

# Performance Improvement in Alternate Mainbeam Nulling by Adaptive Estimation of Convergence Parameters in Linearly Constrained Adaptive Arrays

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**Abstract**—A novel approach is presented to improve the array performance of the alternate mainbeam nulling in a linearly constrained adaptive array processor in coherent environment. The convergence parameters in the linearly constrained LMS algorithm with a unit gain constraint and a null constraint in the direction of the desired signal are adaptively estimated to reduce the error power between the desired signal and the array output in the 2-dimensional convergence parameter space.

It is shown that the case for estimating the convergence parameter for the unit gain constraint with that for null constraint fixed performs best. Also, it is observed that the proposed method performs significantly better than conventional methods as the number of coherent interferences increases.

**Index Terms** – Adaptive Array, Convergence Parameter, Constraint, Estimation, Mainbeam, Nulling

## I. Introduction

Linearly constrained adaptive array processing has been widely investigated last a few decades. In the linearly constrained adaptive array processor, the directional and spectral informations for the desired signal is assumed to be known a priori, while those for interference signals are unknown. Also,

the desired signal and the interference signals are assumed to be mutually uncorrelated. The desired signal is estimated in a least mean square sense such that the

array output power is minimized while maintaining a unit gain in the look direction (i.e., the direction of the desired signal).

In the linearly constrained adaptive array processor proposed by O. L. Frost, the desired signal is estimated such that the array weights are updated iteratively by the linearly constrained LMS adaptive algorithm[1]. It has been shown that the algorithm performs well if the desired signal and the interference signals are uncorrelated. Also, it has self-correcting capability for roundoff or truncation errors in digital implementation. But, it was shown that the signal interaction inherent in the algorithm causes partial cancellation of the desired signal in the array output even though the desired signal and the interferences are uncorrelated[2]. If the interference signals are correlated with the desired signal, the signal interaction causes the desired signal to be partially or totally cancelled in the array output depending on the extent of correlation between the desired signal and the interferences.

To avoid the signal cancellation, a variety of methods have been proposed[2-6]. A master-slave type array processor with subtractive preprocessing was proposed to eliminate the signal interaction between the desired signal and the interferences during adaptive process[2]. In this method, the desired signal is eliminated through subtractive preprocessing and thus the array weights in the master processor are updated by the linearly constrained LMS algorithm using only subtractive interferences which yields no signal interaction, resulting in improved array performance.

A spatial smoothing approach[3] employs subarray preprocessing to decorrelate coherent input signals. In this approach, the average of the input correlation matrix of each subarray (i.e., spatially smoothed correlation matrix) yields an input correlation matrix similar to a diagonal matrix as if all the incident input signals were uncorrelated. Thus the coherent interferences are successfully eliminated in the array output.

Recently, an alternate mainbeam nulling method has been proposed to prevent the signal interaction in the linearly constrained LMS algorithm[6]. The basic idea of the method is that the desired signal is alternately removed during adaptive process so that the effect of the signal interaction may be reduced. To this end, the

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null constraint is employed in the look direction in addition to the unit gain constraints such that the linearly constrained LMS algorithm with the unit gain constraint and that with the null constraint are implemented alternately.

In this paper, a novel approach of estimation of convergence parameters is proposed to improve the performance of the alternate mainbeam nulling method. One factor that affects the performance of the alternate mainbeam nulling method is the convergence parameters employed in the alternate implementation of the algorithms. The values of the convergence parameters which yield a better performance are shown to depend on the magnitude of the incoming interferences[6]. To determine the optimum pair of the convergence parameters in a given interference environment, the convergence parameters are estimated such that the mean square error between the desired signal and the array output is minimized, assuming that a training signal or a signal which is highly correlated with the desired signal is available. It is shown that the estimated convergence parameters yield a better performance than the arbitrarily specified values of convergence parameters used in the alternate mainbeam nulling method.

The proposed method may be applied in the area of the smart antenna systems of the base station in GSM system to eliminate interference signals[7].

## II. Linearly Constrained LMS Narrowband Array Processor

A narrowband linear array with  $N$  equispaced sensor elements is employed to examine the performance of the proposed method. Assume that a desired signal and interference signals are incident on the array. We want to estimate the desired signal as closely as possible by minimizing the effect of the undesired interference signals in the array output in a least mean square sense. To this end, a linearly constrained LMS algorithm was proposed[1], which is represented by

$$\mathbf{w}_{k+1} = \left[ \mathbf{I} - \frac{\mathbf{ss}^H}{N} \right] [\mathbf{w}_k - \mu \mathbf{y}_k^* \mathbf{x}_k] + \frac{s}{N} \quad (1)$$

where  $\mathbf{w}_k$  is the weight vector at the  $k$ th iteration, i.e.,  $[\mathbf{w}_{0k} \ \mathbf{w}_{1k} \ \dots \ \mathbf{w}_{(N-1)k}]^T$ ,  $\mathbf{I}$  is the  $N \times N$  identity matrix,  $\mathbf{x}_k$  is the input signal vector at the  $k$ th iteration, i.e.,  $[\mathbf{x}_{0k} \ \mathbf{x}_{1k} \ \dots \ \mathbf{x}_{(N-1)k}]^T$ ,  $\mathbf{y}_k$  is the array output at the  $k$ th iteration,  $\mu$  is convergence parameter,  $s$  is the  $N \times 1$  steering vector for the desired signal, i.e.,  $[\mathbf{1} \ e^{-j\beta\tau_0} \ e^{-j2\beta\tau_0} \ \dots \ e^{-j(N-1)\beta\tau_0}]^T$ ,  $\beta$  is the radian frequency of the desired signal,  $\tau_0 = \mathbf{d} \sin \theta_0 / v$ ,  $\theta_0$  is the incident angle from the array

normal,  $\mathbf{d}$  is interelement spacing,  $v$  is the propagation velocity,  $*$  and  $\mathbf{H}$  denotes complex conjugate and complex conjugate transpose respectively, and  $k$  is iteration index. It is assumed that the incident angle of the desired signal is known, and the desired signal and the interferences are uncorrelated. It is to be noted that the unit gain in the look direction is maintained during implementation of the algorithm.

In the linearly constrained LMS algorithm in (1), it is inherent that the interaction occurs between the desired signal and the interferences during adaptive process, which results in cancellation of the desired signal. It is known that the signal cancellation phenomenon occurs whether the desired signal and the interferences are correlated or not[2]. If the interferences are coherent with the desired signal, the desired signal will be totally cancelled in the array output.

To prevent the signal cancellation phenomenon in the linearly constrained adaptive array processor, the alternate mainbeam nulling method has been proposed[6].

## III. Alternate Mainbeam Nulling Method for Linearly Constrained Adaptive Array Processor

In the linearly constrained LMS algorithm, the desired signal may not be estimated properly due to the signal cancellation phenomenon, the main cause of which is the interaction between the desired signal and the interferences. One way of reducing the signal interaction is to partially remove the desired signal during adaptive process by employing a null constraint in the look direction. The linearly constrained LMS algorithm with a null constraint in the look direction is shown to be expressed as [6]

$$\mathbf{w}_{k+1} = \left[ \mathbf{I} - \frac{\mathbf{ss}^H}{N} \right] [\mathbf{w}_k - \mu \mathbf{y}_k^* \mathbf{x}_k] \quad (2)$$

In the alternate mainbeam nulling method, the iterative equation (2) is alternately implemented with that in (1) with fixed convergence parameters. The convergence parameters for the two adaptive algorithms in (1) and (2) may be different.

In the weight vector space, the weight vector updated by (1) stays on the constraint plane defined by  $\mathbf{w}^H \mathbf{s} = 1$  while that updated by (2) stays on the constraint subspace defined by  $\mathbf{w}^H \mathbf{s} = 0$ . Note that the constraint plane and the constraint subspace are parallel to each other. During implementation of the alternate mainbeam nulling method, the weight vector exchanges between the constraint plane and the constraint subspace. Since the weight vector on

$\mathbf{w}^H \mathbf{s} = \mathbf{0}$  is orthogonal to the steering vector in the look direction (i.e.,  $s$ ), the effect of the desired signal will be reduced in the signal interaction if the weight vector updated by (2) is applied to (1). It is noted that the array output  $\mathbf{y}_k$  employed in (1) is produced by the resulting weight  $\mathbf{w}_{k+1}$  in (2) (i.e.,  $\mathbf{y}_k = \mathbf{w}_{k+1}^H \mathbf{x}_{k+1}$ , where the iteration index has no sequential meaning). Thus, the array output produced by the weight vector in (1) yields less signal interaction than that by the original linearly constrained LMS algorithm.

It was shown that the performance of the alternate mainbeam nulling method depends on the values of the convergence parameters in (1) and (2). It was experimentally shown that their optimal values which yield the best array performances depend on the magnitude and the number of interferences [6]. Thus, it is not easy to find the optimal values of convergence parameters for a given signal environment.

To estimate the optimal values of convergence parameters, we minimize the mean squared error between the array output and a desired response (i.e., a training signal or a signal highly correlated with the desired signal) using the steepest descent method [8] in the 2-dimensional convergence parameter space.

#### IV. Performance Improvement by Adaptive Estimation of Convergence Parameters

To improve the performance of the alternate mainbeam nulling method, the convergence parameters are adaptively estimated such that the mean squared error between the array output and a desired response is minimized in the 2-dimensional convergence parameter space of the convergence parameter for the unit gain constraint  $\mu_u$  and that for the null constraint  $\mu_n$ .

The steepest descent method is employed to search the suboptimal values of convergence parameters. The gradients of the mean square error in terms of the convergence parameters  $\mu_u$  and  $\mu_n$  are numerically estimated by measuring the mean squared errors with respect to a specified convergence parameter interval in the  $\mu_u$ - $\mu_n$  axes.

The iterative equation to update the convergence parameters is given by

$$\boldsymbol{\mu}_{k+1} = \boldsymbol{\mu}_k - \eta \hat{\mathbf{v}}_k \quad (3)$$

where  $\boldsymbol{\mu}_k$  is a convergence parameter vector at the  $k$ th iteration, which is given by  $[\mu_{u,k} \ \mu_{n,k}]^T$ ,  $\eta$  is another convergence parameter for estimating the convergence parameters  $\mu_{u,k}$  and  $\mu_{n,k}$ .  $\hat{\mathbf{v}}_k$  is estimated gradient vector which is given by

$$\hat{\mathbf{v}}_k = [\hat{\mathbf{v}}_{u,k} \ \hat{\mathbf{v}}_{n,k}] \quad (4)$$

Where  $\hat{\mathbf{v}}_{u,k}$  and  $\hat{\mathbf{v}}_{n,k}$  are gradient estimates of the mean square error with respect to  $\mu_{u,k}$  and  $\mu_{n,k}$ , which are represented by

$$\hat{\mathbf{v}}_{u,k} = \frac{\Delta e_{u,k}^2}{\Delta \mu_u} \quad (5)$$

and

$$\hat{\mathbf{v}}_{n,k} = \frac{\Delta e_{n,k}^2}{\Delta \mu_n} \quad (6)$$

where  $\Delta e_{u,k}^2$  and  $\Delta e_{n,k}^2$  are the difference of the average squared errors at two points spaced by  $\Delta \mu_u$  and  $\Delta \mu_n$  along the  $\mu_u$  and  $\mu_n$  axes respectively.

The estimation of the convergence parameters is initiated when the steady state is reasonably reached with initial convergence parameters fixed. It is shown that the array performance is degraded if the convergence parameters are updated in the transient state. It is observed that  $\mu_{u,k}$  decreases and  $\mu_{n,k}$  increases as the power of the input signals increases. Thus, the array performance may be improved by setting the initial values of the convergence parameters based on the estimation of the input signal power at the sensor element.

The weight update is done as follows. The iterative equation for the unit gain constraint with estimated  $\mu_{u,k}$  is applied to an input signal sample vector, i.e.,

$$\mathbf{w}_{u,k+1} = \left[ \mathbf{I} - \frac{ss^H}{N} \right] [\mathbf{w}_{n,k} - \mu_{u,k} \mathbf{y}_{n,k}^* \mathbf{x}_k] + \frac{s}{N} \quad (7)$$

where  $\mathbf{y}_{n,k}$  is the array output generated by  $\mathbf{w}_{n,k}$ . For the next input signal sample vector, the iterative equation for the null constraint with the estimated  $\mu_{n,k}$  is applied as follows.

$$\mathbf{w}_{n,k+1} = \left[ \mathbf{I} - \frac{ss^H}{N} \right] [\mathbf{w}_{u,k} - \mu_{n,k} \mathbf{y}_{u,k}^* \mathbf{x}_k] \quad (8)$$

where  $\mathbf{y}_{u,k}$  is the array output generated by  $\mathbf{w}_{u,k}$ . In this way, the (7) and (8) are alternately implemented. The array output is generated by linearly interpolating the  $\mathbf{y}_{u,k}$ .

#### V. Simulation Results

The proposed method is implemented in the narrowband linear array to examine its performance. The array consists of 7 equispaced sensor elements (i.e.,  $N = 7$ ), where each element is followed by a complex

weight. A desired sinusoid of amplitude 0.1 is incident at array normal(i.e.,  $\mathbf{s} = [1 \ 1 \ \dots \ 1]^T$ ) while coherent interference signals are coming from other directions. The interelement spacing is assumed to be a half wavelength of the desired signal. The cases from 2 to 40 coherent interferences are simulated. A set of 20 randomly chosen angular directions are generated for each case for the number of interferences. Thus, 780 cases of incident interferences are simulated.

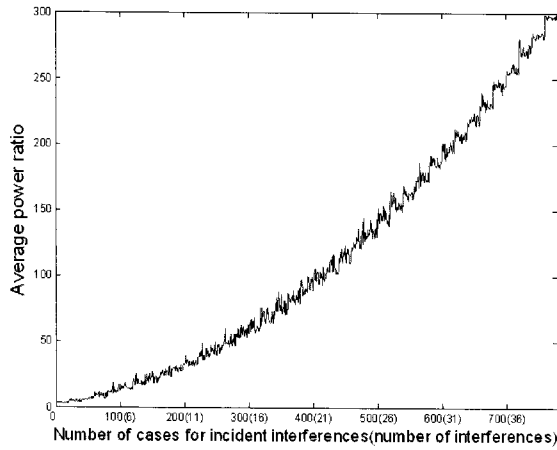


Fig. 1 Variation of average power ratios of interferences to the desired signal.

The average power ratios of the interferences over all the antenna elements to the desired signal are shown in Fig. 1.

The initial array weights  $\mathbf{w}_1$  is assumed to be

$$\mathbf{w}_1 = \left[ \frac{1}{7} \ \frac{1}{7} \ \dots \ \frac{1}{7} \right]^T \tag{9}$$

which satisfies the unit gain constraint (i.e.,  $\mathbf{w}_1^H \mathbf{s} = 1$ ). The initial convergence parameters  $\mu_{u,1}$  and  $\mu_{n,1}$  are 0.0015.

The convergence parameter  $\eta$  for estimation of  $\mu_{u,k}$  and  $\mu_{n,k}$  is fixed as  $10^{-4}$ . The convergence parameter interval  $\Delta\mu_u$  and  $\Delta\mu_n$  for gradient estimation is set as  $1.2 \times 10^{-4}$  along the  $\mu_u$  and  $\mu_n$  axes.

The performance surface of the mean square error in the 2-dimensional  $\mu_u - \mu_n$  space is shown in Fig. 2, where 12 coherent interferences are incident at the array. The track of  $\mu_{n,k}$  and  $\mu_{u,k}$  during adaptive estimation is shown with the level curves of the performance surface in Fig. 3. It is observed that there exists a global minimum and the estimated convergence parameters get closer to the global minimum(i.e.  $\mu_u = 0.000358$ ,  $\mu_n = 0.00198$ ).

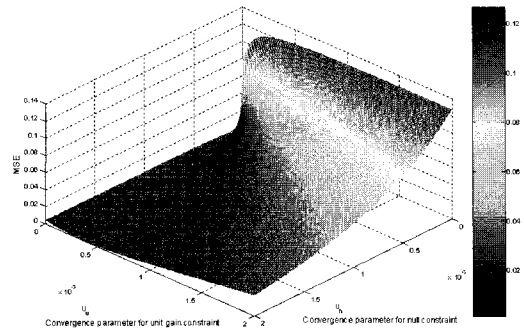


Fig. 2 Performance surface of the mean square error.

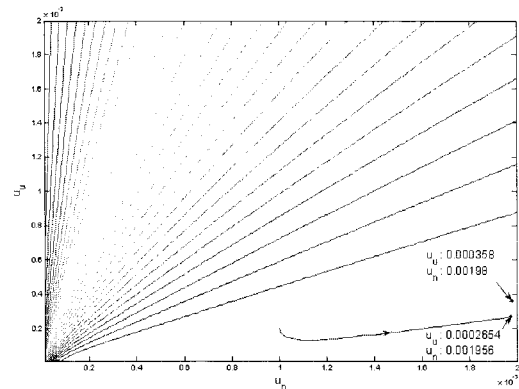


Fig. 3 Trajectory of the convergence parameters with level curves.

Two cases of estimating the convergence parameters are simulated for the proposed method. One case is to estimate both of  $\mu_u$  and  $\mu_n$ . The other case is to estimate  $\mu_u$  while  $\mu_n$  is fixed.

The performances of the two cases are compared with that of the alternate mainbeam nulling method(i.e., fixed  $\mu_u$  and  $\mu_n$ ) in terms of the mean square error in Fig. 4. It is shown that the case of estimating  $\mu_u$  with  $\mu_n$  fixed yields the least amount of the mean square error over almost all cases with respect to the number of interferences while the case of estimating both of  $\mu_u$  and  $\mu_n$  performs better than the alternate mainbeam nulling method.

It is observed that the case of estimating  $\mu_u$  with  $\mu_n$  fixed performs better as the number of interferences increases while other cases perform worse.

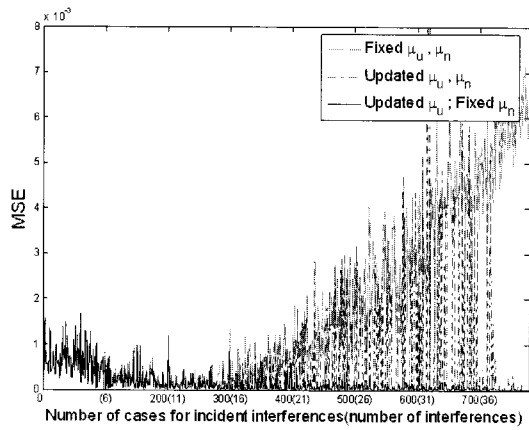


Fig. 4 Comparison of the mean square errors in terms of convergence parameter variations.

The case of estimating  $\mu_u$  with  $\mu_n$  fixed is employed to compare the performance of the proposed method to that of the conventional methods. It is assumed that 40 coherent interferences are incident at the array.

The output signals are compared with the desired signal in Fig. 5. It is shown that the output signal of the proposed method(i.e., captioned by AMN) is in phase and of almost same magnitude with the desired signal while that of the master-slave type method is inverted with reduced magnitude compared to the desired signal and that of the spatial smoothing approach is a little delayed.

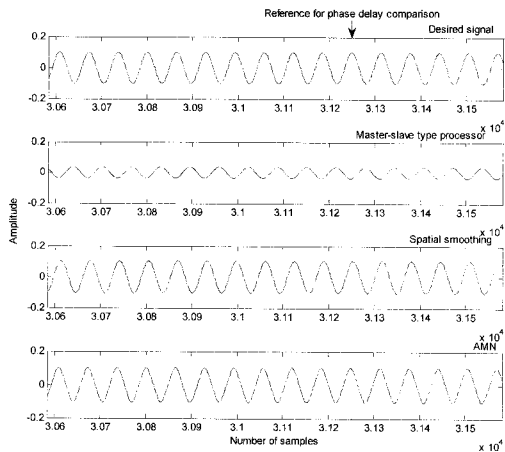


Fig. 5 Comparison of output signals with the desired signal.

The relevant error signals are displayed in Fig. 6. It is observed that the error signal of the proposed method is shown to be significantly small(about 0.01) compared with those of other methods(about 0.15 for the master-slave type processor and about 0.05 for the spatial

smoothing) . Therefore, the desired signal is more precisely estimated by the proposed method than by the conventional methods.

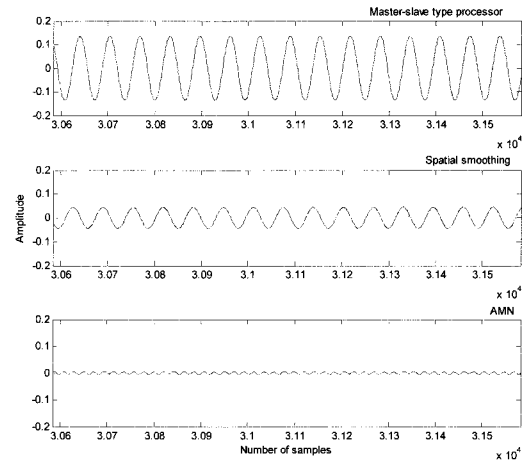


Fig. 6 Relevant error signals.

The beam patterns are compared in Fig. 7, where the angular directions of the interferences are denoted by vertical arrows. It is observed that the sidelobes

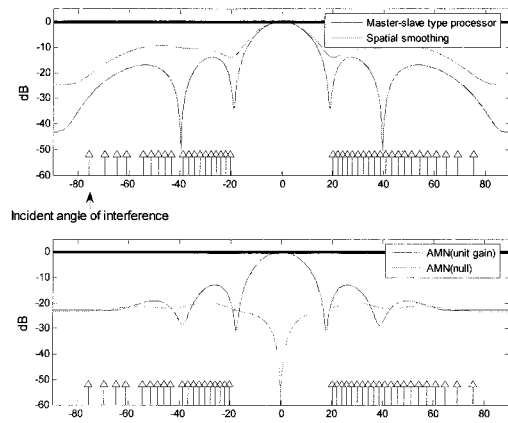


Fig. 7 Comparison of beam patterns.

of the proposed method with null constraint are relatively low(i.e., about -20 dB) and evenly distributed around the first sidelobe compared with those of the conventional methods, which yields more reduction of the interferences incident around the first sidelobes. Also, the array performances are shown to be not proportional to the gains of the beam patterns in the conventional methods.

To Find the performance of the proposed method in terms of signal-to-interference ratio(SIR), the output SIR of the proposed method is compared with that of the spatial smoothing approach with respect to the

number of interferences in Fig. 8. It is observed that the proposed method performs better than the spatial smoothing approach except the number of interferences are 8 and 9.

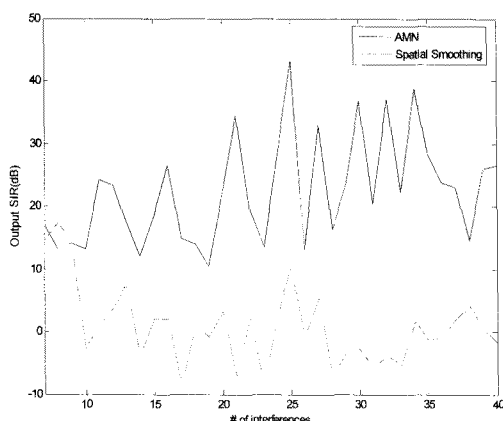


Fig.8 Comparison of signal-to-interference ratios.

## VI. Conclusions

The convergence parameters for the unit gain and null constraints in the linearly constrained LMS algorithm for the linear array are adaptively estimated to improve the performance of the alternate mainbeam nulling method. The convergence parameters are adjusted by the steepest descent method using the estimated gradient of the mean square error between the output signal and the desired response in the two dimensional convergence parameter space.

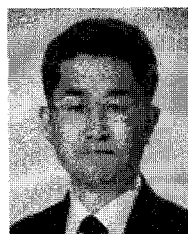
It is shown that estimating the convergence parameter for the unit gain constraint with that for the null constraint fixed yields better array performance than estimating both of convergence parameters. It is also shown that adaptive estimation of the convergence parameters improves the performance of the alternate mainbeam nulling method. The nulling performance of the proposed method is compared with that of the conventional methods in coherent signal environment.

## VI. Acknowledgement

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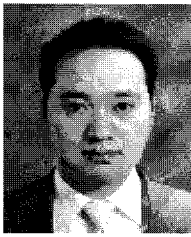
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