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Korean and Hong Kong Student Teachers' Content Knowledge for Teaching Mathematics[1)]

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The purpose of this study is to probe into student teachers' understanding of mathematics content knowledge and to identify the features of knowledge which is required to be emphasized in the elementary teacher education. For this, student teachers attending teacher preparation courses in Korea and Hong Kong were interviewed on tasks encompassing the 'what', 'why' and 'how' aspects of elementary mathematics. It was found that for the student teachers in the sample, their understanding of the concepts behind elementary mathematical topics was not very thorough. They were unable to retrieve the advanced mathematics that they learned in their advanced mathematics courses. It is suggested that for student teachers in mathematics, it is essential that the advanced mathematics they learn be explicitly related to the elementary mathematics they have learned in school.

I. Introduction

In recent years, there has been a resurgence of interest in teachers' knowledge of mathematics (Chinnappan and Lawson, 2005; Hill et al, 2004; Kahan et al, 2003), and one reason for such renewed interest is that teachers' knowledge has been found to have a major impact on teaching practice (Ma, 1999; Sánchez & Llinares, 2003; An et al, 2004). For example, in the comparative study of middle school mathematics teachers in China and the U.S., An et al (2004) found that the pedagogical content knowledge of mathematics teachers in the two countries differed remarkably. Consequently, in their teaching, the Chinese teachers emphasized developing procedural and conceptual knowledge through reliance on traditional and more rigid practices, while American teachers gave emphasis to a variety of activities designed to promote creativity and inquiry in attempting to develop students' understanding of mathematical concepts.

Teachers' knowledge of mathematics has been also found to be significantly related to students' achievement (Mullens et al, 1996; Rowan et al, 1997). In view of East Asian students' superior performance in international comparative studies of mathematics achievement (Beaton et al, 1997; Mullis et al, 2004, 2008; OECD, 2004, 2007), one would expect East Asian teachers to be knowledgeable in

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mathematics. Indeed, in Ma's study found that the Shanghai teachers were more competent than their American counterparts in terms of a "profound understanding of fundamental mathematics" and the related pedagogy. In a replication of Ma's study in Korea and Hong Kong (Leung & Park, 2002), it was found that most of the East Asian teachers in the study had a good grasp of the underlying concepts of elementary mathematics. The teachers were also found to be proficient in mathematics calculations, although compared to the Shanghai teachers in Ma's study, they were weak in their ability to guide students in genuine mathematical investigations.

In a series of international comparative studies of mathematics achievement, the students of Korea and Hong Kong showed high achievement. Then, the natural assumption is that Korean and Hong Kong teachers have more profound knowledge in school mathematics compare to other countries. To study the competence of prospective Korean and Hong Kong teachers, student teachers attending teacher preparation courses were interviewed on their understanding of elementary mathematics.

II. Theoretical Background

1. Content Knowledge for Teaching Mathematics

Shulman is perhaps the first modern scholar to systematically look into the knowledge base of teachers, and he categorized teachers' knowledge content knowledge (1987, p. 8). As far as a particular subject domain such as mathematics is concerned, Shulman proposed three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (1986, p. 9). Shulman's work has made significant contribution to establishing a theoretical framework for teacher knowledge. The work highlights the existence of a specific field of knowledge beyond 'mathe- matics just as a discipline'. However, the line between content knowledge and pedagogical content knowledge may not be clear cut (Marks, 1990). For example, McEwan and Bull (1991) opined that all subject matter knowledge is pedagogical, and argued that the "distinction between content knowledge and pedagogical content knowledge introduces an unnecessary and untenable complication into the conceptual framework" (p. 318).

Notwithstanding these criticisms on the notion of pedagogical content knowledge, the effort of clarifying and determining the nature of this specific field of knowledge beyond 'mathematics just as a discipline' continued. Hill et al (2005) coined the term Content Knowledge for Teaching Mathematics (CKT-M) to describe the specialized content knowledge the teacher, in contrast to the mathematics knowledge an adult or a mathe- matician possessed. This specialized field of content knowledge is needed by the teacher to handle the problems s/he faces and the questions addressed to his/her during the everyday experience of teaching.

2. Knowledge of What, Why and How

Although Hill et al (2005) emphasized that the specialized knowledge discussed above "is mathematical knowledge, not pedagogy", the fact that it is mathematical knowledge the teacher "used to carry out the work of teaching mathematics" (p. 373) implies that this specific field of content knowledge involves not only the content of the knowledge (the 'what' aspect of mathematics), but also the rationale for the content (the 'why') and how the content is to be delivered to students in a way that is mathematically correct and understandable to students (the 'how' aspect of mathematics). Among the three aspects of mathematical knowledge, the aspect of 'what' is considered to be the most fundamental. Without knowing what the mathematics concept is, it is not possible to even start asking for the rationale of the concept, and without knowing the mathematics concept and its rationale, it is not possible to consider 'how' best to deliver the concept to students.

In this paper, results of a small scale study on the content knowledge of prospective elementary mathematics teachers in Korea and Hong Kong will be reported. Content knowledge in this paper is taken to mean what Hill et al (2005) termed as Content Knowledge for Teaching Mathematics (CKT-M). It is the specialized kind of mathematics knowledge that teachers need to master in order to teach mathematics well, and it encompasses the 'what', 'why' and 'how' aspects of mathematics.

III. Methodology

To probe into student teachers' understanding of mathematics content knowledge, an instrument which fits the conception of CKT-M as discussed above is needed. One such instrument was designed in a study by Fung (1999). In this study, Fung's study was replicated for a matching sample of prospective elementary school teachers in Korea. Following his methodology, the Korean student teachers were administered a set of items on elementary mathematics and mathematics teaching, and were interviewed on those items. In order to conduct a more realistic assessment of student teachers' competence in elementary mathematics, a new set of criteria for the assessment instrument was established after re-examining the criteria used by Fung, and was applied to the Korean study. Fung's data set was then re-scored according to the new criteria so that the results for the Hong Kong sample and the Korean sample are comparable.

1. Sample

The sample in Fung's study consisted of 17 students studying in a pre-service teacher education course in a tertiary teacher education institution in Hong Kong. The entry requirements for the course included a pass in mathematics at the Hong Kong Certificate of Education Examination, the public examination taken at the end of secondary schooling. All student teachers in the course needed to take a minimum number of credit point in mathematics, but some would opt for studying mathematics beyond the compulsory requirement. Students invited to participate in the study were chosen among this latter group, so theoretically, they should be more competent and more interested in mathematics among their peers.

To facilitate comparability, a matching sample of 17 Korean student teachers was invited to join the study. The students were studying in a pre-service teacher education program in a national university of education. For entry qualifications into the course, there was no special requirement for mathematics itself, but most students who entered that university were high achievers in the College Scholastic Aptitude Test (CSAT), the national college entrance examination in Korea.

2. Instrument

The instrument consisted of five items which fit the idea of content knowledge for teaching mathematics. These five questions do not constitute a comprehensive mapping of the mathematical preparation expected of a teacher, but the questions serve to provide a partial picture of how well teachers are prepared regarding what they are supposed to teach.

Question 1: What is direct proportion?

Direct proportion is a common topic in the elementary mathematics curriculum, and in this first question, student teachers were asked what direct proportion was in order to see whether they had a clear understanding of this elementary concept.

Question 2: Is it possible to find two positive integers whose quotient is an infinite non-recurring decimal? Why?

The term 'rational numbers' and its full meaning are not introduced at elementary school level. However, to teach the concepts of natural numbers and fractions properly, the teacher needs a clear understanding of the concepts of rational numbers. This question was used to probe into whether student teachers were able to link the elementary mathematics they were going to teach with the mathematics knowledge they had acquired in their secondary school studies.

Question 3: The following calculation is done to find the LCM of three positive integers.

$$\begin{array}{r|lll} 2 & 12 & 20 & 90 \\ \hline 2 & 6 & 10 & 45 \\ \hline 3 & 3 & 5 & 45 \\ \hline 3 & 1 & 5 & 15 \\ \hline 5 & 1 & 1 & 5 \\ \hline & 1 & 1 & 1 \end{array}$$

Thus the LCM of 12, 20 and 90 is $2\cdot2\cdot3\cdot3\cdot5 = 180$.

Why is that it works? Can we apply this method to find the LCM of more than three numbers?

In Korean and the Hong Kong elementary mathematics syllabuses, the use of this method to find the LCM of numbers is limited to finding the LCM of two or three numbers only, and in this question, student teachers were asked whether the method could be applied to more than three numbers. Student teachers should know that the algorithm is simply a way to extract all the factors of the numbers concerned, and the algorithm works as a consequence of the Unique Prime Factorization Theorem, a fact that they should have learned in the Number Theory courses that they studied in advanced mathematics. So this question explored into the linkage between elementary mathematics and advanced mathematics.

Question 4: Why is that when we divide two proper fractions, we simply 'invert and multiply'?

Students are taught a large number of rules and algorithms in mathematics, and this 'invert and multiply' rule is one of them. In fact, there have been several researches on this topic including Borko et al (1992) and Tiroish (2000). For the rule of division of fractions to be taught effectively, the teacher needs a thorough understanding of why these rules work. This question probed into whether the prospective teachers in the study knew why this particular 'invert and multiply' rule worked.

Question 5: How do you explain the meaning of the division algorithm with the help of counting cubes (of different colors)?

$$
\begin{array}{r}
35 \\
6\,\overline{)\,213} \\
\underline{18} \\
33 \\
\underline{30} \\
3
\end{array}
$$

In the teaching of elementary school mathematics, teaching aids such as real life objects and models are often used. For this question, in asking student teachers to explain the meaning of the division algorithm with the help of counting cubes, their understanding of the meaning of division and the digits of the numbers was explored. Their answers will also show whether they know the mathematics well enough to be able to incorporate manipulatives into effective teaching of mathematical concepts and skills.

It can be seen from the descriptions of the five questions above that they address the 'what', 'why' and 'how' of the content knowledge for teaching mathematics. Questions 1, 2, and 3 deal with knowing the mathematics concepts. Questions 2, 3 and 4 tap into the reasons behind mathematical rules and algorithms. And question 5 touches on how to present mathematical concepts.

The five questions also cover various aspects of the elementary school mathematics curriculum. Altogether, they address issues including the understanding of elementary school mathematics concepts (Question 1), the linkage between elementary school mathematics on the one hand and secondary school (Question 2) and advanced mathematics (Question 3) on the other, the understanding of rules and algorithms taught in the elementary classroom (Question 4), and the understanding of the mathematics behind the use of manipulatives (Question 5).

IV. Results

1. Results of question 1

It is surprising to see that many student teachers could not explain clearly what is meant by direct proportion. Only 6 Korean teachers and 1 Hong Kong teacher thoroughly understood what direct proportion is. According to the criteria, a correct answer should look like the following response given by a Hong Kong student teacher:

> When A increases by a certain multiple, B increases in the same multiple. When A decreases by a certain multiple, B decreases in the same multiple.

Two Korean student teachers used symbols to explain direct proportion. Here is one example:

> The relation in direct proportion can be written as y=kx. Here x is the domain and y is the co-domain, and these two variables are direct proportional. In this formula, k determines the slope, and the value of k can be either positive or negative.

The most common erroneous way of defining direct proportion was to say "when one (variable) increases, the other (variable) increases". This may not be true since the proportionality constant may be negative. The number of Korean and Hong Kong teachers defined direct proportion in this ways was 10 and 2 respectively. As one Hong Kong student teacher put it:

> When two numbers change in such a way that when one increases, the other also increases, we say it is direct proportion.

The wrong or incomplete answers of the student teachers in the sample indicate that they did not fully understand this simple concept in elementary mathematics.

2. Results of question 2

In learning fractions and decimals in elementary mathematics, students are often asked to convert fractions to decimals. A common question that may come across the minds of the more inquisitive students is whether the quotient is always a terminating decimal or a recurring decimal. Of course, elementary school students need not learn the full concepts of rational numbers at this stage, but any teacher with a clear understanding rational numbers should be able to come up with a satisfactory answer to this question.

It is disappointing to see many student teachers could not offer a satisfactory answer to this question. Only 9 Korean teachers and 3 Hong Kong teachers responded to this question correctly. Many Hong Kong student teachers (5 out of 17) offered 22/7 as an example of two positive integers whose quotient is an infinite non-recurring decimal. This mistake may have arisen because in Hong Kong elementary schools, 22/7 is often used as an approximation for π, and these student teachers may have missed the point that it is only an approximation, albeit a very good approximation. In contrast, Korean textbooks do not use 22/7 for π, and as a consequence, none of the Korean student teachers referred to 22/7. The results confirm the strong influence of textbooks on students' understanding of mathematics concepts.

Some Korean student teachers mentioned prime numbers or numbers that are relatively prime in answering this question. For example, two student teachers gave "two numbers which are both prime numbers" and one student teacher gave "two numbers which are relatively prime" as examples of two positive integers whose quotient is an infinite non-recurring decimal.

The following response from a student teacher in Korea shows that she had a clear understanding of the meaning of rational and irrational numbers:

> We learned that irrational numbers can be written as infinite non-recurring decimals. This is a very basic fact dealt with in the 8th grade. Another definition of an irrational number is a number which cannot be transformed into a fraction. Thus, it's obvious that finding two positive integers whose quotient is an infinite non-recurring decimal is impossible.

Another Korean student teacher who showed full understanding used an 'informal proof' to show that it is not possible to find two positive integers whose quotient is an infinite non-recurring decimal:

> Let a, b be natural numbers. If a/b = k is an infinite non-recurring decimal, then a=bk. The product of a natural number and an infinite non-recurring decimal should be also infinite non-recurring decimal. Since a is a natural number, this leads to a contradiction.

Among the answers with partial understanding, some student teachers responded with only the correct answer without explanation. Some student teachers' understanding of the concept of rational numbers was not very stable. For example, this is the answer given by one Hong Kong student teacher:

> 7/4. Oh, terminating. ... Maybe 7/3. ... Oh, wrong again.

After a while, she claimed that it could not be done since infinite non-recurring decimals are irrational numbers.

3. Results of question 3

Although all except two student teachers in Hong Kong knew that the algorithm (so-called 'short division' method) works, many did not know why it works. Student teachers' responses to this item show that knowing how to apply an algorithm does not necessarily imply knowing why the algorithm works. Here is a typical answer coded as a partially correct from student teacher in Korea.

> Don't know why it works. But it works for more numbers as well.

In scoring this item, an answer coded as a correct one requires some explicit explanation of the rationale behind the algorithm. For example, one of the Hong Kong student teachers, after separately computing the prime factorization for 12, 20, and 90, she multiplied all prime factors for the three numbers together, then crossed out the repeated duplicated ones and said, This method finds a common factor of all in one shot, thus giving no duplication. Then collect the remaining. It can be used for more numbers.

The algorithm of finding the LCM of numbers in question 3 works as a consequence of the Fundamental Theorem of Arithmetic[2]. The algorithm is just a means of extracting all the factors of the numbers and disregarding the overlapping ones. Yet none of the student teachers interviewed was able to relate this elementary mathematics algorithm to the advanced mathematics that they had learned, showing that the advanced mathematics that they learned did not really help them to understand the rationale behind this algorithm better.

4. Results of question 4

In order to be able to explain why the 'invert and multiply' rule works, one needs a thorough understanding of the meaning of division of fractions. In this study, this particular rule is used as an example to find out whether student teachers are reflective in their learning and teaching of mathematics.

The responses of student teachers showed that they understood the division of fractions in different ways, and as long as they were able to provide mathematically justified explanations, the answers would be classified as 'satisfactory'. Only 1 out of 17 the Hong Kong student teachers gave a satisfactory answer who used the following 'area model' in conceptualizing division of fraction. By dividing two identical rectangles into 4 and 3 equal parts respectively, shading the corresponding fractions, then further subdividing each cell into smaller equal parts (each equivalent to $\frac{1}{12}$ of the original rectangle), he successfully explained that $\frac{3}{4}\left(\frac{1}{4} \div \frac{1}{3}\right)$ was the correct answer.

Among 17 teachers, the responses from 4 Korean student teachers were classified as 'satisfactory'. What follows are the sample of satisfactory responses who used the 'ratio model' and ''induction model'. The following answers show that division of fractions may be understood in a number of legitimate ways

Let's interpret division as a ratio. The meaning of a division where the dividend is 1 and the divisor is 1/2 is to find the dividend of an equivalent number when the divisor is one. I mean, 1: 1/2 = 2: 1. So, we can conclude that the meaning of division is to calculate the dividend in a ratio when the divisor is one. In the above formula, we also notice that division by a fraction is to multiply the reciprocal of the fraction.

If we divide the numerator 1 by the denominator 5, then we get 1/5. Thus we can notice that division by a certain number is the same as multiplication by the reciprocal of that number. Now we generalize this rule to the case when both the numerator and denominator are fractions.

The failure to explain the rule satisfactorily may indicate that one lacks a thorough understanding of the concept of division of fractions. For example, one Korea student

2) This is also called as Unique Prime Factorization Theorem; any integer greater than 1 can be written as a unique product (up to ordering of the terms) of prime numbers.

teacher explained the rule as follows:

> If we change the division of fractions to 'invert and multiply', then we can do the elimination and the computation becomes easier.

She did not realize that this is a consequence of the rule rather than its justification.

5. Results of question 5

Counting cubes of different colors are one of the common teaching aids. For those who understand both the meaning of division and place value of numbers, and the role place value plays in the division algorithm, they should be able to make use of the characteristics of the manipulatives to illustrate the meaning behind the division algorithm. In the scoring, no distinction was made between those who used cubes of three different colors from those who used only two colors.

The number of Korean student teachers who understood division and place value and were able to use cubes in different colors to explain division was 10. Six out of those 10 Korean student teachers explained the division with cubes in three different colors, and the remaining four used cubes in two different colors. Here is an example of a Korean student teacher explaining the division algorithm using cubes of three different colors:

> Assign three different colors for the place values. For example, tell the students that red cubes represent hundreds, blue represent tens, and yellow represent ones. Start with 2 red cubes, 1 blue cube, and 3 yellow cubes. First we can't divide 2 red cubes into 6, so exchange 2 red cubes into 20 blue cubes, now we have 21 blue cubes⋯.

Two out of 17 Hong Kong student teachers showed full understanding of this task. One of them was able to use cubes of three different colors to explain the division, and the other student teacher used 21 cubes of one color and 3 cubes of another color as follows:

> 21 cubes represent 210, which means 1 cube represents 10. Put them 6 in a group, we get 3 groups, 3 cubes remained. These 3 cubes represent 30, together with the 3, we have 33. Further divide it into 6 shares, 3 cubes remain.

The use of manipulatives to illustrate the relevant mathematics concepts reflects one's level of understanding of the concepts. For example, some student teachers suggested using 213 cubes of the same color to illustrate the idea of the division. This shows that they understood the meaning of the division as sharing or grouping, but they may not be able to integrate this understanding of division with the notion of place value in the long division algorithm. Here is the answer from one Hong Kong student teacher who understood the meaning of division, but with no sign of understanding place value, and thus used the cubes of one color.

> I need 213 counting cubes and divide them into 6 equal shares. But then I will not use cubes of different colors.

V. Discussion

During the interview in the study, all the student teachers agreed that the questions posed to them were important and relevant to elementary mathematics teaching, and hence the content covered by the questions should constitute part of the specific field of content knowledge for teaching mathematics as defined for this study. Yet, even with a set of less stringent criteria than that used by Fung (1999), these student teachers did not do very well on the questions. They confessed that they did not know the answers to many of the questions or had not even thought about these questions.

1. The 'what' aspect of mathematics

The mathematics concepts covered by the first three items are rather simple ones. If student teachers do not even have a clear knowledge of what these simple mathematics concepts are, they will not be able to ask why these concepts hold, and will not know how to explain these concepts clearly to students when they become teachers.

Take question 1 as an example. One of the possible reasons for the misconceptions on direct proportion shown in the results is the 'localization' of the concept, i.e., through being exposed only to typical examples of the direct proportion (e.g., the distance traveled being proportional to the time taken when the speed is constant) rather than a full characterization of that concept, students only know some specific case of the concept and not the full concept itself. This shows the importance of comprehensive characterization of concepts for students at elementary school level.

According to Dienes' mathematical variability principle (Dienes, 1969), in order to maximize the generalizability of a mathematical concept, as many irrelevant mathematical variables as possible should be varied while at the same time keeping the relevant variables intact. In the context of teaching direct proportion, this means that the teacher should vary irrelevant variables such as increase and decrease of the quantities while keeping the relevant variable (changing according to the same ratio) invariant. Examples should of course be introduced to helps students understand the concept, but the critical attributes of the concept should also be explicitly pointed out. It cannot be assumed that students can automatically crystallize the critical points of a concept simply by being exposed to examples of the concept. As to the judgment on what constitute the critical points for a certain concept, it requires a teacher with thorough understanding of the content.

2. The 'why' aspect of mathematics

This study shows that there are some mathematics concepts which student teachers did not know well, but there are many other mathematics concepts which they did know well. Yet, knowing what those concepts are does not necessarily imply knowing why the concepts are true. Without a full understanding of the rationale behind the mathematics concepts and rules (the 'why'), it is unlikely that student teachers will be able to explain the meaning of the concepts and rules (the 'what') well in their future teaching. As a consequence, their teaching, and hence their students' learning, can only be very superficial. At best, they may be able to produce students who can perform the algorithms proficiently but who do not know why the algorithms work. And this 'ignorance cycle' may repeat itself when their students in turn become teachers in the future.

On the part of the student teachers, this failure to link advanced mathematics with elementary mathematics to provide the rationale for the latter is a sign of an absence of a reflective attitude in asking 'why' in their learning and teaching of mathematics. It is therefore important that in designing the curriculum of advanced mathematics courses in a teacher education programme, specific attention should be paid to the links between the advanced mathematics and the elementary mathematics that students already know. Student teachers should also be explicitly encouraged to build up the relevant links, and to develop a reflective attitude when they take the advanced mathematics courses.

3. The 'how' aspect of mathematics

In line with a constructivistic view of learning, there is much stress on hands-on experience in learning mathematics, especial at elementary school level. Manipulating concrete objects is supposed to help students construct their own knowledge and hence understand the underlying mathematical ideas better. However, using manipulatives to teach mathematics is not simply a 'method of teaching and learning'. The ability to use manipulatives properly to illustrate the relevant mathematics concepts is intricately related to one's level of understanding of the concepts. As the results of question 5 of this study show, without a solid knowledge of the relevant mathematical concepts, it is doubtful whether teachers are able to direct their students to manipulate concrete objects in order to learn the relevant mathematical ideas. And without well designed activities based on sound mathematics concepts, it is doubtful whether students will really learn the mathematics through manipulating the relevant concrete objects.

VI. Conclusion

In this study, it is found that Korean and Hong Kong student teachers' understanding of the concepts behind elementary mathematical topics was not very thorough. They were unable to access the advanced mathematics that they had learned in their teacher education course to inform the elementary mathematics topics that they are going to teach when they become teachers. This lack of understanding hampers teachers' capability in conveying mathematical concepts clearly to students, resulting in only superficial understanding of mathematics. At best, such superficial understanding may give rise to students' competence in routine mathematical skills, but it is not conducive to a thorough understanding of the relevant mathematics concepts.

Although the primary purpose of this study is not to compare the performance of student teachers in Hong Kong and Korea, it is obvious from the results presented above that the Korean student teachers performed better than their Hong Kong counterparts in the tasks during the interview. Could this difference in performance be due to differences in the teacher education programmes in the two places?

Park (2005), in a study that compared the elementary mathematics education curricula across East Asian countries, found that while Korea and Hong Kong had roughly the same percentage (29%) of their teacher education curricula devoted to pedagogical content knowledge, the Korean curriculum devoted a higher percentage (31%) of time to content knowledge than the Hong Kong one (25%). This difference in emphasis on content knowledge may be one of the reasons for the difference in performance of the Korean and Hong Kong student teachers in this study.

But more importantly, the responses of the student teachers in this study point to the kind of content knowledge that is needed for effective teaching in elementary school mathematics. This may be even more important than the quantity of content knowledge covered in the teacher education programme. For example, an obvious kind of content knowledge needed is that which relates advanced mathematics specifically to elementary mathematics (e.g., when teaching unique prime factorization of numbers, relate it to LCM explicitly). Another kind of content knowledge is a revisit of some elementary school mathematics concepts (e.g., direct proportion) which are not explicitly dealt with in advanced mathematics.

For student teachers preparing for their mathematics teaching career, it is essential that the advanced mathematics they learn be explicitly related to the elementary mathematics they have learned in school. This will enhance their appreciation of both the advanced mathematics they are learning and the elementary mathematics they have learned in school.

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한국과 홍콩 예비교사의 학교수학에 대한 이해 분석 연구

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이 연구의 목적은 한국과 홍콩 초등학교 예비교사들의 학교수학에 대한 이해 정도를 조사하고, 교사교육 과정에서 강조되어야 할 지식의 성격에 대한 시사점을 얻기 위한 것이다. 이를 위해 초등학교 수학 교수(敎授)와 관련되면서 '무엇을', '왜', '어떻게'의 측면을 포괄하는 다섯 개의 문항을 선정하고, 한국과 홍콩의 초등학교 예비교사를 대상으로 심층면접을 실시하였다. 면접 결과 한국과 홍콩의 초등학교 예비교사들은 학교 수학의 원리에 대해 충분히 이해하고 있지 못하며, 초등학교 수학을 중등학교 수학 및 대학 수준의 수학과 연결시켜 이해하는 능력이 부족한 것으로 나타났다. 이 결과에 비추어볼 때 초등학교 예비교사 교육에서 더욱 강조되어야 할 것은 중등학교 및 대학 수준의 수학을 초등학교 수학과 연계시키는 것이라고 할 수 있다.

* **Key Words** : 교수학습(teaching and learning); content knowledge for teaching mathematics(수학 교수위한 내용지식); pedagogical content knowledge (내용 교수 지식); teacher education program(교사 교육 프로그램)

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