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A Variable Step-Size NLMS Algorithm with Low Complexity

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Abstract

In this paper, we propose a new VSS-NLMS algorithm through a simple modification of the conventional NLMS algorithm, which leads to a low complexity algorithm with enhanced performance. The step size of the proposed algorithm becomes smaller as the error signal is getting orthogonal to the input vector. We also show that the proposed algorithm is an approximated normalized version of the KZ-algorithm and requires less computation than the KZ-algorithm. We carried out a performance comparison of the proposed algorithm with the conventional NLMS and other VSS algorithms using an adaptive channel equalization model. It is shown that the proposed algorithm presents good convergence characteristics under both stationary and non-stationary environments despites its low complexity.

Keywords: *Adaptive filters, Normalized least mean square (NLMS), Variable step-size NLMS (VSS-NLMS)*

I. Introduction

Adaptive filters are widely employed in many signal processing applications. One of the commonly used adaptive algorithms is the normalized least-mean square (NLMS) algorithm[1],[2]. This is mainly due to its robustness and the simplicity of its implementation. It is well known that the choice of the step size governs the speed of convergence and the misadjustment which is a steady-state performance with the filter weights fully adjusted. Many different schemes which enhance these two convergence characteristics have been proposed. Harris et al. [3] and Kwong et al. [4] proposed variable step size (VSS) LMS algorithms where the step size is adjusted to improve performance, which led to less misadjustment and better tracking property than the conventional LMS algorithm. More recently, several VSS NLMS algorithms were introduced and these algorithms also show better performance than the conventional NLMS, with rapid convergence, good tracking, and low misadjustment [5]-[7].

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In order to get both rapid convergence and small misadjustment, it is necessary to control the step size μ in such a way that for rapid convergence the values of μ keep relatively large when the filter weights are being converged, while those are getting smaller after convergence is achieved for reducing misadjustment without affecting tracking performance to non-stationary inputs. For this purpose, Karni and Zeng [8] proposed a new adaptive damped convergence factor based on the principle of orthogonality. In this work, we introduce a new VSS-NLMS algorithm where the step-size is controlled by the degree of orthogonality through a simple modification of the conventional NLMS algorithm. However, the algorithm is more computationally efficient and gives as good performance as Karni-Zeng's algorithm in the stationary environment and better in the non-stationary environment.

In Section II, we formulate the proposed VSS-NLMS algorithm and its justification is discussed. In Section III, we apply the proposed adaptation algorithm to an adaptive equalization problem and simulation results obtained are compared to existing algorithms. Section IV contains the conclusions.

II. Proposed VSS-NLMS Algorithm

According to the conventional NLMS algorithm, the adaptive filter weights vector is updated as follows:

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i + \frac{\mu}{\|\boldsymbol{x}_i\|^2} e_i \boldsymbol{x}_i, \quad 0 < \mu < 2 \tag{1}$$

where $\boldsymbol{w}_i$ is the filter weights vector, $\boldsymbol{x}_i = [x_i \; x_{i-1} \cdots x_{i-M+1}]^T$ is the input vector, e_i is the adaptive filter error, μ is the step size, and M is the number of filter weights. $\|\cdot\|$ is the Euclidean norm of a vector. First, we introduce an update equation for the proposed VSS-NLMS algorithm and then we discuss its validity and justification later. The algorithm is as follows:

◎ VSS-NLMS I :

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i + \frac{\mu}{\|\boldsymbol{x}_i\|^2} e_i \boldsymbol{x}_i, \quad \text{if } \alpha e_i^2 \|\boldsymbol{x}_i\|^2 \geq 1 \tag{2a}$$

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i + \mu\alpha e_i^3 \boldsymbol{x}_i, \quad \text{if } \alpha e_i^2 \|\boldsymbol{x}_i\|^2 < 1 \tag{2b}$$

where α is a sort of control factor which will be referred to later. From (2a), one can see the VSS-NLMS I algorithm is the same as the conventional NLMS when $\alpha e_i^2 \|\boldsymbol{x}_i\|^2 \geq 1$, but in case of $\alpha e_i^2 \|\boldsymbol{x}_i\|^2 < 1$, the algorithm is updated using (2b).

Equation (2b) can be rewritten as

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i + \frac{\mu\alpha e_i^2 \|\boldsymbol{x}_i\|^2}{\|\boldsymbol{x}_i\|^2} e_k \boldsymbol{x}_k = \boldsymbol{w}_k + \frac{\mu\alpha \|e_i \boldsymbol{x}_i\|^2}{\|\boldsymbol{x}_i\|^2} e_i \boldsymbol{x}_i, \quad \text{if } \alpha \|e_i \boldsymbol{x}_i\|^2 \leq 1 \tag{3}$$

If the adaptive filter is operating in the vicinity of optimum point on the performance surface, $\|e_i \boldsymbol{x}_i\|^2 \cong 0$ according to the principle of orthogonality[1]. Therefore, due to small values of $\mu\alpha\|e_i \boldsymbol{x}_i\|^2$ the filter weights can be adjusted enough to reach the optimum point. From the fact that $\|e_i \boldsymbol{x}_i\|^2$ is related to the amount of being converged, we can see that in the state of convergence, multiplying the normal step size μ by $\|e_i \boldsymbol{x}_i\|^2$ will make the actual step size smaller and contribute to reduce the misadjustment. In the meanwhile, Karni and Zeng also introduced a VSS-LMS algorithm using the principle of orthoghonality[8], which has been originally developed for standard LMS algorithm. They suggested the following step size.

$$\hat{\mu}_k = \mu\left(1 - e^{-\alpha\|e_k \boldsymbol{x}_k\|^2}\right) \tag{4}$$

In (4), the term inside the parenthesis can be expanded using Taylor expansion as follows:

$$\left(1 - e^{-\alpha\|e_k \boldsymbol{x}_k\|^2}\right) = \alpha\|e_k \boldsymbol{x}_k\|^2 - \frac{\alpha^2\|e_k \boldsymbol{x}_k\|^4}{2!} + \frac{\alpha^3\|e_k \boldsymbol{x}_k\|^6}{3!} - \cdots \tag{5}$$

We see that equation (3) adopts only the first term of the Taylor expansion. That is, (3) is an NLMS-type algorithm using the approximated step size which was proposed by Karni and Zeng. However, for valid approximation, $\alpha\|e_k \boldsymbol{x}_k\|^2$ needs to be small. So we assume that $\alpha\|e_k \boldsymbol{x}_k\|^2$ should be smaller than 1, which leads to the inequality in (3). With this assumption and using the first term of the Taylor expansion, we will show the VSS-NLMS I algorithm has almost the same performance as the Karni and Zeng's algorithm (KZ-algorithm) through simulation in Section III. As we see, the VSS-NLMS I has less computational load than the KZ-algorithm which includes computation of exponential function[1].

Next, we consider the proper selection of α in

1) The basic idea of the VSS-NLMS I has been originally proposed in [9] written in Korean.

(2a) and (2b). Actually, the VSS-NLMS I algorithm and KZ-algorithm work well over the wide range of values of α. However, if the input signal has small variance, $\alpha\|e_k x_k\|^2$ becomes relatively small and this implies that the convergence might be achieved, although the algorithm has not been fully converged yet. This misevaluation may prevent the filter weights from being adapted further. Fig. 3 shows the convergence behaviors of the VSS-NLMS I as the input variance is varied with α being fixed. (The KZ-algorithm also shows similar behaviors.) From Fig. 3, since the inputs of relatively small variance give poor convergence characteristics for an adaptive filter, we need to choose large values for α in compensation for the small input variance. To the contrary, for inputs of large variance, α should be small for convergence itself and low misadjustment. These facts imply that α is needed to be normalized. So we modified (2b) and the modified algorithm is as follows:

◎ VSS-NLMS II:

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i + \mu\alpha_i e_i^3 \boldsymbol{x}_i, \quad \text{if } \alpha_i e_i^2 \|\boldsymbol{x}_i\|^2 \le 1 \quad \text{where } \alpha_i = \frac{M\beta}{\|\boldsymbol{x}_i\|^2} \tag{6}$$

where β is a control factor which may have similar values to the range of α. The above equation is rewritten as

$$\boldsymbol{w}_{i+1} = \boldsymbol{w}_i + \frac{\mu'}{\|\boldsymbol{x}_i\|^2} e_i^3 \boldsymbol{x}_i, \quad \text{if } e_i^2 \le \frac{1}{M\beta} \tag{7}$$

where $\mu' = M\beta\mu$.

From (1) and (7), one can see that there is a minor difference that e_i is replaced by e_i^3 and one comparison operation is added. Therefore, additional computation required is trivial as compared to that of the NLMS. The proposed algorithm requires $3M+3$ multiplications, $3M-1$ additions and one comparison operation, while the NLMS requires $3M+1$ multiplications and $3M-1$ additions[2]. We will show that this modification enhances convergence performance through simulation in Section III.

III. Simulation Results and Discussions

In this section, we demonstrate the performance of the proposed algorithm by carrying out computer simulations using a simple channel equalization model, and compare their convergence characteristics with other VSS NLMS algorithms. In the simulation, a random signal d_i with equiprobably generated ± 1 level is used as input data and it is filtered through the intersymbol interference channel which has the following impulse response[1]:

$$h_i = \begin{cases} 0.5[1 + \cos\{2\pi(i-2)/3.1\}], & i = 1,2,3 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

White Gaussian noise with variance $\sigma_n^2 = 0.001$ is added to the filtered signal. The number of the adaptive filter weights, M was 16 and we chose the step size $\mu = 1.0$ for stable convergence.

Fig. 1 compares the convergence of the VSS-NLMS I with the NLMS and the KZ-algorithm. For meaningful comparison, the KZ-algorithm was normalized (KZ-NLMS) by dividing $\hat{\mu}_i$ in (4) by $\|x_i\|^2$. The mean squared error (MSE) is calculated and averaged over 500 independent trials as a performance measure. It is clear from the figure that while the convergence speed of the VSS-NLMS I is the same as that of the NLMS, the final performance after convergence is better by about 3 dB than the NLMS. As aforementioned in Section II, Fig. 1 also indicates that the VSS-NLMS I and the KZ-NLMS have similar convergence behaviors.

Fig. 2 shows the non-stationary characteristics of the three algorithms. For the non-stationary environment, a random signal d_i with equiprobably generated ± 0.5 level is input before the number of adaptation is 900, and after then the input data is abruptly changed into a signal with equiprobably generated ± 1 level. As we can see from Fig. 2, the VSS-NLMS I has as good tracking capabilities as the NLMS and better steady-state convergence characteristics as well. However, we may notice that the VSS-NLMS I and the KZ

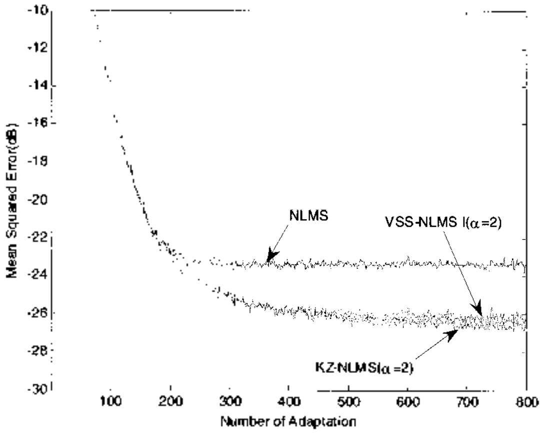

Fig. 1. Comparison of the NLMS, KZ-NLMS and the VSS-NLMS I in stationary environment.

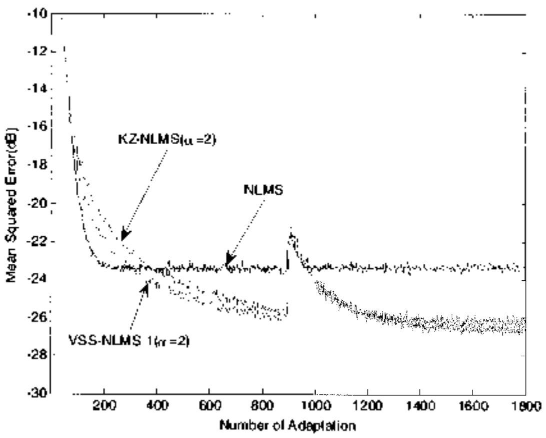

Fig. 2. Comparison of the NLMS, KZ-NLMS and the VSS-NLMS I in non-stationary environment.

–NLMS show slower convergence speed than the NLMS before about 300 iterations, where the input signal d_i has ± 0.5 level. This degradation is due to a relatively small input variance as we explained in Section II.

Fig. 3 shows how sensitive the algorithm is to the input variance. The figures in Fig. 3 denote various levels of the input signal d_i . In order to alleviate this sensitivity, α should be properly selected so that it can compensate for the small input variance. To over–come this problem, we proposed the VSS–NLMS II where α is normalized in Section II.

Fig. 4 and Fig. 5 show the stationary and non–stationary performances of the VSS–NLMS II. Note that the VSS–NLMS I and the VSS–NLMS II show almost the same stationary convergence behaviors, but in case of non–stationary environment, the VSS–

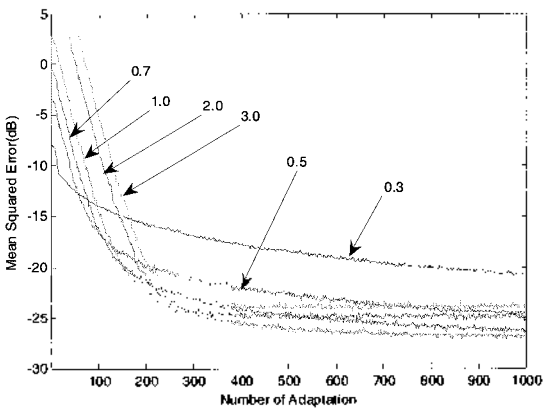

Fig. 3. Convergence behavior of the VSS-NLMS I for various input variance.($\alpha=1$)

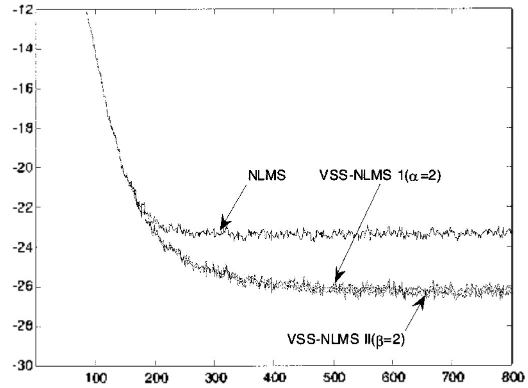

Fig. 4. Comparison of the NLMS, the VSS-NLMS I and VSS-NLMS II in stationary environment.

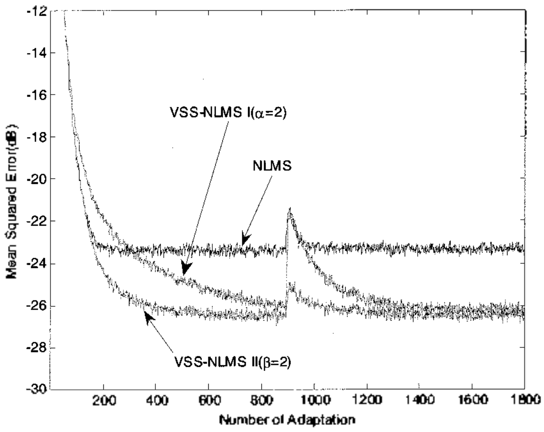

Fig. 5. Comparison of the NLMS, the VSS-NLMS I and VSS-NLMS II in non-stationary environment.

NLMS II doesn't show performance degradation caused by small input variance any more, and so outperforms the VSS–NLMS I. Fig. 6 clearly shows the convert–gence behaviors of the VSS–NLMS II with various input variances. Comparing Fig. 6 with Fig. 3, unlike the VSS–NLMS I, we see that the VSS–NLMS II

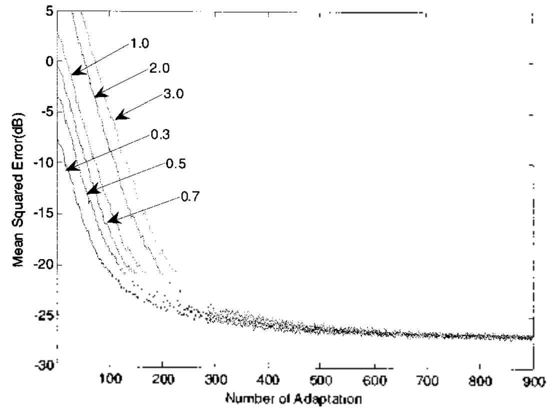

Fig. 6. Convergence behavior of the VSS-NLMS II for various input variances.(β = 1)

shows good convergence performance regardless of input variances.

Next, we present the simulation results of the comparison with other two VSS NLMS algorithms, Set Membership NLMS (SM-NLMS) [5] and VSS Affine Projection (VSS-AP) [6]. For reader's convenience, these two algorithms are presented as follows:

◎ SM-NLMS algorithm :

$$w_{i+1} = w_i + \frac{\mu\alpha_i}{\|x_i\|^2} e_i x_i,$$
$$\text{where} \quad \alpha_i = \begin{cases} 1 - \frac{\gamma}{|e_i|}, & \text{if} \quad |e_i| > \gamma \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

◎ VSS-AP algorithm :

$$w_{i+1} = w_i + \frac{\mu\alpha_i}{\|x_i\|^2} e_i x_i, \qquad \alpha_i = \frac{\|p_i\|^2}{(\|p_i\|^2 + C)},$$
$$p_i = \beta p_{i-1} + (1-\beta) e_i \frac{x_i}{\|x_i\|^2} \tag{10}$$

In case of the VSS-AP shown in (10), the projection order was set to 1 for minimum computation. For the simulation, we chose μ = 1.0, γ = 0.035, C = 0.0001, β = 0.988.

As illustrated in Fig. 7 and Fig. 8, the overall performance of the VSS-NLMS II is relatively good, even though the performance of the VSS-AP is better than that of the VSS-NLMS II. However,

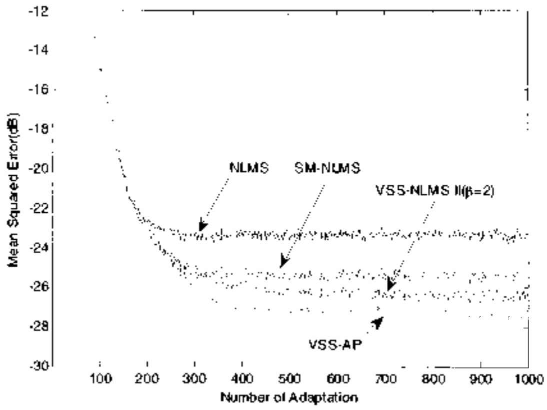

Fig. 7. Comparison of the NLMS, the VSS-NLMS II, the SM-NLMS and VSS-AP in stationary environment.

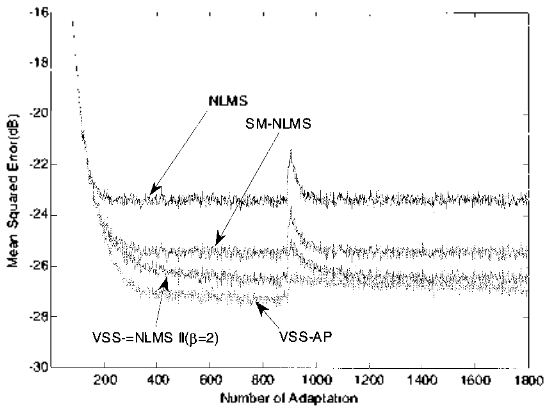

Fig. 8. Comparison of the NLMS, the VSS-NLMS II, the SM-NLMS and VSS-AP in non-stationary environment.

comparing the computational burdens of the two algorithms, we believe that the proposed VSS-NLMS II will be an attractive algorithm in practice.

IV. Conclusion

In this work, we proposed a new VSS-NLMS algorithm based on the principle of orthogonality. The more orthogonal to the input vector the error signal is, the smaller the convergence factor of the proposed algorithm is. We also showed that the proposed algorithm is an approximated normalized version of the KZ-algorithm and requires less computation than the KZ-algorithm.

We carried out a performance comparison of the proposed algorithm with the conventional NLMS and other VSS algorithms using an adaptive channel equalization model. It is shown that the proposed algorithm presents good convergence characteristics under both stationary and non-stationary environments despites its low complexity.

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[Profile]

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