required beam section sizes are expressed by the plastic section modulus which are obtained from collapse equations Section 2.

The basic principle for the beam section design is an iterative method to find the minimum beam sectional area for a given plastic section modulus. The thicknesses of flange and web, and the ratios of thickness to depth are given as constraints in the iteration.

As shown in Fig. 4 and Table 1, the iteration process starts with an ordinary T-bar section to find an appropriate web thickness  $T_w(TB)$  with a given  $D_w/T_w(TB)$ ,  $T_f/T_w(TB)$  and  $D_f/T_f(TB)$ . When  $T_w(TB)$  reaches the maximum  $T_w(TB)$ , then a deep girder section should be selected. Another possibility is that the required section modulus may be too big to be selected from T-bar when the web thickness reaches the minimum  $T_w(TB)$ , then a flat bar section should be tried. The iteration process for deep girder section is similar to ordinary T-bar sections with different constraint values. With girder sections, there is no limit on web thickness. For flat bar sections, the basic constraint is  $D_w/T_w(TB)$ . When the web thickness reaches the minimum  $T_w(TB)$ , the depth of web is reduced until the required section modulus is satisfied.

### 소성해석법의 선체 GRILLAGE 설계에의 응용



Fig. 4 Flow chart for beam design procedure

ltem	Girder (TG)	T-Section (TB)	Flat-bar ( <i>FB</i> )
$D_w/T_w$	150.0	50.0	18.0
$T_{f}/T_{f}$	2.5	1.5	-
$D_{f}/T_{f}$	10.0	18.0	-
Min. $T_w \ (mm)$	-	5.0	5.0
Max. $T_w$ (mm)	-	15.0	-
Min. $D_w \ (mm)$	-	150.0	-

Table 1 Standard beam data

 $D_w$  = Depth of web (mm)

 $T_w$  = Thickness of web (mm)

 $D_{\rm f}$  = Width of flange (mm)

 $T_{f}$  = Thickness of flange (mm)

# 4. Grillage Design Process

The design process may be summarised by the following flow chart in Fig. 5. In this flow chart, all the information required is obtained from the appropriate sections. Design data for grillage design of transverse bulkheads are usually as follows:

B = Grillage panel breadth (mm).

- D = Grillage panel depth (mm).
- $\alpha$  = Ratio of intersection reaction to one web space line load (0-1)
- Ht = Design oil head at top of bulkhead (mm).
- LF(P) = Load factor for plate thickness design.
- LF(S) = Load factor for stiffener design.
- LF(G) = Load factor for primary heavy member design.
- $\sigma_y$  = Material yield stress ( $N/mm^2$ )
- IGB = Boundary condition for gitder ends.
  - [IGB=0: simple ends, IGB=1: fixed ends]
- IWB = Boundary condition for web ends [IWB=0: simple ends, IWB=1: fixed ends]
- $w_c$  = Weight density of oil cargo  $(N\!/mm^2)$
- $w_s$  = Weight density of steel  $(N\!/mm^2)$
- $\ensuremath{\mathsf{TPCA}}$  = Corrosion allowance for plate (mm)



Fig. 5 Flow chart for grillage design process

# 5. Application to Transverse Oil-tight Bulkhead Design of VLCC

### (1) Design Problem

Design example chosen for the application of the plastic analysis method is a transverse oil-tight bulkhead of the centre tank of a VLCC. The bulkhead model in this example was originally designed for Lloyd's Classification and the structural arrangement of this bulkhead is shown in Fig. 6. In this figure, the bulkhead dimensions are 18.8m in breadth and 25.5m in depth. The stiffeners are constant cross-section T-bars. The three horizontal girders are of similar scantlings. Two side vertical webs are slightly lighter than the horizontal girders. On the other hand, the central vertical web consists of two built sections on each side of the plate. One of them has section dimensions of 3480×12.5+540×35 in mm. The other one has section dimensions of 1090×12.5+540×35 in mm. Therefore this central web may be considered as a fixed boundary dividing the centre tank bulkhead into two identical grillage models (each 9.4 m × 25.5 m). In the present design study, this half centre bulkhead of the existing design are shown in Table 2.

The main emphasis of the present design study is placed



Fig. 6 Structural arrangement of transverse bulkhead

Table 2	2 Scaltlings	s of oil-tight	t bulkhead	for	existing	design
---------	--------------	----------------	------------	-----	----------	--------

Unit : tonnes

Members	Dimension (mm)	No.	Weight	Sub-Tot
Stiffeners	545×10.5+150×24.5	8	15.30	15.30
Girder (30,43)	1790×13.0+300×35.0	2	5.00	
Girder (36)	1790×13.0+300×30.0	1	2.40	7.40
Web (Side)	1240×10.5+250×25.4	1	3.90	3.90
Plate	15.5×9400×25500		29.20	29.20
Total				55.80

- B × D = 9.4m × 25.5m.

- W  $\times$  G  $\times$  S = 1  $\times$  3  $\times$  8 (web  $\times$  girders  $\times$  stiffeners).

- The above dimensions are with corrosion control.

- Flat bar connection between stiffener and girders.

- I-web longl stiffener on web (10.5  $\times$  150).

on the examination of the new design concept to the actual design problem of bulkhead. Therefore, the design data and beam data are chosen to generate a similar configuration to the existing design.

### (2) Design Methods for Structural Members

For the purpose of comparison, between the existing design and new designs, the design data is chosen as closely as possible to the existing one. Design load is taken by Lloyd's rule requirements, which is due to the oil pressure, up to deck level, of a full tank.

Mild steel material is used for new designs as in the existing design. The beam ends are considered fixed, as the beam ends are connected to the adjacent similar structural members by heavy brackets, and this type of end connection would resist the full plastic bending moment.

The plate thickness and beam sizes are calculated according to the procedures in Section 2 and Fig. 4 respectively. In the present design study, only regular beam spaces are considered. Section sizes of stiffeners are varied at each girder space.

For the DESIGN DATA in Fig. 5, numerical values in Table 3 are used. In this table, a grillage intersection reaction value for primary member design (the intersection reactions are expressed by  $\alpha$  as in Section 2.3) is chosen to give similar scantlings as in the existing design. This  $\alpha$  value is not necessarily optimal when minimum weight is considered.

Table 3	Design	data	for	oil-tight	bulkhea	d
---------	--------	------	-----	-----------	---------	---

	-	
ltem	Values	Comments
B D LF(P) LF(S) LF(G)	9400 25500 1.6 3.4	<ul> <li>Breadth of bulkhead (mm).</li> <li>Depth of bulkhead (mm).</li> <li>Load factor for plate design.</li> <li>Load factor for stiffener design.</li> <li>Load factor for girder and web design.</li> </ul>
$\sigma_y$ IGB IGW	240 I	<ul> <li>Yield stress for material (N/mm<sup>2</sup>).</li> <li>Girder end conditions (0=simple, l=fixed).</li> <li>Web end conditions (0=simple, l=fixed).</li> </ul>
$W_c$ $W_s$ TPCA	0.833E-5 0.785E-8 2.5	<ul> <li>Weight density of cargo oil (N/mm<sup>3</sup>).</li> <li>Weight density of steel (N/mm<sup>3</sup>).</li> <li>Corrosion allowance for plate (mm).</li> </ul>
a	0.07	- Non-dimensional intersection reaction value.

(3) Results and Discussions

The design results for the oil-tight bulkhead are shown in Figs. 7 for structural weight. The structural weight and scantlings of new designs are shown in Tables 4 and 5 for similar stiffener space to the existing design.

In Table 4, the similarity of the present with the existing design can be noticed when  $W\times G\times S=1\times 3\times 4$ . In this case,

#### 소성해석법의 선체 GRILLAGE 설계에의 응용

the new design gives a total weight of 53.1 tonnes which is slightly lighter than the existing design total weight of 55.8 tonnes. The similarity of the member scantlings in the new design can also be seen by comparing Tables 2 and 5.



Fig. 7(a) Weight variation of oil-tight bulkhead with web=0



Table 4	Weights	of	structure	members	of	oil-tight
	bulkhead					

						-	-			
W	G	S	S	Ws	Wg	Ww	Wb	Wp	Wt	Wp/Wt
0	1 2 3	9	940	35.1 24.2 15.7	4.6 6.8 8.3		39.6 30.9 23.9	27.4	67.0 58.4 51.4	0.41 0.47 0.53
1	1 2 3	4	940	31.2 21.5 13.9	4.3 6.4 7.9	3.9	39.4 31.8 25.8	27.4	66.8 59.2 53.1	0.41 0.46 0.52
W =	W = Number of webs.									

G = Number of girder.

S = Number of stiffeners between webs

s = Stiffener space (mm).

Ws = Stiffener weight (tonnes).

Wg = Girder weight (tonnes).

Ww = Web weight (tonnes).

Wb = Total beam weight (Ws+Wg+Ww).

Wp = Plate weight (tonnes).

Wt = Total bulkhead weight (Wb+Wp).

Table 4 also shows the possibility of reducing the total weight of the bulkhead when a two-directional grillage system is adopted without a vertical web, i.e. when  $W\times G\times S=0\times 3\times 9$ , the total weight is reduced to 51.4 tonnes.

Figs. 7 show the total weight variations of the bulkheads for different numbers of girders and webs. The general trend of these results is that the total weight decreases when girder numbers increase with a given number of webs. In these figures, the sudden jump in the weight curve with a given number of girders results from the change of stiffener configuration from deep girder section to ordinary T-section (i.e. from Dw/Tw=150.0 to Dw/Tw=50.0).

Therefore, the general conclusions from the oil-tight bulkhead design study are as follows:

(a) Minimum weight design leads to a two-directional grillage system of heavy horizontal girders along the shorter span and vertical small stiffeners.

(b) However, the number of heavy primary members (horizontal girders and vertical webs) and also the number of stiffeners should be decided by considering the whole structural continuity and other functional requirements.

### 6. Irregular Grillages

A design method for grillages of bulkheads was dealt with in Section 2.3 In that section, the primary heavy members (horizontal girders and vertical webs) were designed by introducing constant intersection reactions along the horizontal girders. In that method, all the vertical webs had the same sizes due to the constant internal reactions along the horizontal girders. In this section, a special case of the above method

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	10/			Stf No To (mm)		Stiffener sections (mm)					
$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	VV	G	3	Sti NO.	тр (mm)	Tw	Dw	Tf	Df	As	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1	9/4	(1)	11.3	14.0	698.5	21.0	377.2	17662.0	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $				(2)	17.8	10.0	1500.0	25.0	250.0	21250.0	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $				(1)	9.7	8.5	426.0	12.8	230.0	6569.4	
O or 1         (3)         18.7         14.7         733.5         22.0         396.1         19476           3         9/4         (1)         6.3         6.3         317.5         9.5         171.4         3642           3         9/4         (2)         9.1         9.1         455.0         13.6         245.7         7494           (3)         10.8         10.8         539.0         16.2         291.1         10516           (4)         12.0         12.0         602.5         18.1         325.3         1314C           W         G         Grd No.         S         Tp (mm)         Tw         Dw         Tf         Df         As           1         (1)         9         15.0         17.0         2557.5         42.6         426.2         61774           2         (1)         9         12.7         12.9         1930.5         32.2         321.7         35197           3         (2)         9         15.0         17.4         2007.0         334.5         380.0         56597           3         (2)         9         15.0         13.4         2007.0         334.5         380.42         50135 <td></td> <td>2 9/4</td> <td>9/4</td> <td>(2)</td> <td>15.0</td> <td>12.5</td> <td>626.5</td> <td>18.8</td> <td>338.3</td> <td>14208.6</td>		2 9/4	9/4	(2)	15.0	12.5	626.5	18.8	338.3	14208.6	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	0 or 1			(3)	18.7	14.7	733.5	22.0	396.1	19476.4	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $				(1)	6.3	6.3	317.5	9.5	171.4	3649.2	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		3	9/4	(2)	9.1	9.1	455.0	13.6	245.7	7494.3	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		0	0, 1	(3)	10.8	10.8	539.0	16.2	291.1	10516.8	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $				(4)	12.0	12.0	602.5	18.1	325.3	13140.8	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	۱۸/	G	Grd No	¢	To (mm)		Stiffe	ener sections (i	mm)		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	vv	G	GIÙ NU.	5		Tw	Dw	Tf	Df	As	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		1	(1)	9	15.0	17.0	2557.5	42.6	426.2	61774.1	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		0	(1)	9	12.7	12.9	1930.5	32.2	321.7	35197.8	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	2	(2)		16.9	16.3	2448.0	40.8	408.0	56597.6	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	0		(1)		11.3	10.6	1593.0	265.5	265.5	23966.7	
(3)         17.8         15.4         2304.0         384.0         384.0         50135           1         (1)         4         15.0         16.6         2490.0         415.0         415.0         58556           2         (1)         4         12.7         12.6         1884.3         314.0         314.0         33533           2         (2)         4         16.9         15.9         2383.5         397.2         397.2         53654           3         (2)         4         15.0         13.1         1960.5         326.7         326.7         36300		3	(2)	9	15.0	13.4	2007.0	334.5	334.5	38042.7	
1         (1)         4         15.0         16.6         2490.0         415.0         415.0         5856           2         (1)         4         12.7         12.6         1884.3         314.0         314.0         33533           2         (2)         4         12.7         12.6         1884.3         314.0         314.0         33533           3         (2)         4         15.9         2383.5         397.2         397.2         53654           3         (2)         4         15.0         13.1         1960.5         326.7         326.7         36300			(3)		17.8	15.4	2304.0	384.0	384.0	50135.0	
2         (1) (2)         4         12.7 16.9         12.6 15.9         1884.3 2383.5         314.0 397.2         314.0 397.2         33533 397.2           1         (1)         11.3         10.4         1555.5         259.2         259.2         22851           3         (2)         4         15.0         13.1         1960.5         326.7         326.7         36300		1	(1)	4	15.0	16.6	2490.0	415.0	415.0	58556.4	
1         2         (2)         4         16.9         15.9         2383.5         397.2         397.2         53654           (1)         11.3         10.4         1555.5         259.2         259.2         22851           3         (2)         4         15.0         13.1         1960.5         326.7         326.7         36300		0	(1)	4	12.7	12.6	1884.3	314.0	314.0	33533.3	
(1)         11.3         10.4         1555.5         259.2         259.2         22851           3         (2)         4         15.0         13.1         1960.5         326.7         326.7         36300	1	2	(2)	4	16.9	15.9	2383.5	397.2	397.2	53654.5	
3 (2) 4 15.0 13.1 1960.5 326.7 326.7 36300	1		(1)		11.3	10.4	1555.5	259.2	259.2	22851.6	
		3	(2)	4	15.0	13.1	1960.5	326.7	326.7	36300.3	
(3) 17.8 15.0 2245.5 374.2 374.2 47621			(3)		17.8	15.0	2245.5	374.2	374.2	47621.4	
Stiffener sections (mm)	\\/	0	0	To (	~~)		Stiffe	ener sections (i	mm)		
Tw Dw Tf Df As	VV	G	3	ip (		Tw	Dw	Tf	Df	As	
1 9.6 1437.0 23.9 239.5 19502.5		1				9.6	1437.0	23.9	239.5	19502.5	
1 2 4 15.0 9.6 1435.5 23.9 239.2 19461.8	1	2	4	15	5.0	9.6	1435.5	23.9	239.2	19461.8	
3 9.6 1438.9 24.0 239.8 19555.4		3				9.6	1438.9	24.0	239.8	19555.4	

Table 5 Designed beam section scantlings of oil-tight bulkhead

dealing with different sizes of vertical webs, and with particular reference to 3 vertical webs, is discussed. In a practical case, the number of vertical webs is not likely to exceed three, and when the number of vertical webs is one or two, the design method in Section 2.3 is directly applicable.

An example chosen for the present study is the oil-tight bulkhead in Fig. 8 which is the same model shown in Fig. 1. In this structure, the grillage consists of 3 horizontal girders of similar size with different girder spaces which consists of a very heavy centre web and two side webs which have similar cross-sections to the horizontal girders. However, in the present study, only constant beam spaces are considered with 3 girders and 3 vertical webs. The design of stiffeners is excluded from this study, and the relevant design method can be referred to Section 2.3.

As was the case in Section 2.3, it is assumed that the vertical webs support the horizontal girders by means of intersection reactions. The external load is assumed to be taken by horizontal girders in the form of line loads, these loads differ depending on the position of the girders. In other

words, with the vertical stiffening system, the external load is firstly applied to the stiffeners and these loads are transferred to the horizontal girders by means of stiffener end reactions. Then, at the intersection points between horizontal girders he position of th,girders by means of exist. For c ovenience of as bysis, the intersection means of sre express otherthe total line load on a girder between webs. The general arrangement of this grillage system is shown in Fig. 8(a).

The design method for this grillage system is similar to the method in Section 2.3 except that, here, there are differing values of intersection reactions along each girder. By following the normal practice for 3 vertical webs, the central vertical web is chosen to be stronger than the two side webs of the same size. Referring to Figs. 8(b) and (c), the required beam sizes are determined as follows in terms of the required plastic bending moments,

#### (1) Horizontal Girders

The collapse modes and analysis method are shown in Fig. 9. The actual collapse mode and load can be found by



(a) Loading and Internal Reactions



(b) Collapse Modes of Cirders



(c) Collapse Modes of Webs

Fig. 8 Grillage system of oil-tight bulkhead



Fig. 9 Collapse modes of horizontal girders (fixed ends)

placing the two central hinge positions to give the highest plastic bending moment as in the following equation:

$$M_{pg} = \frac{x}{4} [W(1-x) - \sum R(i) \cdot d(i)]$$
(9)

Where,

 $M_{pq}$  = Plastic bending moment on girder

W = Line load on the girder l = length of the girder R(i) = Reactions at intersection i d(i) = Deflections at intersection ix = Assumed hinge position

### (2) Vertical Webs

The general design method for each vertical web is discussed in Section 2.3 and Fig. 9. In the present case, the sizes of the central web are different from the side webs due to different intersection reactions along the horizontal girder. In the abe a equations for the minimum required plastic bending moments (i.e. beam sizes), the magnitudes of the intersection reactions have a crucial influence on the size of beams and collapse modes for girders and webs. As mentioned earlier, intersection reaction values are expressed in terms of the girder line load of one web space and to make the central vertical web stronger than side vertical webs, the central intersection reaction value should be greater than those values on the side intersections. Then the maximum possible reaction values can be expressed by the following equations:

$$\alpha(1) = \alpha(3) \le 1.0$$
  

$$\alpha(2) = 1.0 + 1.0 - \alpha(1)$$
(10)

Where,  $\alpha(i)$  = Ratio of reaction value at *i* intersection to one web space girder load.

When the  $\alpha(2)$  value takes an artificial reaction value greater than the maximum possible value of the above equation, the collapse mode changes from two positive hinge along a beam to one negative hinge at the centre intersection. Although this case is possible in practice, the increase of reaction value above the maximum value is not necessary, because this would result in a waste of material by increasing the web size unnecessarily (Zaslavsky, 1962).

Some of the examples of the collapse modes for 3-supports with external line load are shown in Figs. 10. In these figures, the top lines represent plastic moments resulting from external load; bottom lines, plastic moments resulting from reactions; middle lines, the resultant bending moments of the two. The possible hinge points are shown by \* signs.

Now the above design method is applied to the oil-tight bulkhead of Fig. 8. The main scantlings of the existing design are shown in Table 6. For comparison between the existing design and the new designs, the design data and beam data



The results of the designs for various reaction values are shown in Table 7. Each case represents different reaction values at 3 supports along a girder. In this table, case (VI) represents a similar design to the existing one. The designed beam scantlings in case (VI) are shown in Table 8.

The conclusion from these results, therefore, is that the

as in Table 1.

are chosen as closely as possible to the existing design.

The girders and side webs are designed according to the

beam data in Table 1 and the heavier central web is designed

with Dw/Tw = 400 and the other ratios are kept the same

o

Table 7 Design weight of primary members of oil-tight bulkhead

Case	$\alpha(1\&3)$	$\alpha(2)$	Wg	$W_w$	$W_t$
	0.8	1.5	11.2	35.8	47.0
-	0.4	1.6	12.2	33.8	46.0
	0.3	1.7	13.2	31.5	44.8
IV	0.2	1.8	14.3	28.9	43.2
V	0.15	1.85	14.9	27.3	42.2
VI	0.1	1.9	15.5	25.6	41.2
VII	0.05	1.95	15.9	26.1	42.0

 $\alpha(i)$  = Non-dimensional internal reactions at intersection i

Wg = Weight of girder (tonnes).

 $W_w$  = Weight of webs (tonnes)

 $W_t$  = Total weight (Wg + Ww) (tonnes)

#### Table 8 Designed beam section for Case VI

(a) H	a) Horizontal girders (u								
G	Zpg	Тр	Dw/Tw	Tw	Dw	Tf	Df	As	
1	2.251E7	11.3	150	10.2	1537.5	25.6	256.2	22325.8	
2	4.502E7	15.0	150	12.9	1938.0	32.3	323.0	35471.8	
3	6.753E7	17.8	150	14.8	2218.5	37.0	369.7	46483.1	
(b) Vertical webs (unit: mn									
(b) \	Vertical v	webs					(ur	nit: mm)	
(b) \ W	Vertical v Zpw	webs Tp	Dw/Tw	Tw	Dw	Tf	<b>(ur</b> Df	nit: mm) As	
(b) V W 1	Zpw 1.523E7	webs Tp 15.0	Dw/Tw 150	Tw 9.0	Dw 1342.9	Tf 22.4	(ur Df 223.8	nit: mm) As 17033.2	
(b) V W 1 2	Zpw 1.523E7 2.893E8	<b>vebs</b> Tp 15.0 15.0	Dw/Tw 150 400	Tw 9.0 14.2	Dw 1342.9 5692.0	Tf 22.4 35.6	(ur Df 223.8 355.7	nit: mm) As 17033.2 93652.8	

simple design method for heavy primary members in grillages which utilities the different intersection reaction values to give different beam sizes may be used for general grillage system and also the designer can control the beam sizes by choosing appropriate reaction values.

# 7. Concluding Remarks

The basic concept of plastic design method of ship grillage is introduced with the example of transverse oil tight bulkhead design.

The results shows that minimum weight design leads to a two-directional grillage system of heavy horizontal girders along the shorter span and vertical small stiffeners.

The simple design method for heavy primary members in grillages which utilities the different intersection reaction values to give different beam sizes may be used for general grillage system and also the designer can control the beam sizes by choosing appropriate reaction values. However, the number of heavy primary members (horizontal girders and vertical webs) and also the number of stiffeners should be decided by considering the whole structural continuity and other functional requirements.

### Acknowledgements

The work described in this paper is supported by the research fund from Inha University.

### References

- Broughton, W.J., 1962. Limit Design of Stiffened Rectangular Plates under Lateral Loading with Application to Ship Bulkheads. *Society of Naval Architets and Marine Engineers.*
- Chowdhury, M., 1977. *The Optimal Design of Ship Structures.* Ph.D. Thesis, University of Newcastle upon Tyne, U.K.
- Kim, K.S., 1982. Strength Analysis as a Basis for Structural Design and Optimisation. Ph.D. Thesis, University of Newcastle upon Tyne, U.K.
- Kim, K.S. & Hong, K.S., 2004. Formulation of General Equations for Plastic Collapse Loads of Grillages under Lateral Point Load. *Journal of the Society of Naval Architects of Korea*, 41(6), pp.91-101.
- Kim, K.S. Hong, K.S. & Lee, S.C., 2001. Plastic Analysis of Simply-supported Grillages under a Point Load. *Journal of Offshore and Polar Engineering*, 11(4), pp.310-314.
- Kim, K.S. & Jin, J., 2009. Multi-criteria Structural Optimization Methods and their Applications. *Journal of the Society of Naval Architects of Korea*, 46(4), pp.409-416.
- Kim, K.S. Kim, K.S. & Park, H.J., 2004. A Study on the Analysis and Design of Grillages under a Worst Point Load. *Key Engineering Materials*, 261-263, pp.783-788.
- Kim, K.S. & Park, Y.H., 1995. A Plastic Design Method of Grillages under a Lateral Point Load. *Journal of Hydrospace Technology*, 1(2), pp.100-115.
- Zaslavsky, A., 1962. Limit Design of Crossed Steel Beams. *Proc.* of American Society of Civil Engineers, No. ST1, pp.135-169.

