Performance Analysis of Chained Amplifier Systems for Metropolitan Optical Network Applications

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Abstract— In this paper, theoretical analysis for metropolitan optical networks is performed. First, analytical optical SNR is derived assuming each node consists of an EDFA, an optical filter, an optical switch, and a VOA, and then the relationship between OSNR and BER is studied. In a metropolitan optical network, an optical signal can be dropped to deliver data, and we also studied the effect of drop loss on system performance. When the drop loss is relatively small, the receiver structure of the node can be treated as a preamplifier receiver which is widely used in long-haul systems. In that case, ASE noise from EDFAs is the dominant noise source in the receiver. However, system performance is relatively insensitive to OSNR when the drop loss is significant because of the noise sources in the receiver (thermal and shot noise).

Index Terms—OSNR, BER, Q-factor, EDFA, Metropolitan Networks, ASE noise.

I. INTRODUCTION

Traditionally metropolitan networks are based almost exclusively on Synchronous Optical Network (SONET) ring architectures. While these networks may efficiently transport low-bandwidth, non-IP voice telephony traffic, they are significantly less effective with high-capacity IP services. Nowadays, many metropolitan core networks utilize existing DWDM systems engineered for the long-haul network[1,2].

In an optical metropolitan network, the optical signal is amplified by EDFAs to compensate path losses while travelling many nodes. In amplification processes, amplifiers add optical noise (ASE noise: amplified spontaneous emission noise) to signal, and the amount of noise introduced to signal depends on the characteristics of amplifiers. In this paper, we assume that the optical amplifiers operate in a constant gain mode and the input power to amplifiers is constant by VOAs (variable optical attenuators). (See Figure 1.)

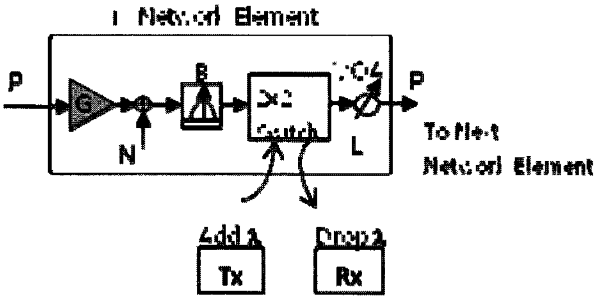

Fig.1 Network Element Model

Figure 1 shows the i^{th} network element model used in this paper. It is assumed that each network element can add and/or drop new wavelengths to achieve rearrangement and reconfiguration of the wavelength. In section II, the analytical expressions for the OSNR(optical signal-to-noise ratio) evolution with and without the effect of optical filters are derived. More ultimate system metric, BER, is studied in the following section including the effect of drop loss. Conclusions follow in the section IV.

II. OSNR EVOLUTION IN CHAINED AMPLIFIER SYSTEMS

A. OSNR Ignoring the Effect of Optical Filters

By assuming that all the amplifiers in each stage have the same gain (G) and the same noise power density (N_{ASE}), the output of i_{th} stage can be expressed as

$$P_{i+1} = L_i\{GP_i + N_{ASE}B_o\} \tag{1}$$

where B_o is the bandwidth of optical filter.

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Since VOAs control output power constant by forcing all the input power to EDFAs are same ($P_{i+1}=P_i=P_{const}$), the loss at each stage should be

$$L_i = \frac{P_{const}}{GP_{const} + N_{ASE}B_o}, \text{ i} = 1,2,\ldots \qquad (2)$$

L_i can be interpreted as the sum of path loss between EDFAs and the controlled VOA loss.

With loss given by (2), the added noise by each stage is given by

$$\begin{aligned}\Delta N &= N_{ASE}B_o \frac{P_{const}}{GP_{const} + N_{ASE}B_o} \\ &= \frac{P_{const}}{1 + GP_{const}/N_{ASE}B_o} \qquad \ldots\ldots\ldots (3)\end{aligned}$$

Since the optical noise is additive, ASE noise power will be increased n times after n_{th} stage. However, signal power will be decreased by the same amount because VOAs control the total output power of each stage constant. Therefore optical signal to noise ratio after n_{th} stage can be expressed as

$$\left.\frac{S}{N}\right|_{nth} = \frac{P_1 - n\Delta N}{n\Delta N} \qquad (4)$$

In eq.(4), it is assumed the input power to the first stage is signal power only (negligible noise power).

The above results are similar to published results by others [3-5] except the input power to the amplifiers in this study is kept constant by VOAs.

B. OSNR With Considering the Effect of Optical Filters

Effect of cascaded optical filters ignored so far. However, concatenated optical filters can affect noise power quite differently than a single optical filter.

Figure 2 shows amplifier noise path in the optical amplifier chain of Figure 1. While noise in the last stage go through a single optical filter, noise in the first amplifier must go through many optical filters and their effect on output signal to noise ratio can then be quite different.

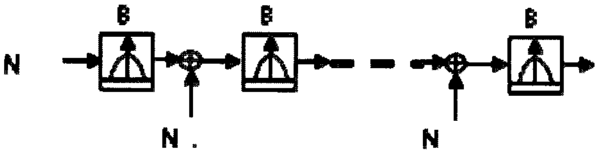


Fig. 2 Noise path in an optical amplifier chain.

It is assumed that all the optical filters have the same Gaussian field transfer function as follows.

$$H(f) = \exp\left(-\ln\sqrt{2}\left(\frac{f}{\Delta f_{3dB}/2}\right)^2\right) \qquad (5)$$

where Δf_{3dB} is 3dB bandwidth of the filter.

When the gaussian filter described by eq.(5) is cascaded by n times, the equivalent transfer function is

$$H^{(n)}(f) = \exp\left(-\ln\sqrt{2n}\left(\frac{f}{\Delta f_{3dB}/2}\right)^2\right) \qquad (6)$$

Then the equivalent noise bandwidth [6] of the cascaded filters is

$$B_{eq}^{(n)} = \sqrt{\frac{(\Delta f_{3dB})^2\pi}{8n\ln 2}} = \frac{0.753}{\sqrt{n}}\Delta f_{3dB} \qquad (7)$$

Therefore the optical signal-to-noise ratio expressed in eq.(4) requires a slight modification to include filtering effect on noise.

Noise power after n_{th} stage is now

$$\begin{aligned}N^{(n)} &= \frac{P_{const}}{1 + GP_{const}\Big/\left[N_{ASE}B_{eq}\left(1 + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{n}}\right)\right]} \\ &= \frac{P_{const}}{1 + GP_{const}\Big/\left[N_{ASE}B_{eq}\sum_{k=1}^{n}\frac{1}{\sqrt{k}}\right]} \qquad \ldots\ldots (8)\end{aligned}$$

and then the optical SNR is given by

$$\left.\frac{S}{N}\right|_{nth} = \frac{P_1 - N^{(n)}}{N^{(n)}} \qquad (9)$$

In eq.(9), the signal spectrum is assumed not altered by filters.

C. Calculation of OSNR

It is well known that the spectral shape of EDFA spontaneous emission noise is nearly flat over 30nm, and the spectral density is given by [3]

$$N_{ASE} = 2n_{sp}(G-1)hf \qquad (10)$$

where G is the amplifier gain, h is the Plank constant(=6.626×10^{-34} J·s), f is optical frequency, and n_{sp} is the inversion parameter of amplifier.

We need to know the value of n_{sp} to estimate the amount of noise power introduced to optical signal. n_{sp} depends on the population inversion in the amplifier as well as the absorption and emission cross sections $\sigma_a(\lambda)$ and $\sigma_e(\lambda)$ at the signal wavelength λ [3],

$$n_{sp} = \frac{N_2}{N_2 - \frac{\sigma_a(\lambda)}{\sigma_e(\lambda)} N_1} \tag{11}$$

where N_1 = lower-sate population and N_2 = upper-state population in the two-level system

However, n_{sp} is often expressed in terms of noise figure(NF) which can be measured by a relatively easy way. NF is approximately given by [7-9]

$$\begin{aligned} NF &\approx 2n_{sp}\frac{G-1}{G} + \frac{1}{G} \\ &\approx 2n_{sp} \quad \text{if } G >> 1 \end{aligned} \tag{12}$$

Noise figure of commercially available EDFAs is usually in the range of 4 to 6dB. If we take NF = 5.5dB, (12) gives $2n_{sp} = 3.55$.

We now can evaluate eq. (4) and (9). Figure 3 shows OSNR evolutions with and without the cascade effect of optical filters. In both cases, NF = 5.5dB, G = 23dB, P_{in} = -22dBm, and $f_{\Delta 3dB}$ = 36GHz are used.

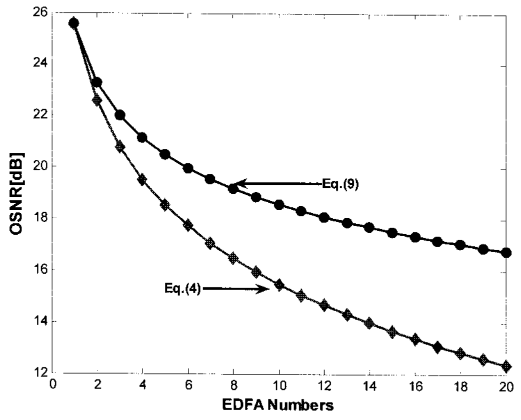


Fig. 3 Optical Signal-to-Noise Ratio with and without the cascade effect

OSNR with the cascade effect tends to saturate around 16.8dB after 20 cascaded amplifier stages while OSNR without the cascade effect is around 12.4dB. This result means that the accumulated ASE noise is effectively reduced by the cascade effect, that is, by the reduction of the equivalent noise bandwidth of the cascaded filters.

III. EVALUATION OF BER AND THE EFFECT OF DROP LOSS

ASE noise from EDFAs will degrade optical signal quality as described in (9). However, the ultimate system performance will be measured by BER of photo-detected signal. Therefore it is required to study how ASE noise degrades signal quality of photo-detected *electrical* signal. BER of a direct detection system can be calculated by assuming noises in the receiver have Gaussian statistics. The BER with the optimum setting of the decision threshold is then given by [8]

$$BER = \frac{1}{2}erfc\left(\frac{1}{\sqrt{2}}\frac{I_1 - I_0}{\sigma_1 + \sigma_0}\right) = \frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right) \tag{13}$$

where $erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-y^2)dy$, I_1 and I_0 are the average photocurrents (or equivalent electrical signal) generated by a 1 bit and a 0 bit, respectively, and σ_0^2 and σ_1^2 are the noise variance for a 0 bit and a 1 bit, respectively.

ASE noises from EDFAs will affect noise variances of photo-detected signals, and thus BER as expressed in (13). To study the effects of ASE noise on BER (or Q), let's consider a pre-amplifier structure receiver shown in Fig. 1. The noise variance or noise power can be obtained by integrating the spectral density of noise up to receiver bandwidth, B_e. However, in a chained amplifier system, ASE noise accumulates within signal band. To include the effect of accumulated ASE noise, we define the effective inversion parameter $n_{sp}^{(n)}$ such as $n_{sp}^{(n)} = n_{sp}\sum_{k=1}^{n}\frac{1}{\sqrt{k}}$ (n = number of chained amplifiers). Then, the beat noise variances including the effect of accumulated ASE noise are

$$\sigma_{s-sp}^2 = 4qRGP_s n_{sp}^{(n)}(G-1)B_e \tag{14}$$

$$\sigma_{sp-sp}^2 = 4q^2\left(n_{sp}^{(n)}\right)^2(G-1)^2 B_o B_e\left(1 - \frac{B_e}{2B_o}\right) \tag{15}$$

Here, σ^2_{s-sp} is the signal-spontaneous beat noise variance, and σ^2_{sp-sp} is the spontaneous-spontaneous term. In the following discussion, other noise powers (thermal and shot noise powers) are ignored since they are usually negligible compared to the ASE beat noises in a pre-amplifier receiver.
Figure 4 shows the two noise powers as a function of the cascaded amplifier number at 2.5Gb/s and 10Gb/s. 10Gb/s system is more vulnerable than 2.5Gb/s because 10Gb/s system requires about 4 times larger electrical bandwidth than 2.5Gb/s system in the receiver.

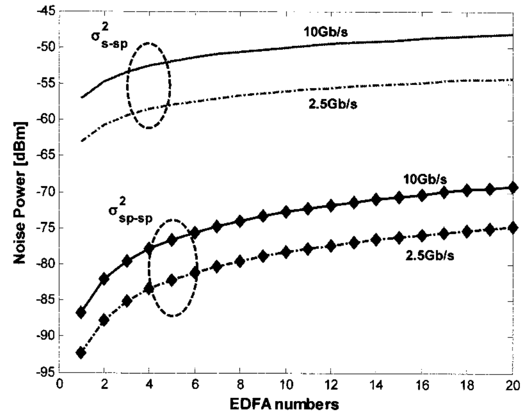


Fig. 4 Dominant Noise Powers to Evaluate Q-factors ($R = 1$ [A/W], $G = 23$dB, NF = 5.5dB, $P_s = -22$dBm, $B_e = 0.7\times R_b$, $B_o = 0.753\times 36$GHz)

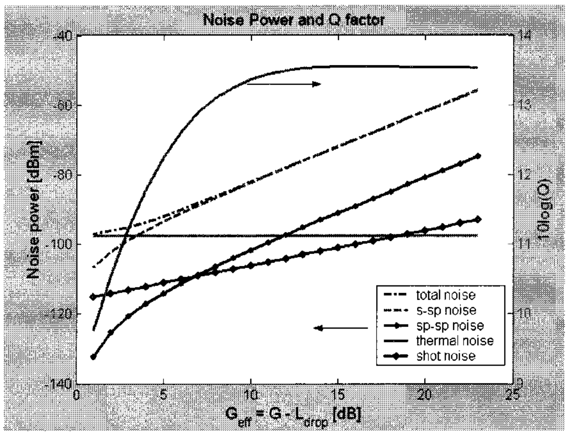
When signal-ASE beat noise and ASE-ASE beat noise are dominant over thermal noise and shot noise, Q value can be simplified in terms of *optical* signal-to-noise ratio. For reasonably large ONSR (> 10 or 10dB), we can maintain a 1:1 scaling between Q_{dB} and $OSNR_{dB}$ as below [10]

$$
\begin{aligned}
Q_{dB} &\approx 20\log\left(\sqrt{OSNR}\sqrt{\frac{B_o}{B_e}}\right) \\
&= OSNR_{dB} + 10\log\left(\frac{B_o}{B_e}\right)
\end{aligned}
\tag{16}
$$

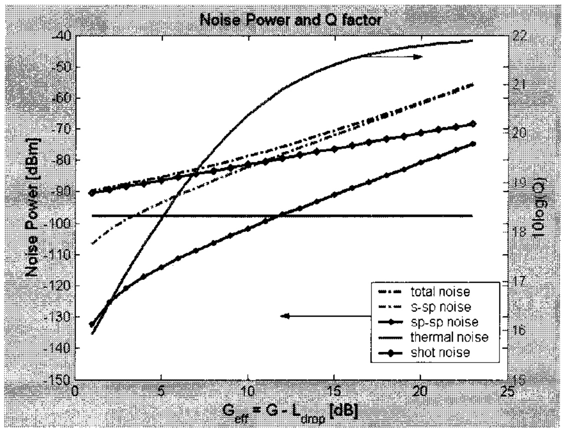
Therefore, we can expect that 1dB degradation of OSNR due to the increased number of amplifiers cascaded will cause 1dB penalty of Q_{dB} (= 20log(Q)). However, the network element shown in Fig.1 can have a significantly large drop loss between the output of preamplifier and the input of photo-detector due to the optical switch for reconfiguration of wavelengths. The optical signal-to-noise ratio before and after the drop loss will remain exactly same, however, the relative impact of optical noise in the system performance may be quite different than a preamplifier receiver without a drop loss. To see the effect of the drop loss, we define the effective gain of the preamplifier such as

$$G_{eff}[dB] = G[dB] - L_{drop}[dB] \tag{17}$$

Figure 5(a) shows various noise powers in a PIN receiver and the resulting Q-factor as a function of G_{eff}, and Figure 5(b) is when the receiver is APD. Both cases assume R_b = 2.5Gb/s and 19 amplifiers are cascaded before the pre-amplifier. (total 20 EDFAs cascaded.) The thermal noise density in the receiver is assumed 1×10^{-16}mA^2/Hz, and $M = 7$, $F_A = 5.97$ is assumed for APD receiver.



(a)



(b)

Fig. 5 Noise Power and Q factor at R_b = 2.5Gb/s, (a) PIN receiver (b) APD receiver

From Figure 5, we can observe that when G_{eff} is large (small drop loss), signal-spontaneous beat noise is dominant and Q-factor is nearly constant. (Note that signal is also amplified by G_{eff}.) In such a case, system performance will depend only on the received optical signal-to-noise ratio as predicted by (16). Conversely, when the drop loss is significant, the system performance will be relatively insensitive to the received optical signal-to-noise ratio (relative significance of thermal noise and shot noise increases.), but will be more dependable on the received power.

Figure 6 highlights the above statement. It shows Q-factor performance at 2.5Gb/s as a function of the received OSNR when the PIN receiver is used. With 3dB of the drop loss, 10log(Q) is a linear function of the received OSNR, which means the system performance is ASE-noise limited. The slope of OSNR[dB] versus 10×log(Q) is calculated as 1.88. Therefore when ASE-noise limited, we can have a close 1:1 scaling between Q_{dB} (=20log(Q)) and $OSNR_{dB}$ as predicted in eq.(16). However, when the drop loss is significant, the system performance is insensitive to $OSNR_{dB}$, and OSNR is not a good system metric in this case.

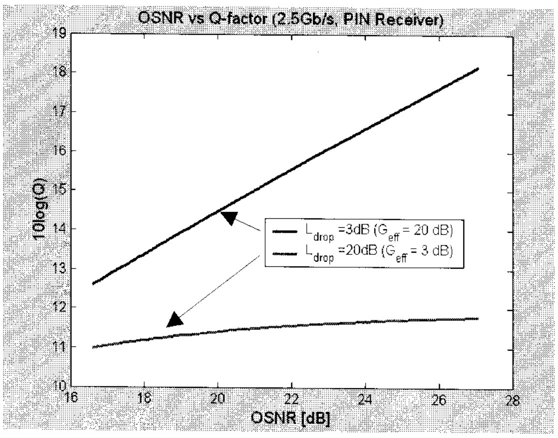


Fig. 6 Received OSNR vs Q-factor with a PIN receiver

IV. CONCLUSIONS

We derived some of important analytical expressions to estimate system performance of the metropolitan network which consists of a cascaded network element shown in Fig.1. Those results will give us a better physical insight to identify limiting factors of the network element for wavelength rearrangement and reconfiguration. We show that cascaded optical filters effectively reduce noise power, and thus improve OSNR. The improved OSNR will give us a better performance (lower BER). However the improvement depends on the drop loss because the system performance is relatively insensitive to OSNR when the drop loss is significant. When the drop loss is relatively small, the receiver structure of the network element can be treated as a preamplifier receiver which is widely used in long-haul systems. In that case, ASE noise from EDFAs is a limiting noise source in the receiver.

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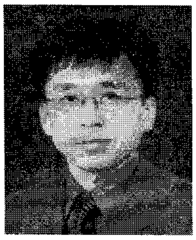

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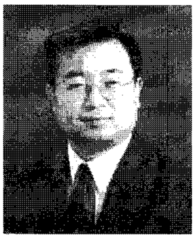

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