Nominal States Relationship and Its Sliding Mode Control Application

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***Abstract*—**A novel method to derive a Nominal States Relationship (NSR) of a control system is proposed. The obtained relationship is used to design a sliding surface which can have the characteristic of a nominal system. With this sliding surface, a Sliding Mode Control (SMC) system which has the characteristics of the nominal system controlled by pole placement is designed for an uncertain system.

***Index Terms*—**Nominal State Relationship, Pole Placement Control, Robust Control, Sliding Mode Control.

I. INTRODUCTION

Conventional sliding surface cannot have the nominal system characteristic because of the linearly dependent property of states [1]-[3]. This means that the robustness of the SMC cannot be added to the other control method besides SMC. To overcome this problem, a special SMC with a virtual state was proposed for combining the robustness of SMC with linear control theory. However, the introduction of a virtual state increases the order of a controller[4]-[7].

In this paper, a novel NSR is derived for a nominal system and used to design a sliding surface which has the characteristic of the nominal system. The relationship has been considered very difficult because of its nonlinearity even in the case of linear systems. The proposed method can be applied for a system with different eigenvalues except the case of second order system with complex eigenvalues. The SMC with the proposed sliding surface is applied to pole placement control and computer simulation shows its effectiveness.

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This paper is organized as follows. Nominal states relationship for higher order system is derived in Section 2. In Section 3, noble sliding surface is designed by using the NSR. In Section 4, computer simulation is carried out to verify the NSR and the robustness of the pole placement with the SMC proposed in this paper. Finally, some conclusions are presented in Section 5.

II. NSR FOR HIGHER ORDER SYSTEM

It could be considered as a very difficult work to obtain a NSR in a higher order system. This paper proposes a simple method to obtain a states relationship in a higher order system. Let us consider the following fourth order system with two different real eigenvalues and complex eigenvalues. The result can be extended to higher order systems easily without loss of generality

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}\lambda_1 & 0 & 0 & 0\\ 0 & \lambda_1 & 0 & 0\\ 0 & 0 & \sigma & \omega\\ 0 & 0 & -\omega & \sigma\end{bmatrix}\mathbf{x}(t) \quad (1)$$

The solutions of Eq.(1) are as follows.

$$x_1(t)=e^{\lambda_1 t}x_1(0), \quad x_2(t)=e^{\lambda_2 t}x_2(0) \quad (2)$$

$$\begin{aligned}x_3(t)&=e^{\sigma t}(\cos(wt)x_3(0)+\sin(wt)x_4(0))\\ x_4(t)&=e^{\sigma t}(\cos(wt)x_4(0)-\sin(wt)x_3(0))\end{aligned} \quad (3)$$

To obtain a states relationship, the equations of Eq.(2) is changed as follows.

$$t=\ln\left(\left(\frac{x_1(t)}{x_1(0)}\right)^{\frac{1}{\lambda_1}}\right), \quad t=\ln\left(\left(\frac{x_2(t)}{x_2(0)}\right)^{\frac{1}{\lambda_2}}\right) \quad (4)$$

From Eq.(3), the following equation is obtained.

$$x_3^2(t) + x_4^2(t) = e^{2\sigma t}(x_3^2(0) + x_4^2(0)) \quad (5)$$

Eq.(5) is changed to the following equation.

$$t = \ln\left(\left(\frac{x_3^2(t) + x_4^2(t)}{x_3^2(0) + x_4^2(0)}\right)^{\frac{1}{2\sigma}}\right) \quad (6)$$

From the above Eq.(4) and Eq.(6), we obtain the following equation.

$$\begin{aligned} a\,t &= a\ln\left(\left(\frac{x_3^2(t) + x_4^2(t)}{x_3^2(0) + x_4^2(0)}\right)^{\frac{1}{2\sigma}}\right) \\ &= b\ln\left(\left(\frac{x_1(t)}{x_1(0)}\right)^{\frac{1}{\lambda_1}}\right) + c\ln\left(\left(\frac{x_2(t)}{x_2(0)}\right)^{\frac{1}{\lambda_2}}\right) \end{aligned} \quad (7)$$

where a, b and c are constants which satisfy $a = b + c$

Therefore, the following relationship among the states of Eq.(1) is derived.

$$\left(\frac{x_3^2(t) + x_4^2(t)}{x_3^2(0) + x_4^2(0)}\right)^{\frac{a}{2\sigma}} = \left(\frac{x_1(t)}{x_1(0)}\right)^{\frac{b}{\lambda_1}}\left(\frac{x_2(t)}{x_2(0)}\right)^{\frac{c}{\lambda_2}} \quad (8)$$

The above result can be extended to general systems without difficulty. The usefulness of the relationship will be shown through a SMC application. A sliding surface designed by using the above equation can have the characteristic of a nominal system.

Note that NSR without time variable cannot be derived for a second order system with complex conjugate eigenvalues.

III. SMC WITH NOVEL SLIDING SURFACE

Consider the following uncertain system. The control objective is to make the following system have the same characteristic with Eq.(1).

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \sigma & \omega \\ 0 & 0 & -\omega & \sigma \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}(u_{SMC}(t) + h(t)) \quad (9)$$

where $u_{SMC}(t)$ is SMC input and h(t) is disturbance.

By using Eq.(8), a sliding surface is designed as follows.

$$s = \left(\frac{x_3^2(t) + x_4^2(t)}{x_3^2(0) + x_4^2(0)}\right)^{\frac{a}{2\sigma}} - \left(\frac{x_1(t)}{x_1(0)}\right)^{\frac{b}{\lambda_1}}\left(\frac{x_2(t)}{x_2(0)}\right)^{\frac{c}{\lambda_2}} \quad (10)$$

To make the states stay on the sliding surface, the following SMC input is derived from the hitting condition $s\dot{s} < 0$.

For simplicity, we set a=2, b=1 and c=1.

$$u_{SMC}(t) = \frac{1}{D(t)}(-2\sigma\lambda_1\lambda_2 s) - h_{max}\,\mathrm{sign}(s\,D(t)) \quad (11)$$

where

$$\begin{aligned} D(t) = &\left(2\lambda_1\lambda_2\left(\frac{x_3^2(t)+x_4^2(t)}{x_3^2(0)+x_4^2(0)}\right)^{\lambda_1\lambda_2}(x_3^2(t)+x_4^2(t))^{-1}(b_3x_3(t)+b_4x_4(t))\right. \\ &\left. -\sigma\left(\frac{x_1(t)}{x_1(0)}\right)^{\sigma\lambda_1}\left(\frac{x_2(t)}{x_2(0)}\right)^{\sigma\lambda_2}(\lambda_2 b_1 x_1(t)^{-1} + \lambda_1 b_2 x_2(t)^{-1}\right) \end{aligned} \quad (12)$$

IV. COMPUTER SIMULATION

Consider the following fourth-order system.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} -0.95 & -0.40 & 0.38 & 0.11 \\ 5.704 & -8.64 & 4.14 & 6.90 \\ 6.30 & -9.93 & 5.56 & 9.56 \\ 1.68 & -0.75 & -0.24 & 0.03 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}u(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}h(t) \quad (13)$$

where $u(t) = u_0(t) + u_{SMC}(t)$, $h(t) = \sin(5\,t)$ and $\mathbf{x}(0) = [4\ \ 3\ \ 2\ \ 1]^T$.

Nominal control input $u_0(t) = -[0.21\ -0.20\ 0.38\ 0.61]\mathbf{x}(t)$ makes the eigenvales of the nominal system equal to -1, -2, -1+j and -1-j.

Diagonalization of Eq.(13) is achieved as follows.

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \\ \dot{z}_3(t) \\ \dot{z}_4(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix} + \begin{bmatrix} 15.77 \\ 15.00 \\ 13.42 \\ -3.39 \end{bmatrix}u_{SMC}(t) + \begin{bmatrix} 15.77 \\ 15.00 \\ 13.42 \\ -3.39 \end{bmatrix}h(t) \quad (14)$$

The proposed sliding surface is defined as follows.

$$\begin{aligned} s &= \left(\frac{z_3^2(t)+z_4^2(t)}{z_3^2(0)+z_4^2(0)}\right)^{\lambda_1\lambda_2} - \left(\frac{z_1(t)}{z_1(0)}\right)^{\sigma\lambda_2}\left(\frac{z_2(t)}{z_2(0)}\right)^{\sigma\lambda_1} \\ &= \left(\frac{z_3^2(t)+z_4^2(t)}{5}\right)^2 - \left(\frac{z_1(t)}{4}\right)^2\left(\frac{z_2(t)}{3}\right) = 0 \end{aligned} \quad (15)$$

From Eq.(11), sliding mode control input is calculated as follows.

$$u_{\mathrm{SMC}}(t) = \frac{1}{\mathrm{D}(t)}(4s) - h_{\max}\,\mathrm{sign}(s\,\mathrm{D}(t)) \qquad (16)$$

where

$$D(t) = \left(4\left(\frac{z_3^2(t)+z_4^2(t)}{5}\right)^2 (z_3^2(t)+z_4^2(t))^{-1}(13.4192 z_3(t) - 3.3919 z_4(t)) + \left(\frac{z_1(t)}{4}\right)^2\left(\frac{z_2(t)}{3}\right)(-31.5298 z_1(t)^{-1} - 15.0030 z_2(t)^{-1}\right)$$

and $h_{\max}=1.1$.

In Eq.(16), the input chattering is occurred. It causes an undesirable high frequency component in the state trajectory.

To solve this problem, the following saturation function can be obtained.

$$\mathrm{sat}\left[\frac{s}{\phi}\right] = \begin{cases} \dfrac{s}{\phi} & \|s\| \le \varphi \\ \mathrm{sgn}(s) & \|s\| > \varphi \end{cases} \qquad (17)$$

where ϕ is the boundary layer thickness neighboring the sliding surface.

Fig. 1 and Fig. 2 show that the proposed SMC gives the nominal system responses for the uncertain system Eq.(13). It is clear that the dynamics of the system has the dynamics as these of the nominal system. This means that the proposed sliding mode controller has the robustness for parameter uncertainty and disturbance. The proposed sliding surface and the proposed sliding mode control input are shown in Fig. 3 and Fig. 4 respectively. Fig. 3 shows that the reaching phase is eliminated.

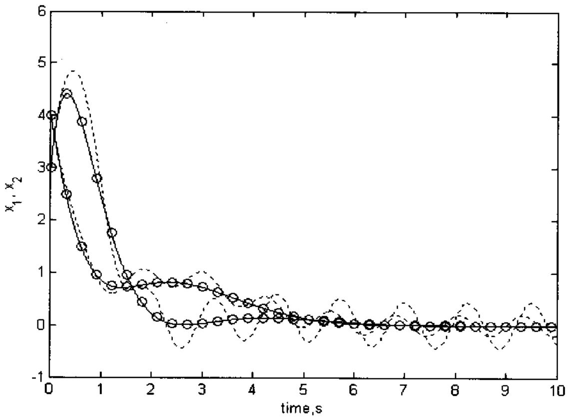

Fig. 1 State trajectories of x_1, x_2.
—— pole placement control without disturbance
------ pole placement control with disturbance
O proposed SMC with disturbance

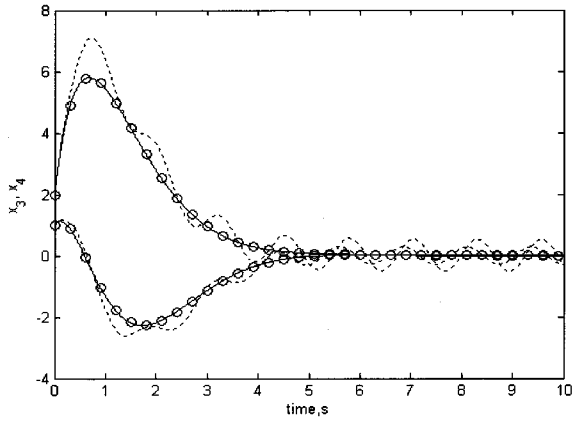

Fig. 2 State trajectories of x_3, x_4.
—— pole placement control without disturbance
------ pole placement control with disturbance
O proposed SMC with disturbance

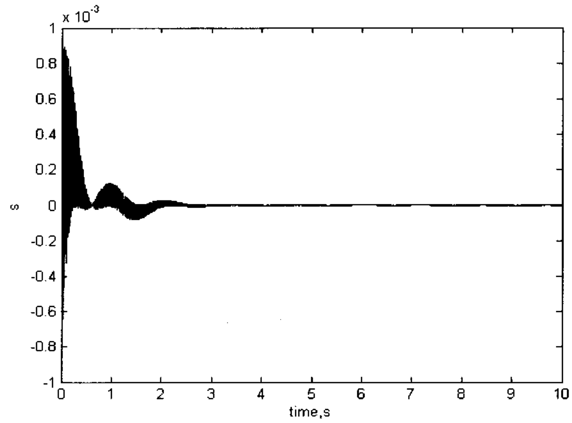

Fig. 3 Sliding surface of the proposed SMC.

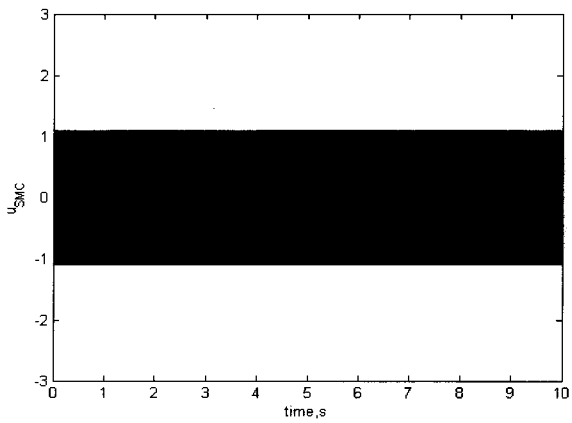

Fig. 4 Control input of the proposed SMC.

V. CONCLUSIONS

A novel deriving method of NSR was proposed and its usefulness was shown through the SMC with a sliding surface designed by using NSR. The SMC system shows the characteristic of the nominal system for the uncertain system. Therefore the SMC with the proposed sliding surface can add the robustness of SMC to pole placement control.

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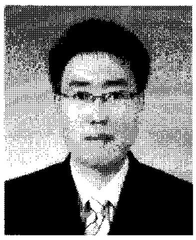

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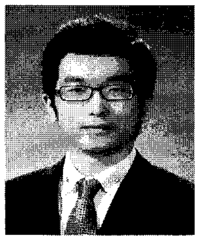

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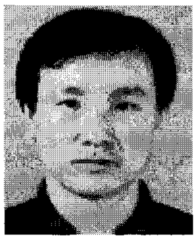

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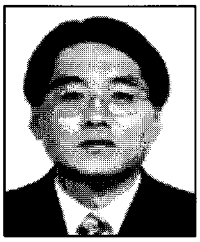

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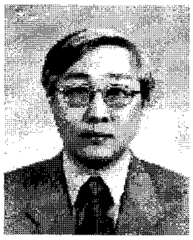

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