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A Comparative Study on the Forecasting Performance of Range Volatility Estimators using KOSPI 200 Tick Data

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〈abstract〉

This study is on the forecasting performance analysis of range volatility estimators(Parkinson, Garman and Klass, and Rogers and Satchell) relative to historical one using two-scale realized volatility estimator as a benchmark.

American sub-prime mortgage loan shock to Korean stock markets happened in sample period(January 2, 2006～March 10, 2008), so the structural change somewhere within this period can make a huge influence on the results. Therefore sample was divided into two sub-samples by May 30, 2007 according to Zivot and Andrews unit root test results. As expected, the second sub-sample was much more volatile than the first sub-sample.

As a result of forecasting performance analysis, Rogers and Satchell volatility estimator showed the best forecasting performance in the full sample and relatively better forecasting performance than other estimators in sub-samples. Range volatility estimators showed better forecasting performance than historical volatility estimator during the period before the outbreak of structural change(the first sub-sample). On the contrary, the forecasting performance of range volatility estimators couldn't beat that of historical volatility estimator during the period after this event(the second sub-sample). The main culprit of this result seems to be the increment of range volatility caused by that of intraday volatility after structural change.

Keywords : Range Volatility Estimator, Two-scale Realized Volatility Estimator, Forecasting Performance, KOSPI 200

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I. Introduction

The volatility of various asset returns lies in the center of option pricing, portfolio allocation, and risk management problems. Any financial economist or expert has paid a huge amount of attention to the study on the measurement and forecasting methods of volatility. Historical volatility, most frequently used one, can be estimated as the simple standard deviation of returns based on closing prices for a certain period. Its computational convenience is dwarfed by an amount of noise attached to true volatility (Andersen and Bollerslev, 1998).

An alternative to this weakness of historical volatility estimator has been suggested in the name of “range volatility estimators” introduced by Parkinson (1980) for the first time and followed by Garman and Klass (1980), and Rogers and Satchell (1991). They were evaluated to be 5 to 14 times more efficient compared to historical volatility estimator (Vipul and Jacob, 2007). Another merit of range volatility estimators could be the greater informational contents because they are calculated with opening prices, the highest prices, the lowest prices as well as closing prices.

In spite of these appeals, the strict assumptions of log-normal asset returns distribution and continuous trading have been a big obstacle for attracting enough attention. For example Marsh and Rosenfeld (1986) and Wiggins (1991, 1992) show that the analyses using range volatility estimators succeed in enhancing efficiency but fail to reduce bias. The main cause of this finding is the low liquidity of the assets under their studies which lead to the violation of continuous trading assumption.

Recent development of IT (Information Technology) and advance of financial statistics have made researchers jump over these obstacles with the availability of high frequency data. Realized volatility can be calculated with high frequency data. Furthermore various estimators can be compared by using realized volatility as a benchmark, which is a clear contrast to earlier studies in which historical volatility has been used as a benchmark. Quite useful results have been reported that the assumption of log-normal asset returns distribution is not necessary (Andersen and Bollerslev, 1998; Andersen, Bollerslev, Diebold, and Labys, 2001; Andreou and Ghysels, 2002; Barndorff-Nielsen and Shephard, 2002). For example Bali and Weinbaum (2005) and Shu and Zhang (2006) found that the range volatility estimators were not significantly biased and were also robust to microstructure errors like bid-ask spread. The relative efficiency and simplicity of range volatility estimators make a strong case for evaluating their performance further (Vipul and Jacob, 2007).

This analysis contributes to the literature by investigating the ability of range volatility estimators to forecast the future realized volatility compared to historical volatility estimator using KOSPI 200 tick data. KOSPI 200 is the underlying asset of KOSPI 200 options, the most hot-selling financial contract in the world, so the analysis on this spot market deserves the attention from academic and business fields owing to the international importance of its derivatives. In addition, the sample period of this analysis includes the outbreak of American sub-prime mortgage loan shock, so two sharply different situations by this catastrophe can be tested for finding out which volatility estimator shows better forecasting performance between conventionally used historical volatility estimator and potentially powerful range volatility estimators. Finally, the realized volatility adopted here is the two-scale realized volatility of Zhang, Mykland, and Aït-Sahalia (2005) which combines the estimates of the variance from high frequency domain and low frequency domain respectively for the purpose of reducing the effect of noise. ARMA filters are added in this study on KOSPI 200 to the forecasting filters used in Vipul and Jacob (2007) because these two studies deal with different market data.

The remainder of this analysis is organized as follows. In section 2 the five volatility estimators including three range volatility estimators, historical volatility estimator, and two-scale realized volatility estimator are introduced. In section 3 the basic analysis of data, the detailed explanation of forecasting filters and the criteria of forecasting performance, the interpretation of empirical analysis and its results we've got. The conclusions are presented in the final section.

Ⅱ. Volatility Estimators

1. Range Volatility Estimators

1) Parkinson Volatility Estimator

Parkinson (1980) proposed range volatility estimator as an alternative to conventional volatility estimator for the first time. Parkinson suggested using range (the highest value – the lowest value) variance instead of a widely used method for estimating variance of log-transformed stock returns. He found that the standard deviation of his novel method was 2.5 to 5 times smaller than that of conventional method. He also gave an illustration of how to use this novel method in estimating returns of common stocks.

His estimator is given by;

$$\hat{\sigma}^2_{PK} = \left(\frac{1}{4\ln 2}\right)\frac{1}{n}\sum_{t=1}^{n}(H_t - L_t)^2 \quad (1)$$

where H_t and L_t are the log-transformed highest and lowest prices observed during the trading day t, and n is the number of trading days.

2) Garman and Klass Volatility Estimator

Garman and Klass (1980) showed that discrete trading could induce the downward bias of Parkinson range volatility estimator using simulation due to the impossible observation of true highs and lows. They devised a volatility estimator of equation (2) by adding the opening and closing prices to the price range.

$$\hat{\sigma}^2_{GK} = \frac{1}{n}\sum_{t=1}^{n}\left[0.511(H_t - L_t)^2 - 0.019\{(C_t - O_t)(H_t + L_t - 2O_t) - 2(H_t - O_t)(L_t - O_t)\} - 0.383(C_t - O_t)^2\right] \quad (2)$$

where H_t, L_t, O_t, and C_t are the log-transformed highest, lowest, opening, and closing prices observed during the trading day t, and n is the number of trading days.

While Garman and Klass volatility estimator (GK, hereafter) is claimed to be more efficient than Parkinson volatility estimator (PK, hereafter), both face the problem of overestimation caused by the assumption of driftless price process. This problem could be serious especially when the security prices exhibit a distinct trend as in a bearish or a bullish market.

3) Rogers and Satchell Volatility Estimator

Rogers and Satchell (1991) developed a range volatility estimator of equation (3) by including the drift in the price process to settle the overestimation of PK, and GK volatility estimators. Rogers and Satchell volatility estimator (RS, hereafter) is given by;

$$\hat{\sigma}^2_{RS} = \frac{1}{n}\sum_{t=1}^{n}\left[(H_t - C_t)(H_t - O_t) + (L_t - C_t)(L_t - O_t)\right] \tag{3}$$

where H_t, L_t, O_t, and C_t are the log-transformed highest, lowest, opening, and closing prices observed during the trading day t, and n is the number of trading days.

2. Conventional Historical Volatility Estimator

The historical variance for a period of T trading days can be estimated as the sum of daily variance and one-lag covariance like Equation (4).

$$\hat{\sigma}^2_{H} = \sum_{t=1}^{T} r_t^2 + \sum_{t=1}^{T-1} r_t r_{t+1} \tag{4}$$

where r_t is the close-to-close daily return and r_t^2 is the close-to-close daily variance based on the closing price of the trading day t.

3. The Measurement of Realized Volatility

The realized volatility for a day is the sum of the squared finely sampled intraday returns, as the covariances can be ignored. When the sampling intervals approach zero, the realized volatility provides an unbiased estimate of the latent variance (Andersen et al., 2002). It could be ideal that the sampling intervals should be as short as possible like the use of tick level data. However, the realized volatility estimator estimated with fine sampling intervals had a tendency to make bias bigger (Brown, 1990). The market microstructure such as bid-ask spread could be a major cause of this problem. As price had the finer sampling intervals, the stronger market microstructure effect appeared.

So the researchers preferred relatively longer sampling intervals like 5 to 30 minutes in the case of foreign exchange data, but this could cause the following problems in respect to financial statistics. First, sampling in the longer time intervals means the loss of available information. Second, it can just reduce the extent of market micro-structure effect, leaving the distortion of volatility estimate unsettled.

Zhang et al. (2005) suggested a two-scale realized volatility estimator (TSRV, here-after) to resolve these matters. They assumed that the microstructure noise was in-dependent of the price process and demonstrated that the estimate of realized variance with $n+1$ price observations also contained $2n$ times the variance of the noise. This implies that the realized variance, based on a very high frequency data, would be domi-nated by the variance of the noise. Therefore they recommended combining the estimates of the variance at relatively low and high frequencies like equation (5) to relieve the effect of noise.

$$\hat{\sigma}_t^2 = \frac{N}{(N - \overline{n})}\left[\overline{\sigma^2_{5\mathrm{min},t}} - \frac{\overline{n}}{N}\left(\sigma^2_{2\mathrm{sec},t}\right)\right] \tag{5}$$

where N is the total number of returns at the high frequency and $\overline{n}$ is the average number of returns across all the sub-samples at the low frequency.

The TSRV is composed of two variances estimated at low and high frequencies. The variance at high frequency $\left(\sigma^2_{2\mathrm{sec},t}\right)$ is estimated by using all the data which corresponds to 2-second intervals in the case of KOSPI 200 tick level data here. The variance at low frequency is estimated in the form of average by sampling KOSPI 200 tick level data at every 5 minute.

Ⅲ. Empirical Analysis

1. Data

1) Raw Data

The data dealt with in this study is KOSPI 200 (KOrea Stock Price Index 200). The type of raw data is tick which has begun to be quoted from November 25, 1996. <Table 1> shows the changes of quoting time intervals of KOSPI 200 tick data between November 25, 1996 and May 10, 2009.

<Table 1> The changes of quoting time intervals of KOSPI 200 tick level data

(as of May, 2009)

Time period	Quoting time interval
11/25/1996~12/05/1998	60 seconds
12/07/1998~8/19/2002	30 seconds
8/20/2002~12/31/2005	10 seconds
1/02/2006~5/10/2009	2 seconds

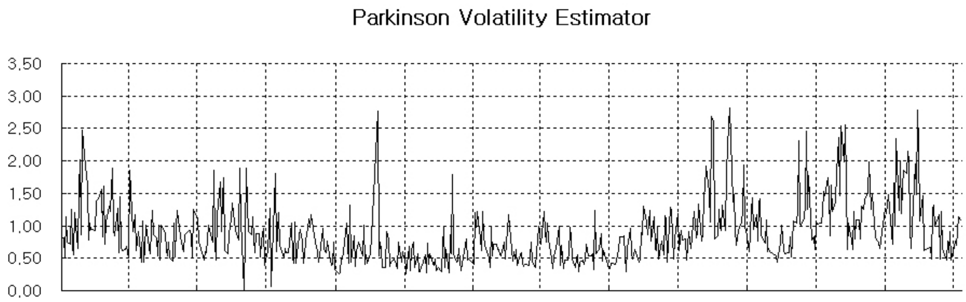
We used the data from January 2, 2006 to March 10, 2008 as high frequency raw data in this study because of their shortest quoting time interval, 2 seconds. The first quoting time is 9 : 00 a.m. and the last one is 2 : 50 p.m. in a trading day.

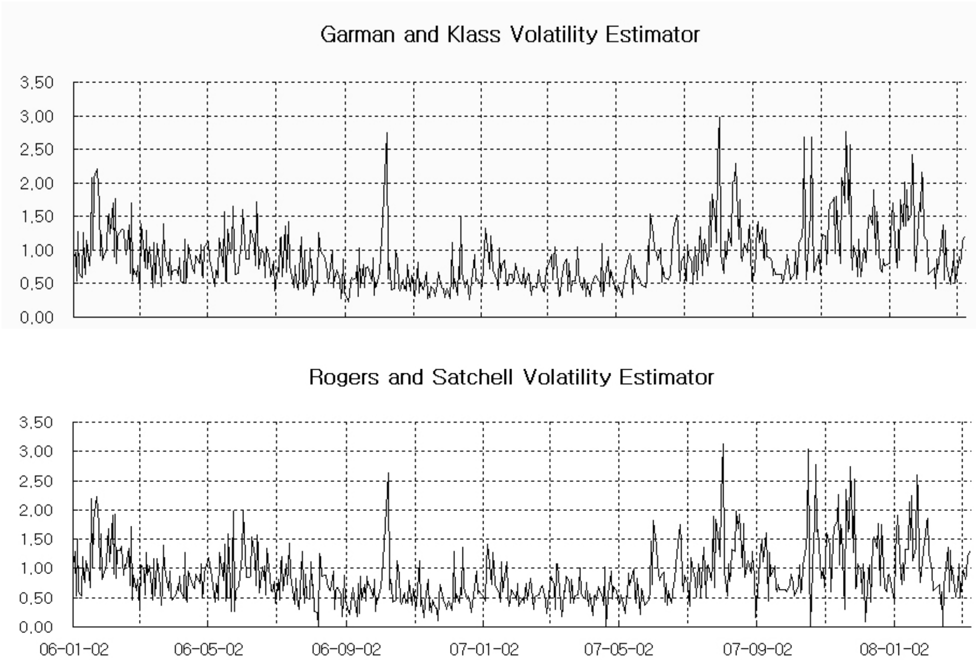
2) The Data of Range Volatility Estimators

[Figure 1] is the plot of three daily range volatility estimators. Three sub-plots all have the same volatility scale on each y-axis from 0 to 3.5. It makes us to compare the pattern and magnitude of the three range volatility estimators with ease.

The roughly similar pattern and magnitude in terms of volatility by the naked eye we've got. More specific differences could be drawn only by statistical analyses after-

[Figure 1] Range volatility estimators





ward, but one common thing the three range volatility estimators have is that the latter part of the time series is more volatile than the earlier part.

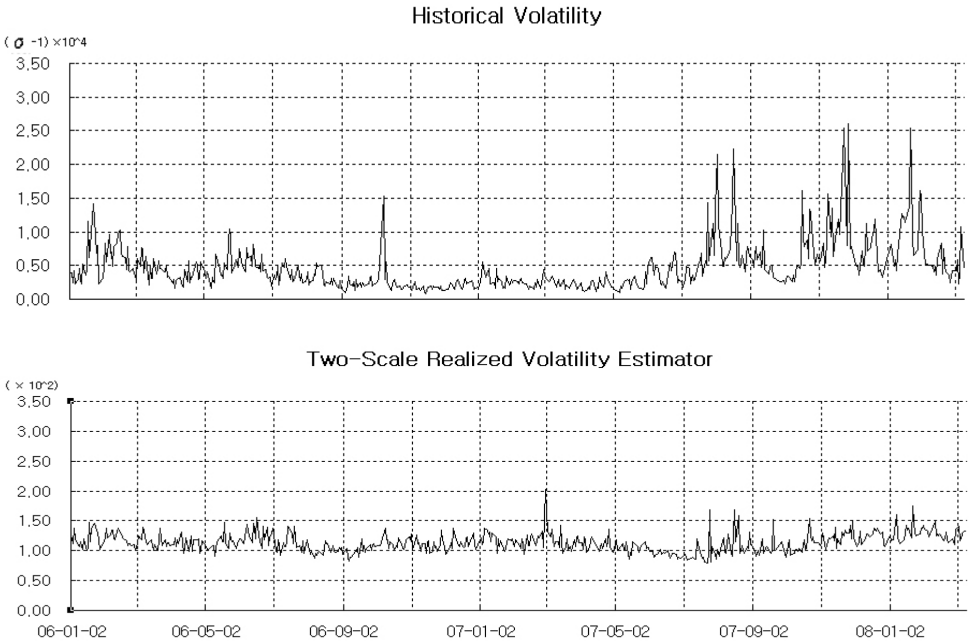
3) The Data of Historical Volatility and Two-Scale Realized Volatility Estimators

[Figure 2] is the plot of historical volatility estimator (HV, hereafter) and TSRV. Their values have been calculated by equation (4) and (5). Special attention should be paid to the y-axis scales of the plots of HV and TSRV.

These are different from the scales of the plots of previous range volatility estimators in the following respects. First, HV has the values quite slightly over one, which can be accepted as well-known fact in the financial industry. Second, TSRV has much smaller values due to the calculation formula, equation (5) using the combined data we've got from high frequency and low frequency domains.

The plots of HV and TSRV also show more volatile latter part of the time series like the range volatility estimators. It's especially clear in the plot of HV.

[Figure 2] Historical volatility and two-scale realized volatility estimators



4) The Stationarity Test of Time Series

To check the stationarity of five volatility estimators, we performed two unit root tests. One was Augmented Dickey-Fuller (ADF, hereafter) unit root test ignoring the possibility of structural changes in time series. The other was Zivot and Andrews (ZA, hereafter) unit root test considering the possibility of structural changes in time series.

<Table 2> shows the results of ADF unit root test which has been conducted on the level variables. The results show that the null hypothesis of "this time series has a unit root" or "this time series is not stationary" can be rejected at the 1% significance level in every time series. It means that all of the five volatility estimators are stationary, so the forecasting filters fit for stationary time series should be used in this study.

The plots of volatility estimators seem to have the structural change of volatility somewhere in the latter part of each time series. It could be an important factor in this study, so some further analysis should be done for finding out the exact changing

<Table 2> Augmented Dickey-Fuller unit root test results

This table reports the results of Augmented Dickey-Fuller unit root test in the case of only containing a constant term for checking the stationarity of time series. The criterion for choosing an optimal lag in this test is Schwarz Information Criterion. *** and ** indicate that null hypothesis can be rejected at the 1% and 5% significance levels respectively.

Estimator		ADF t-statistic
Range Volatility Estimator	Parkinson	-7.261***
	Garman and Klass	-4.754***
	Rogers and Satchell	-4.923***
Historical Volatility Estimator		-6.883***
Two-Scale Realized Volatility Estimator		-5.896***

point. ZA unit root test could be a good candidate for detecting one-time unknown changing point (Zivot and Andrews, 1992; Shrestha and Chowdhury, 2005). ZA unit root test statistic is as follows. Equation (6), (7), and (8) are the ZA unit root test statistics in the cases of having only constant term, only trend term, and both constant and trend terms respectively.

$$y_t = \hat{\mu}^A + \hat{\theta} DU_t(\hat{\lambda}) + \hat{\beta}^A t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^A \triangle y_{t-j} + \hat{e}_t \quad (6)$$

$$y_t = \hat{\mu}^B + \hat{\beta}^B t + \hat{\gamma}^B DT_t^*(\hat{\lambda}) + \hat{\alpha}^B y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^B \triangle y_{t-j} + \hat{e}_t \quad (7)$$

$$y_t = \hat{\mu}^C + \hat{\theta}^C DU_t(\hat{\lambda}) + \hat{\beta}^C t + \hat{\gamma}^C DT_t^*(\hat{\lambda}) + \hat{\alpha}^C y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^C \triangle y_{t-j} + \hat{e}_t \quad (8)$$

where $\hat{\mu}$ is a constant term and $\hat{e}_t$ is white noise. If $t > T\lambda$, then $DU_t(\lambda) = 1$. If $t \leq T\lambda$, then $DU_t(\lambda) = 0$. If $t > T\lambda$, then $DT_t^*(\lambda) = t - T\lambda$. If $t \leq T\lambda$, then $DT_t^*(\lambda) = 0$.

<Table 3> shows the ZA unit root test results on the five volatility estimators in this study.

We've got the rejection of null hypothesis as test results. So these five volatility estimators can be said to be stationary with possible break. The structurally changing points

<Table 3> Zivot and Andrews unit root test results

This table reports the results of Zivot and Andrews unit root test for detecting one-time unknown structural changing point. This test allows for break in intercept. Lag selection criterion is BIC (Bayesian Information Criterion). The null hypothesis in this Zivot and Andrews unit root test is that this series is not stationary with one-time unknown structural change. Critical values for 1% and 5% significance levels are -5.43 and -4.80 respectively. *** indicates that null hypothesis can be rejected at the 1% significance level. PK, GK, and RS are the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively.

Estimator		t-statistic	Observation Number of Structural Change
	PK	-9.453***	
Range volatility	GK	-9.299***	350
	RS	-9.723***	
Historical volatility		-14.231***	355
Two-scale realized volatility		-8.031***	352

are still useful despite the above test results of stationarity because we experienced the influence of American sub-prime mortgage loan shock on the Korean stock markets. The minimum observation number of structural changes is 350 that corresponds to May 30, 2007, so full sample can be divided into the two sub-samples by this date.

5) Descriptive Statistics

<Table 4> shows descriptive statistics of the five volatility estimators in full sample and the two sub-samples.

The major difference among three samples lies in mean and standard deviation. Sample 2 remarks the greatest mean and standard deviation level. Full sample and sample 1 are followed. It means that sample 1 which have not been affected by the American sub-prime mortgage loan shock is relatively stationary, but sample 2 becomes much more volatile thanks to this main culprit. Therefore it would be quite interesting to find out how the different nature of sample 1 and sample 2 can make an influence on the performance of forecasting. Finally the results of Jarque-Bera test show that every time series of five volatility estimators does not follow normal distribution.

<Table 4> Descriptive Statistics

This table reports the data description of 3 range volatility estimators, historical volatility estimator and two-scale realized volatility estimator examined across the sample period January 2, 2006 to March 10, 2008. The aim of Jarque-Bera test is to find out the normality of each time series. Its null hypothesis is that this series follows normal distribution and *** indicates that the null hypothesis can be rejected at the 1% significance level. PK, GK, and RS are the estimators of Parkinson, Garman and Klass, and Rogers and Satchell respectively. HV is the Historical Volatility estimator and TSRV is the Two-Scale Realized Volatility estimator. The figures of volatility estimators are given in percentages.

Sample	Statistic	Range Volatility			HV	TSRV
		PK	GK	RS		
Full ample (1/02/2006 ~ 3/10/2008)	Mean	0.890	0.882	0.873	1.000	0.011
	Std. Dev.	0.474	0.452	0.489	0.000	0.002
	Jarque-Bera	338.389***	393.635***	296.356***	2181.407***	181.925***
	Observations	539	539	539	539	539
Sample 1 (1/02/2006 ~ 5/30/2007)	Mean	0.749	0.747	0.740	1.000	0.011
	Std. Dev.	0.377	0.354	0.387	0.000	0.001
	Jarque-Bera	385.061***	383.067***	201.255***	787.808***	453.585***
	Observations	350	350	350	350	350
Sample 2 (5/31/2007 ~ 3/10/2008)	Mean	1.151	1.132	1.117	1.000	0.011
	Std. Dev.	0.523	0.506	0.561	0.000	0.002
	Jarque-Bera	53.429***	63.096***	43.230***	234.034***	10.817***
	Observations	189	189	189	189	189

2. Volatility Forecasting Filters

The volatility forecasting filters used in this study are AR (AutoRegressive), MA (Moving Average), and ARMA (Moving Average AutoRegressive). The specific forms of them are detailed by;

$$\text{AR }(p):u_t=\sum_{i=1}^{p}\rho_i u_{t-i}+\epsilon_t \tag{9}$$

$$\text{MA }(q): u_t = \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} \tag{10}$$

$$\text{ARMA }(p,\ q): u_t = \sum_{i=1}^{p} \rho_i u_{t-i} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} \tag{11}$$

The one-step-ahead forecast u_{t+1} can be attained by using all the parameters estimated in the rolling fixed windows of every volatility forecasting filter, so the parameters have been reestimated as often as the number of rolling fixed windows. The parameters are ρ in AR filter, θ in MA filter, and ρ and θ in ARMA filter. p and q are the lags chosen optimally in each filters and p and q in ARMA filter have a tendency to be smaller than those of AR or MA filter.

3. The Comparison Criteria for Forecasting Performance

The competing estimators and forecasting filters can be evaluated in terms of efficiency and bias. The efficiency has been measured by the mean squared error (MSE, hereafter). The bias has been measured by the mean relative bias (MRB, hereafter). The specific forms of these two measurements are given by;

$$MSE = E\left(\hat{\sigma}_t - \sigma_t\right)^2 \tag{12}$$

$$MRB = E\left[\frac{\left(\hat{\sigma}_t - \sigma_t\right)}{\sigma_t}\right] \tag{13}$$

where the estimates of PK, GK, RS, and HV can be used as $\hat{\sigma}_t$, and the estimate of TSRV can be used as σ_t in the real calculations. MSE can be replaced as RMSE (Root Mean Squared Error, hereafter) for the comparison of PK, GK, RS and HV.

4. Empirical Analysis Findings

1) Volatility Estimation

The performance of various volatility estimators has been evaluated by the efficiency and bias standards. The efficiency and bias of three range volatility estimators can be compared with those of the HV in <Table 5>. For the performance comparison of the three range volatility estimators to HV, Wilcoxon signed-rank test (Diebold and Mariano, 1995) has been conducted because of five volatility estimators' non-normality. The null hypothesis of this test is that a certain range volatility estimator and HV are equally efficient[biased] compared with each other.

<Table 5> Performance of volatility estimators

This table reports the performance comparison of 3 range volatility estimators to historical volatility estimator in terms of efficiency (RMSE) and bias (MRB). For this, Wilcoxon signed-rank test has been conducted. Its null hypothesis is that a certain range volatility estimator and historical volatility estimator are equally efficient[biased] compared with each other. ***, **, and * indicate that null hypothesis can be rejected at the 1%, 5%, and 10% significance levels respectively. The p-values of Wilcoxon signed-rank test were not reported for saving space. RMSE and MRB are given in percentages. PK, GK, and RS are the estimators of Parkinson (1980), Garman and Klass (1980), and Rogers and Satchell (1991) respectively. HV is the Historical Volatility estimator and TSRV is the Two-Scale Realized Volatility estimator.

Performance Criteria			HV	Range Volatility Estimators		
				PK	GK	RS
Full Sample (1/02/2006 ~ 3/10/2008)	Efficiency	RMSE	0.988725	0.997504***	0.980755***	0.990142***
	Bias	MRB	88.914524	77.507644***	76.914792***	76.069645***
Sample1 (1/02/2006 ~ 5/30/2007)	Efficiency	RMSE	0.988716	0.827586***	0.816259***	0.824833***
	Bias	MRB	88.497846	64.842619***	64.714160***	64.057057***
Sample2 (5/31/2007 ~ 3/10/2008)	Efficiency	RMSE	0.988741	1.252718***	1.228532***	1.239353***
	Bias	MRB	89.686151	100.961395**	99.508555*	98.315177*

The results of the performance of volatility estimators can be summarized as follows.

First, the range volatility estimators in sample 1 rank the highest performance, and those in full sample except PK of RMSE and sample 2 are followed. This result is easily expected because sample 1 is a relatively stationary period compared with sample 2 in respect of volatility.

Second, all of the performance criteria in sample 1 and full sample have the zero *p*-values of Wilcoxon signed-rank test, which means that range volatility estimators in sample 1 and full sample have higher efficiency and lower bias compared with HV. In light of these findings, the intuition that the price range contains more information about volatility than the closing prices, appears to be valid. These results are in line with those of Bali and Weinbaum (2005) and Vipul and Jacob (2007).

Finally, the three range volatility estimators cannot beat HV on performance in sample 2.

2) Volatility Forecasting[1)]

<Table 6> reports the forecasting performance of three range volatility estimators and HV, and the range volatility estimators' forecasting performance relative to HV according to forecasting filters using TSRV as a benchmark. RMSE and MRB have been used as forecasting performance comparison criteria for efficiency and bias. The range estimators' forecasting performance relative to HV was expressed as percentage change in MSE (efficiency) and MRB (bias), and its statistical significance could be decided by Wilcoxon signed-rank test. So shaded areas in <Table 6> point to "this range volatility estimator is more efficient[less biased] compared with HV in forecasting performance using this forecasting filter."

The results can be summarized as follows.

First, sample 1 shows the superior forecasting performance of range volatility estimators to HV in all the forecasting filters except ARMA(1, 1). Specifically AR(2) in RS is the most efficient forecasting filter and MA(3) in RS is the least biased forecasting filter among all the cases except ARMA(1, 1) in sample 1.

Second, RS in full sample and sample 1 is the superior forecast to HV among all the filters except ARMA(1, 1). RS is also effective in respect to efficiency and bias in AR(1) of sample 2.

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1) Estimation of all the filters on three extreme-value volatility estimators and historical one has been done before forecasting. The only filters and lags having statistically significant parameters were reported. Each sample has been divided into two spans. The former 2/3 of the sample has been used for estimation and the latter 1/3 has been used for forecasting in full sample, sample 1, and sample 2.

<Table 6> Forecasting Performance of Volatility Forecasting Methods

This table reports the forecasting performance comparison of 3 range volatility estimators to historical volatility estimator in terms of efficiency (RMSE) and bias (MRB). Change in efficiency[relative bias] represents percentage change in MSE[MRB] of the range forecasts as compared to the forecasts based on the historical volatility. The statistical significance of this difference is decided by Wilcoxon signed-rank test's p-values which were not reported here for saving space. The null hypothesis of Wilcoxon signed-rank test is that a certain range volatility estimator and historical volatility estimator are equally efficient[biased] compared with each other in forecasting performance using a certain forecasting filter. ***, **, and * indicate that the null hypothesis of Wilcoxon signed-rank test can be rejected at the 1%, 5%, and 10% significance levels respectively. PK, GK, and RS are the Parkinson, Garman and Klass, and Rogers and Satchell volatility estimators. HV is the historical volatility estimator.

Sample Period	Forecasting method	RMSE [change (%) in Efficiency (MSE)]				MRB [change (%) in Relative Bias (MRB)]			
		HV	PK	GK	RS	HV	PK	GK	RS
Full Sample 1/02/2006 ~ 3/10/2008	AR(1)	1.541	1.639	1.596	1.440	139.1	140.1	138.8	125.3
			0.063***	0.036*	-0.066***		0.007**	-0.002	-0.099***
	AR(2)	1.615	1.754	1.667	0.542	145.3	148.7	144.8	134.8
			0.086***	0.032	-0.045***		0.023 **	-0.003	-0.072***
	AR(3)	1.637	1.795	1.784	1.599	147.2	153.9	151.9	139.2
			0.097***	0.090**	-0.023***		0.045***	0.032*	-0.054***
	MA(1)	1.425	1.631	1.556	1.384	128.4	138.7	134.1	119.7
			0.144***	0.092***	-0.029 *		0.081***	0.044***	-0.067 **
	MA(2)	1.769	1.989	1.759	1.587	159.1	166.3	152.7	137.5
			0.124***	-0.006	-0.103***		0.045***	-0.040	-0.136***
	MA(3)	2.114	2.296	2.164	1.934	188.9	192.6	185.2	166.0
			0.086**	0.024	-0.085***		0.019	-0.020	-0.121***
	ARMA (1, 1)	1.466	38.794	16.001	2.811	132.0	534.7	248.5	137.1
			25.451***	9.912***	0.917***		3.051***	0.883***	0.039***
Sample 1 1/02/2006 ~ 5/30/2007	AR(1)	1.378	0.733	0.755	0.708	121.9	64.9	66.6	62.3
			-0.468***	-0.452***	-0.486***		-0.467***	-0.453***	-0.489***
	AR(2)	1.460	0.810	0.784	0.687	129.1	71.7	69.2	60.2
			-0.445***	-0.463***	-0.529***		-0.445***	-0.464***	-0.534***
	AR(3)	1.480	0.779	0.765	0.695	131.1	68.9	67.2	59.8
			-0.474***	-0.483***	-0.530***		-0.474***	-0.487***	-0.544***
	MA(1)	1.319	0.704	0.739	0.723	116.5	62.3	65.2	63.4
			-0.466***	-0.440***	-0.452***		-0.465***	-0.440***	-0.455***
	MA(2)	1.647	0.859	0.856	0.711	144.4	75.8	75.3	61.6
			-0.479***	-0.481***	-0.568***		-0.475***	-0.479***	-0.574***
	MA(3)	1.754	0.840	0.816	0.701	152.0	73.7	71.2	58.5
			-0.521***	-0.535***	-0.600***		-0.515***	-0.532***	-0.615***
	ARMA (1, 1)	1.274	12.365	35.413	20.113	112.2	227.6	581.2	382.3
			8.706***	26.798***	14.788***		1.026***	4.182***	2.409***
Sample 2 5/31/2007~ 3/10/2008	AR(1)	1.533	1.761	1.657	1.467	121.4	137.8	129.9	115.5
			0.149***	0.081***	-0.043***		0.135***	0.070***	-0.048***
	MA(1)	1.406	1.577	1.566	1.420	111.0	124.0	123.2	112.0
			0.122***	0.114***	0.010		0.117***	0.109***	0.009

Finally, these differences mentioned above could be caused by much more volatile situation of the latter sample period from May 31, 2007 to March 10, 2008. The worse result than expected in full sample could be caused by the fact that the forecasting span falls to more volatile period while estimation span does to relatively stationary one.

Great attention should be paid to the forecasting performance of optimal volatility forecasting filters by RMSE[MRB] of each volatility estimator. The optimal volatility forecasting filters by RMSE of each volatility estimator are ARMA(1, 1) in HV, MA(1) in PK, GK, and RS of full sample, ARMA(1, 1) in HV, MA(1) in PK and GK, AR(2) in RS of sample 1, and MA(1) of sample 2. The optimal volatility forecasting filters by MRB of each volatility estimator are MA(1) of full sample, ARMA(1, 1) in HV, MA(1) in PK and GK, MA(3) in RS of sample 1, and MA(1) of sample 2.

<Table 7> Forecasting Performance of Optimal Volatility Forecasting Methods by RMSE[MRB] of Each Volatility Estimator

This table reports the forecasting performance comparison of 3 range volatility estimators to historical volatility estimator in terms of efficiency (RMSE) and bias (MRB) with optimal volatility forecasting methods. Change in efficiency[relative bias] represents percentage change in MSE[MRB] of the range forecasts as compared to the forecasts based on the historical volatility. The statistical significance of this difference is decided by Wilcoxon signed-rank test's p-values which were not reported here for saving space. The null hypothesis of Wilcoxon signed-rank test is that a certain range volatility estimator and historical volatility estimator are equally efficient[biased] compared with each other in forecasting performance using a certain forecasting filter. ***, **, and * indicate that the null hypothesis of Wilcoxon signed-rank test can be rejected at the 1%, 5%, and 10% significance levels respectively. PK, GK, and RS are the Parkinson, Garman and Klass, and Rogers and Satchell volatility estimators. HV is the historical volatility estimator.

Sample Period	RMSE [change (%) in Efficiency (MSE)]				MRB [change (%) in Relative Bias (MRB)]			
	HV	PK	GK	RS	HV	PK	GK	RS
Full Sample	1.466	1.631	1.556	1.384	128.4	138.7	134.1	119.7
		0.113***	0.061**	-0.060*		0.081***	0.044***	-0.067**
Sample 1	1.274	0.704	0.739	0.687	112.2	62.3	65.2	58.5
		-0.447***	-0.420***	-0.461***		-0.445***	-0.419***	-0.479***
Sample 2	1.406	1.577	1.566	1.420	111.0	124.0	123.2	112.0
		0.122***	0.114***	0.010		0.117***	0.109***	0.009

<Table 7> shows the results of the forecasting performance of optimal volatility forecasting methods by RMSE[MRB] of each volatility estimator. And the shaded areas in <table 7> points to "this range volatility estimator is more efficient[less biased] compared with HV in forecasting performance using this forecasting filter."

The results in this analysis can be summarized as follows.

First, the best forecasting performance of range volatility estimators relative to HV lies in sample 1 which has relatively stationary period.

Second, only RS can be more efficient and less biased estimator than HV in full sample.

Finally, HV is still the best option for forecasting in relatively volatile sample 2 period compared with range volatility estimators.

Ⅳ. Conclusions

KOSPI 200 tick data for two years and two months has been used to analyze the forecasting performance of three range volatility estimators relative to HV using RSRV as a benchmark. The measurement and forecasting performance of the range volatility estimators of PK, GK, and RS can be evaluated with AR, MA, ARMA-type forecasting filters.

The entire sample period is from January 2, 2006 to March 10, 2008, so the American sub-prime mortgage loan shock happened within this period. This shock was expected to have a considerable influence on the Korean stock markets. Therefore it should be necessary to check whether there is any structural change in all the volatility estimators or not. According to ZA unit root test results, sample was divided into two sub-samples by May 30, 2007. As expected, sample 2 was much more volatile than sample 1. This difference between the two sub-samples has been expected to make a considerable influence on forecasting performance of range volatility estimators.

We've got the following conclusions based on the results of the comparative analysis on forecasting performance between range volatility estimators and HV using TSRV as a benchmark.

First, RS showed the best forecasting performance in the full sample and relatively better forecasting performance than other estimators in sub-samples.

Second, range volatility estimators showed better forecasting performance than HV during the period before the outbreak of structural change (sample 1). On the contrary, the forecasting performance of range volatility estimators couldn't beat that of HV during the period after this event (sample 2). The main culprit of this result seems to be the increment of range volatility caused by that of intraday volatility after structural change.

The aim of this study is to measure volatility using opening prices, the highest prices, the lowest prices as well as closing prices and to find out how accurately range volatility estimators can forecast the actual volatility in the future. We plan to expand the scope of this study by adding forecasting filters such as GARCH-type ones in the further study. The other expansions which can make this study more interesting can be the effects of the asymmetric information of white candlesticks and black ones on the forecasting performance of volatility estimators and the relationship of range volatility estimator and risk premium in the options markets.

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