Research on the Throughput Impact of the Strategic Stabilizing Capacity Constraint Resources by Prolonging Planning Period

Horng-Huei Wu[1], Ching-Piao Chen[2], Chih-Hung Tsai[3†] and Kuo-Wei Huang[1]

[1]Department of Industrial Engineering and System Management Chung-Hua University
No. 707, Sec. 2, WuFu Rd., Hsin-Chu, Taiwan 300, R.O.C

[2]Department of Industrial Engineering and Management Ta-Hwa Institute of Technology
1 Ta-Hwa Road, Chung-Lin Hsin-Chu, Taiwan, ROC.

[3]Department of Information Management Yuan-Pei University
No. 306, Yuan-pei Street, Hsin-Chu, Taiwan, Tel: +886-3-6102338
E-mail: imtch@mail.ypu.edu.tw

Abstract

The issue of capacity constraint resources (CCR) or bottlenecks wandering in product mix decision by applying Theory of Constraints (TOC) management philosophy has been mentioned and demonstrated in several papers. In this study, the effect for prolonging the planning period (PPP) so as to stabilize the CCR is investigated. The results show that the effect for PPP alternative will be positive or negative which is depending on the environment condition. However, a majority cases which have positive effect for PPP alternative can be recognized prior knowing the marketing demand, which is significant in the real application.

Key Words: Theory of Constraints, Product Mix Determination, Strategic Stabilizing CCR

1. Introduction

Product mix problem is one of the most fundamental decision problems confronting a manufacturing company. It is defined as a collection of products or orders competition for bottleneck or capacity constrained resources (CCRs) (Goldratt, 1990). Basic decision of product mix is to determine which product and how many quantities of the product should be produced. The objective of product mix decision is to maximize company profits (which means selecting profitable products) unless a special strategy policy has been established. The profitable product defined here is based on the unit-margin of the product. The higher

† Corresponding Author

unit-margin the more profitable will be. However, the unit-margin is viewed different in cost world concept (conventional cost concept) and throughput world concept (Theory of Constraints (TOC) concept) (Goldratt, 1990). In the cost world concept, the unit-margin of a product is defined either the product margin divided by the total processing time of the product (Goldratt, 1990) or the product margin minus the total allocated overhead of the product (Atwater and Gagne, 1997; Patterson, 1992). The total allocated overhead of a product is equal to the unit-overhead times the total processing time of the product. While in the throughput world concept, since the CCR limited the throughput, the unit-margin of a product is then defined as the product margin divided by the total CCR processing time of the product. The product margin here defined as product selling price minus the material cost of the product. Based on the throughput world or TOC concept, production plant with a functioning CCR exist can reach more profitability (Goldratt, 1990; Luebbe and Finch, 1992; Tsai and Li, 1997). However, this concept revolves around the effective management of CCR and following assumptions: (1) Material is the only true variable cost; (2) Direct labor, in most factories, should be considered a fixed cost in the short run, along with all other operating expenses such as salaries, rents, insurance, etc; (3) Manufacturing overhead can no longer be tied, if indeed it ever accurately could be, to the cost of manufacturing a product; (4) CCR is present in most production plants; (5) CCR operations would remain stable from one production period to the next and within the scheduling time horizon.

The above 1~4 assumptions probably holds true for most production plants in light of current manufacturing environment. However, the assumption 5 is probably not true due to both market demand change and Murphy. The phenomenon of different machines constraining the throughput of the plant at different times is known as CCR wandering. The CCR wandering phenomenon can actually be demonstrated to occur in the most optimally scheduled plant when marketing demand changes (Hurley and Kadipasaoglu, 1998; Lawrence and Buss, 1994). Ironically, CCR wandering would more likely occur for the more stability that a plant has. Plenert (1993) first created an example with CCR wandering behavior to show the inefficiency of the traditional product mix algorithm presented by Goldratt (1990). Lee and Plenert (1993) further presented the CCR wandering issues for new product alternatives exist. They demonstrated that linear programming technique for product mix optimization is more accurate than the traditional algorithm of TOC. In order to improve the inefficiency of the traditional algorithm, several revised algorithms were presented, i.e., Aryanezhad and Komijan (1997), Balakrishnan and Cheng (2000), Fredendall and Lea (1997), Lee and Plenert (1996), Onwubolu (2001), Onwubolu and Mutingi (2001a/b), and Huang *et al.* (2008).

However, knowing whether CCR wandering occurs or not poses yet another problem. The CCR wandering can be known to occur in TOC either only prior to knowing marketing demand or else when Murphy is hitting. This is a reactive concept. Wu and Li (1995a/b) in their study have shown that CCR wandering phenomena and its wandering behaviors have been proved to be existed and can be known prior to knowing the marketing demand. What effect does CCR wandering have on product mix? Would prolonging planning horizon so as to stabilize the CCR be a viable approach? This is the primary focuses of this paper. Although product mix decision with prolonging planning horizon should not consider product margin only, it should include inventory cost, backorder and loss sale. This paper will take product margin as the only considering issue.

2. Concept of Prolonging Planning Period

What does mean to prolong planning period to stable the CCR? An example shown in Figure 1 is used here to address the concept.

In this example, machines A and B are identified as being potential CCRs with the algorithm developed by Wu and Li (1995a) without considering the marketing demand. With the marketing demand P and Q for period 1 and 2 is illustrated in Table 1(a). By performing the loading computation for the potential CCRs machine A and B, in period 1 machine A becomes CCR, while in period 2 machine B is CCR, as shown in Table 1(b). From the throughput world concept, the product mix and total throughput in period 1 and 2 can easily

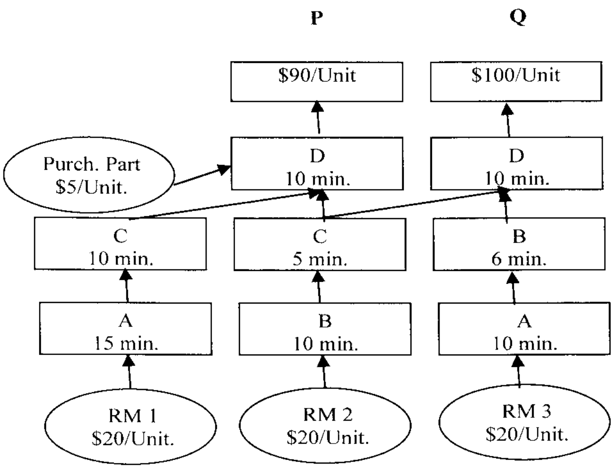

Figure 1. A production plant example

be determined as illustrated in Table 1(c). However, is this right product mix decision? Consider another alternative. If we combine both periods into one period would make more profit? For following discuss, this action of combining several periods into one period is referred as prolong planning period (PPP) alternative and the original alternative is referred as not prolong planning period (NPPP) alternative. The marketing demand and available capacity for PPP alternative is equal to the sum of both periods as shown in Table 1(d). Table 1(e) presents that the loading for the potential CCRs are then computed again and machine A is CCR. The product mix and net profit for PPP alternative are illustrated in Table 1(f). It is obviously the PPP alternative is better than the NPPP one. But can we make conclusion the PPP alternative in a wandering CCR production environment is more profitable? This requires further study.

Table 1. The effect of PPP alternative in Figure 1

(a) Marketing demand in either period

Product	Period 1	Period 2
Q	40	100
P	140	90

(b) The loading in either period

Period	Loading A	Loading B
1	2500*	2040
2	2350	2500*

* is the CCR.

(c) The product mix and total margin in either period

Period	Product mix Q	Product mix P	Total throughput
1	40	133	8,385
2	93	90	9,630
Total	133	223	18,015

(d) Market demand and capacity for PPP alternative

Period	Q	P	Capacity
1+2	140	230	4,800

(e) Loading for PPP alternative

Period	Loading A	Loading B
1 + 2	4,850*	4,540

* is the CCR.

(f) The product mix and total margin for PPP alternative

Period	Product mix Q	Product mix P	Total throughput
1 + 2	140	226	20,370

Basically, study of the effect of PPP against NPPP alternative is simply to compare the throughput between both alternatives. It is because that the net profit equals throughput minus overhead. The more throughputs the more net profit is because the overhead is fixed for both alternatives. Secondly, the complexity of the problem depends on the number of potential CCRs, number of products and number of planning periods and it is a combinatory problem. Table 2 illustrates a combination cases for two potential CCRs, two potential CCRs and two products. Further, the product mix decision is based on the rank of unit-margin of CCR and it is also a combinatory problem. Four cases for two potential CCRs and two products are shown in Table 3. In an environment, it is supposed there are N types of product, M potential CCRs and D planning periods in a planning horizon. In this environment, the number (denoted as L) of CCR will occur for PPP and NPPP alternatives depending on the number of CCR which has occurred in each planning period, as follows:

Table 2. A combination cases for two potential CCRs and two products

	d_1	d_2	$D_{(1+2)}$
case 1	m_1	m_1	m_1
case 2	m_1	m_2	m_1
case 3	m_1	m_2	m_2
case 4	m_2	m_2	m_2
case 5	m_2	m_1	m_2
case 6	m_2	m_1	m_1

Table 3. A combination cases of the rank of unit-margin for two potential CCRs and two products

	m_1	m_2
case 1	$\frac{G_1}{t_{11}} \geq \frac{G_2}{t_{12}}$	$\frac{G_1}{t_{21}} \geq \frac{G_2}{t_{22}}$
case 2	$\frac{G_1}{t_{11}} \geq \frac{G_2}{t_{12}}$	$\frac{G_1}{t_{21}} \leq \frac{G_2}{t_{22}}$
case 3	$\frac{G_1}{t_{11}} \leq \frac{G_2}{t_{12}}$	$\frac{G_1}{t_{21}} \geq \frac{G_2}{t_{22}}$
case 4	$\frac{G_1}{t_{11}} \leq \frac{G_2}{t_{12}}$	$\frac{G_1}{t_{21}} \leq \frac{G_2}{t_{22}}$

Note: Notations used in this paper, see Appendix.

[Case 1] one CCR occurred in each planning period.

There is only this CCR will occur for PPP and NPPP alternatives; in other word, the CCR is stable. This case is excluded to be discussed, i.e., L = 0.

[Case 2] two CCRs occurred in each planning period.

There will occur two or three CCR for PPP and NPPP alternatives, which is depended on the number of potential CCRs, i.e., L = min{2+1, M}. However, for one CCR, there are N! combination of dollar per constraint minute for N types product. Therefore, the total combination is $N!^{L}$ or $N!^{min\{2+1,M\}}$.

[Case 3] three CCR occurred in each planning period.

There will occur four or less CCR for PPP and NPPP alternatives, which is depended on the number of potential CCRs, i.e., min{3+1, M}. However, for one CCR, there are N! combination of dollar per constraint minute for N types of product. Therefore, the total combination is $N!^{L}$ or $N!^{\min\{3+1,\ M\}}$.

[Case X] X CCR occurred in each planning period.

The maximum number of CCR in all planning periods is X = min{D, M}. There will occur X+1 or less CCR for PPP and NPPP alternatives, which is depended on the number of potential CCRs, i.e., min{X+1, M}. However, for one CCR, there are N! combination of dollar per constraint minute for N types product. Therefore, the total combination is $N!^{L}$ or $N!^{min\{X+1,M\}}$. Therefore, the total combinatorial number (TCN) is to sum up all cases discussed above. That is

$$TCN = \sum_{i=2}^{min\{D,\ M\}} N!^{min\{i+1,\ M\}!}$$

Table 4. The combinatorial numbers

N	M	D	TCN	N	M	D	TCN
2	2	2	4	2	2	5	8
3	2	2	36	2	3	5	16
4	2	2	576	2	4	5	40
5	2	2	14400	2	5	5	72
				2	5	2	8
				2	5	3	24
				2	5	4	56

Some combinatorial numbers are illustrated in Table 4.

3. Model for Study the Effect of PPP Alternative

How to model the problem so that we can know in what situation the PPP alternative will make more profit and in what situation will not? This section we will present clearly our concept by using a simple environment-two product type, two potential CCRs and two planning period in the planning horizon, for ease of understanding. Under this simple environment, we assume the CCR combination in different period is case 2 in Table2 and the combination of dollar per resource minute is case 2 in Table 3. That is the CCR in d_1 is m_1, in d_2 is m_2 and in $d_{(1+2)}$ is m1 and the dollar per resource minute relationship is $\frac{G_1}{t_{11}} \geq \frac{G_2}{t_{12}}$ and $\frac{G_1}{t_{21}} \leq \frac{G_2}{t_{22}}$.

According to TOC concept and the above assumptions, the produced quantities in each planning period for NPPP and PPP alternatives are as follows:

$$r_{11} = q_{11} \text{ and } r_{21} = \frac{C - q_{11}t_{11}}{t_{12}} \text{ for } d_1;$$

$$r_{22} = q_{22} \text{ and } r_{12} = \frac{C - q_{22}t_{22}}{t_{21}} \text{ for } d_2; \text{ also}$$

$$r_{1(1+2)} = q_{1(1+2)} = q_{11} + q_{12} \text{ and } r_{2(1+2)} = \frac{2C - (q_{11} + q_{12})t_{11}}{t_{12}} \text{ for } d_{(1+2)}.$$

The total margins for each planning period are

$$T_1 = r_{11}G_1 + r_{21}G_2 = q_{11}G_1 + \left(\frac{C - q_{11}t_{11}}{t_{12}}\right)G_2,$$

$$T_2 = r_{12}G_1 + r_{22}G_2 = q_{22}G_2 + \left(\frac{C - q_{22}t_{22}}{t_{21}}\right)G_1, \text{ and}$$

$$T_{(1+2)} = r_{1(1+2)}G_1 + r_{2(1+2)}G_2$$
$$= (q_{11} + q_{12})G_1 + \left[\frac{2C - (q_{11} + q_{12})t_{11}}{t_{12}}\right]G_2.$$

So,

$$T_{(1+2)} - T_1 - T_2$$

$$= (q_{11}+q_{12})G_1 + \left[\frac{2C-(q_{11}+q_{12})t_{11}}{t_{12}}\right]G_2 - q_{11}G_1$$
$$-\left(\frac{C-q_{11}t_{11}}{t_{12}}\right)G_2 - q_{22}G_2 - \left(\frac{C-q_{22}t_{22}}{t_{21}}\right)G_1$$
$$= q_{12}G_1 + \left(\frac{C-q_{12}t_{11}}{t_{12}}\right)G_2 - q_{22}G_2 - \left(\frac{C-q_{22}t_{22}}{t_{21}}\right)G_1$$
$$= \left(\frac{C-q_{12}t_{11}-q_{22}t_{12}}{t_{12}}\right)G_2 - \left(\frac{C-q_{12}t_{21}-q_{22}t_{22}}{t_{21}}\right)G_1$$
$$= \left(\frac{C-l_{12}}{t_{12}}\right)G_2 + \left(\frac{l_{22}-C}{t_{21}}\right)G_1$$

Because the CCR in d_2 is m_2, $l_{22}-C \geq 0$. However, the term $(C-l_{12})$ which is greater than or less than zero is dependence:

[Case 1] If $C \geq l_{12}$, $(C-l_{12}) \geq 0$. So, $T_{(1+2)} - T_1 - T_2 \geq 0$.

[Case 2] If $C \leq l_{12}$, $(C-l_{12}) \leq 0$. The term $[T_{(1+2)} - T_1 - T_2]$ which is greater than or less than zero is dependence:

[Case 2.1] If $C \leq l_{12} \leq V$, $T_{(1+2)} - T_1 - T_2 \geq 0$;

[Case 2.2] If $V \leq l_{12} \leq l_{22}$, $T_{(1+2)} - T_1 - T_2 \leq 0$;

Where $V = C + \frac{G_1 t_{12}}{G_2 t_{21}}(l_{22}-C)$.

The V is named as a critical point since V determines the effect of PPP alternative. If l_{12} is greater than V then the effect of PPP alternative is negative otherwise the effect is positive. Based on the above analysis, we can summarize the term $[T_{(1+2)} - T_1 - T_2]$ as follows:

$$\text{If } l_{12} \leq V,\ T_{(1+2)} - T_1 - T_2 \geq 0. \quad (1)$$
$$\text{If } V \leq l_{12} \leq l_{22},\ T_{(1+2)} - T_1 - T_2 \leq 0. \quad (2)$$

From Equation (2), we have a necessary condition is $C + \frac{G_1 t_{12}}{G_2 t_{21}}(l_{22}-C) \leq l_{22}$ which can be rearranged as follows

$$(l_{22}-C)\left(1 - \frac{G_1 t_{12}}{G_2 t_{21}}\right) \geq 0.$$

Because $(l_{22} - C) \geq 0,\ \left(1 - \frac{G_1 t_{12}}{G_2 t_{21}}\right) \geq 0,$

So, $G_2 t_{21} \geq G_1 t_{12}$ or $\frac{G_2}{G_1} \geq \frac{t_{12}}{t_{21}}$.(3)

Equation (3) is a necessary condition to have $T_{(1+2)} - T_1 - T_2 \leq 0$.

4. Algorithm for Identifying the Effect

Equation (1) through (3) only show in what condition the effect of the PPP alternative is. In practical application, the following algorithm is developed here which helps identify the effect. This algorithm is divided into two phases. The phase one is used prior to knowing the product marketing demand. The only input of it is the product margin and machining time on all potential CCRs. If the effect can not be determined in phase one, then phase two is applied after knowing the product marketing demand.

[Phase 1: Prior to knowing the marketing demand.]

Step 1: Check if Equation (3) is satisfied. If the environment condition does not satisfy Equation (3), we can conclude that the PPP alternative is positive to increase the total margin, which bears no relation to the marketing demand. Otherwise, the impact can't be concluded in this phase. Go to phase 2.

[Phase 2: After knowing the marketing demand.]

Step 2: Calculate the resource loading in different period, i.e., l_{ik}, $i = 1, 2, \cdots, M$ and $k = 1, 2, \cdots, D$.

Step 3: Calculate the value of critical point V.

Step 4: Evaluate Equation (1) and (2). If Equation (1) is satisfied, we can conclude that PPP alternative is positive to increase the total margin. If Equation (2) is satisfied, the PPP alternative is negative to increase the total margin.

5. Examples and Analysis

[Example 1]

The example illustrated in Figure 1 is used here again for demonstrating the effect for PPP alternative.

Step 1: Check if Equation (3) is satisfied. [Prior to knowing the marketing demand.]

Because $G_2t_{21} = 45 \times 16 = 720$ and $G_1t_{12} = 60 \times 15 = 900$. So, $G_2t_{21} < G_1t_{12}$ which does not satisfy the necessary condition in Equation (3). We can conclude that under the condition of example 1, the total margin for PPP alternative is positive to increase total margin in spite of the marketing demand.

[Example 2]

The margin of product Q illustrated in Figure 1 is further modified to be 40. The effect for PPP alternative is studied as follows:

Step 1: Check if Equation (3) is satisfied.

Because $G_2t_{21} = 45 \times 16 = 720$ and $G_1t_{12} = 40 \times 15 = 600$. So, $G_2t_{21} < G_1t_{12}$ which satisfy the necessary condition in Equation 3. The impact of PPP alternative can't be concluded in this phase. Go to phase 2.

Step 2: Calculate the resource loading in different period.

$l_{11} = 2{,}500$, $l_{21} = 2{,}040$, $l_{12} = 2{,}350$, and $l_{22} = 2{,}500$.

Step 3: Calculate the value of critical point V.

$$V = 2{,}400 + \frac{40 \times 15}{45 \times 16}(2{,}500 - 2{,}400) = 2{,}483.33$$

Step 4: Evaluate Equation (1) and (2).

Equation (1) is satisfied, because $l_{12} < V$.

We can conclude that, under the condition of example 2, the PPP alternative is positive to increase the total margin.

[Example 3]

The marketing demand of product P in planning period 2 illustrated in Figure 1 is further modified to be 119. The PPP alternative is studied as follows:

Step1: Check the necessary condition in Equation (3).

Because $G_2t_{21} = 45 \times 16 = 720$ and $G_1t_{12} = 40 \times 15 = 600$. So, $G_2t_{21} > G_1t_{12}$ which satisfy the necessary condition in Equation (3). The impact of PPP alternative can't be concluded in this phase.

Step 2: Calculate the resource loading in different period.

$l_{11} = 2{,}500$, $l_{21} = 2{,}040$, $l_{12} = 2{,}785$, and $l_{22} = 2{,}790$.

Step 3: Calculate the value of critical point V:

$$V = 2{,}400 + \frac{40 \times 15}{45 \times 16}(2{,}790 - 2{,}400) = 2{,}725$$

Step 4: Evaluate Equation (l) and (2).

Equation (2) is satisfied since $V < l_{12} < l_{22}$

We can conclude that, under the condition of example 3, the PPP alternative is negative to increase the total margin.

6. Conclusions

The issue of CCR wandering in product mix decision by applying TOC management philosophy has been mentioned and demonstrated in several papers. The impact and how to employ the CCR wandering should be studied. In this paper, the effect of PPP alternative has been investigated. An application procedure was also developed and has been demonstrated by some examples. The following conclusions could be made on the basis of above discussion: (1) The effect of PPP alternative is positive or negative to increase total margin depends on the environment condition that is product margin, product machining time and marketing demand; (2) In most case, the PPP alternative is feasible and profitable, which can be identified prior to knowing the marketing demand; (3) There is a few of cases that PPP alternative has negative effect on the total margin. Basically, study of the impacts on PPP alternatives is a combinatorial problem. The case studied in this paper is a simple one to clearly present our concept and for ease of understanding. In practical application, different combinatorial case can be modeled as computer program to eliminate the tedious computation process. Since the impact of the PPP alternative is not only on total margin, our further research efforts would include other factors such as inventory cost and backorder cost.

References

1. Aryanezhad, M. B. and Komijan, A. R.(2004), "An improved algorithm for optimizing product mix under the theory of constraints," *International Journal of Production Research*, Vol. 42, No. 20, pp. 4221-4233.
2. Atwater, B. and Gagne, M. L.(1997), "The Theory of Constraints versus Contribution Margin analysis for Product Mix Decisions," *Journal of Cost Management*, Vol. 11, No. 1, pp. 14-22.

3. Balakrishnan, J. and Cheng, C. C.(2000), "Discussion: Theory of constraints and linear programming: a re-examination," *International Journal of Production Research*, Vol. 38, No. 6, pp. 1459-1463.
4. Fredendall, L. D. and Lea, B. R.(1997), "Improving the product mix heuristic in the theory of constraints," *International Journal of Production Research*, Vol. 35, No. 6, pp. 1535-1544.
5. Goldratt, E. M.(1990), "The haystack syndrome," North River Press, Croton-on-Hudson, New York.
6. Huang, C. Y., C. P. Chen, R. K. Li, and C. H. Tsai(2008), "Applying Theory of Constraint on Logistic Management in Large Scale Construction Sites~ A Case Study of Steel Bar in TFT-LCD Factory Build-Up," *The Asian Journal On Quality*, Vol. 9, No. 1, pp. 68-93.
7. Hurley, S. and Kadipasaoglu, S.(1998), "Wandering bottlenecks: speculating on the true causes," *Production and Inventory Management Journal*, Vol. 39, No. 4, pp. 1-4.
8. Lawrence, S. R. and Buss, A. A.(1994), "Shifting production bottlenecks: cause, cures, and conundrums," *Production and Operations Management*, Vol. 3, No. 1, pp. 21-37.
9. Lee, T. N. and Plenert, G.(1993), "Optimizing theory of constraints when new product alternatives exist," *Production and Inventory Management Journal*, Vol. 34, No. 3, pp. 51-57.
10. Lee, T. and Plenert, G.(1996), "Maximizing product mix profitability-what's the best analysis tool," *Production Planning and Control*, Vol. 7, No. 6, pp. 547-553.
11. Luebbe, R. and Finch, B.(1992), "Theory of constraints and linear programming: a comparison," *International Journal of Production Research*, Vol. 30, No. 6, pp. 1471-1478.
12. Patterson, M. C.(1992), "The product-mix decision: a comparison of theory of constraints and labor based management accounting," *Production and Inventory Management Journal*, Vol. 33, No. 3, pp. 80-85.
13. Plenert, G.(1993), "Optimizing theory of constraints when multiple constrained resources exist," European Journal of Operational Research, Vol. 70, No. 1, pp. 126-133.
14. Onwubolu, G. C.(2001), "Tabu search-based algorithm for the TOC product mix decision," *International Journal of Production Research*, Vol. 39, No. 10, pp. 2065-2076.
15. Onwubolu, G. C. and Mutingi, M.(2001a), "A genetic algorithm approach to the theory of constraints product mix problems," *Production Planning and Control*, Vol. 12, No. 1, pp. 21-27.
16. Onwubolu, G. C. and Mutingi, M.(2001b), "Optimizing the multiple constrained resources product mix problem using genetic algorithms," *International Journal of Production Research*, Vol. 39, No. 9, pp. 1897-1910.
17. Tsai, C. H. and R. K. Li(1997), "Four-Stage Scheduling Method Based on Capacity Constraint

Resources," *Journal of the Chinese Institute of Industrial Engineers*, Vol. 14, No. 3, pp. 305-317.

18. Wu, H. H. and Li, R. K.(1995a), "Capacity constraint resource wandering and its wandering behaviors in a production plant," *Journal of the Chinese Institute of Industrial Engineers*, Vol. 12, No. 1, pp. 63-69.
19. Wu, H. H. and Li, R. K.(1995b), "A new scheduling method for computer based scheduling systems," *International Journal of Production Research*, Vol. 33, No. 8, pp. 2097-2110.

Appendix

C : The unit capacity of each machine.

D : The number of planning periods in a planning horizon.

G_j : The margin for product j.

l_{ik} : The normalized loading on machine group i in the k^{th} planning period, i.e.,

$l_{ik} = \sum_{j=1}^{N} q_{jk} \times t_{ij}$ for

$i = 1, 2, \cdots, M; k = 1, 2, \cdots, D.$

M : The potential CCR groups in the system.

m_i : The number of machines in machine group i.

N : The maximum product types in the system $i = 1, 2, \cdots, M.$

q_{jk} : The marketing demand quantity for product j in the k^{th} planning period.

r_{jk} : The produced quantity for product j in the k^{th} planning period.

T_{ij}: The machining time for product j on machine group i, for $i = 1, 2, \cdots, M; j = 1, 2, \cdots, N.$

t_{ij} : The normalized machining time for product j on machine group i, for $i = 1, 2, \cdots, M; j = 1, 2, \cdots, N$, i.e., $t_{ij} = T_{ij}/m_i$.

T_k : The total margin in the k^{th} planning period.