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On Mode Correlation of Solar Acoustic Oscillations

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Abstract

In helioseismology it is normally assumed that p-mode oscillations are excited in a statistically independent fashion. Unfortunately, however, this issue is not clearly settled down in that two experiments exist, which apparently look in discrepancy. That is, Appourchaux et al. (2000) looked at bin-to-bin correlation and found no evidence that the assumption is invalid. On the other hand, Roth (2001) reported that p-mode pairs with nearby frequencies tend to be anti-correlated, possibly by a mode-coupling effect. This work is motivated by an idea that one may test if there exists an excess of anticorrelated power variations of pairs of solar p-modes. We have analyzed a 72-day MDI spherical-harmonic time series to examine temporal variations of p-mode power and their correlation. The power variation is computed by a running-window method after the previous study by Roth (2001), and then distribution function of power correlation between mode pairs is produced. We have confirmed Roth's result that there is an excess of anti-correlated p-mode pairs with nearby frequencies. On the other hand, the amount of excess was somewhat smaller than the previous study. Moreover, the distribution function does not exhibit significant change when we paired modes with non-nearby frequencies, implying that the excess is not due to mode coupling. We conclude that the origin of this excess of anticorrelations may not be a solar physical process, by pointing out the possibility of statistical bias playing the central role in producing the excess.

Keywords: Sun:oscillations – Sun:interior – methods:data analysis

1. Introduction

In helioseismology it is normally assumed that p-mode oscillations are excited in a statistically independent fashion. If the assumption is invalid, the current analysis procedures have to be modified accordingly. Is there any hint that the assumption is invalid? Appourchaux et al. (2000) looked at bin-to-bin correlation and found no evidence that it is not. On the other hand, Roth (2001) reported that p-mode pairs with nearby frequencies tend to be anti-correlated, possibly a mode-coupling effect. This work is motivated by a paper by Roth (2001) that there exists an excess of anticorrelated power variations of pairs of solar p-modes. He further argued that a time-dependent perturbation theory of the velocity fields is working (Roth & Stix 1999, 2001), in which they have found that coupling p-modes show anticorrelated power variations.

Using data from the Michelson Doppler Imager, we have demonstrated that there does indeed exist an excess of anticorrelated power variations of pairs of solar p-modes with harmonic degree

$l \leq 60$. We have also calculated the correlation between the power of artificial solar oscillations, and adopted various criteria in choice of mode pairs. In agreement with the earlier investigations, we have found that such an appearance of anticorrelation is rather profound. That is, any choice of mode pair reproduces similar behavior in the mode correlation distribution. Therefore, we have concluded that the origin of what Roth (2001) found is the stochastical behavior of mode power excited by the turbulence. To support an idea that the mode anticorrelation is due to a property of random numbers with the Boltzmann distribution, we have done further experiments, which are followed in following sections.

We describe how we analyze the MDI data and what we find from the analysis in §2. We briefly demonstrate with numerical calculations what the results we have may imply in §3. And finally we conclude by summarizing and making some comments in §4.

2. Analysis of Observational Data

We have used time series data of spherical harmonic coefficients with harmonic degree $l \leq 60$ obtained with the Michelson Doppler Imager (MDI) on the *Solar Heliospheric Observatory (SOHO)*. The data set covers 72 days from 1999 April to June. We choose this particular set of data, which overlaps in time with what Roth (2001) used his analysis, so that the solar cycle effect can be ignored in comparison with Roth's results. We calculate the complex Fourier transform of modes for given l and m and the corresponding power spectrum, which is defined by the square of the Fourier transform, and identify the solar modes. Before taking the Fourier transform, we remove bad data points and detrend long term variations in order to increase the signal-to-noise ratio. Having obtained the power spectrum we further select modes with frequencies $2700 \leq \nu \leq 3200$ μHz in calculating the correlation.

To represent time variations of individual mode power, we calculate the power spectrum of a short data string which is predetermined by the boxcar running window, whose length is denoted by W. We numerically integrate the power under a peak over the interval $(\nu_0 - 3\Gamma,\ \nu_0 + 3\Gamma)$, where Γ is the FWHM of the peak and ν_0 is the central frequency of the peak. The local background level is subtracted from the integrated power. We repeat the procedure with windows separated by a certain time step, s, which leads us to end up with the varying power in time. One may wish to estimate the power in another way. For instance, one may analytically extract the power from a best-fitted Lorentz curve. Or, one may adopt another procedure, like the Hilbert transform. We find that a different definition of the power hardly change conclusions of the analysis. Having obtained a variation of the power of a mode as a function of time, we determine the correlation between power variations of two modes selected by a criterion discussed later on. The correlation integral is done over the period of T.

In Figure 1, we show the distribution of correlation values with different mode selection criteria. Following the earlier investigation (Roth 2001), we adopt analysis parameters $s = 8$ hours, $T = 36$ days, and $W = 3$ days for these plots. They have picked up a pair of modes nearby in the frequency domain such that their separation $\delta\nu$ is less than 1.6 μHz, and argued that oscillations nearby in frequency are likely to be related. In Figure 1, solid, dotted, and dashed curves show results from mode pairs whose frequency difference $\delta\nu$ is $\leq$ 1.0 μHz, $\leq$ 3.0 μHz, $395 \leq \delta\nu \leq 400$ μHz, respectively. We confirm in our analysis that the distribution of correlations is indeed asymmetric and there are more anticorrelated pairs than correlated ones. However, in our analysis we find a bit less anticorrelations than Roth claimed. That is, there are 1.33 times more anticorrelations than correlations for $W = 3$ days, and 1.17 for $W = 7$ days. We also confirm that the width of the running window W influences a degree of the skewedness. A broad window yields a broader distribution of

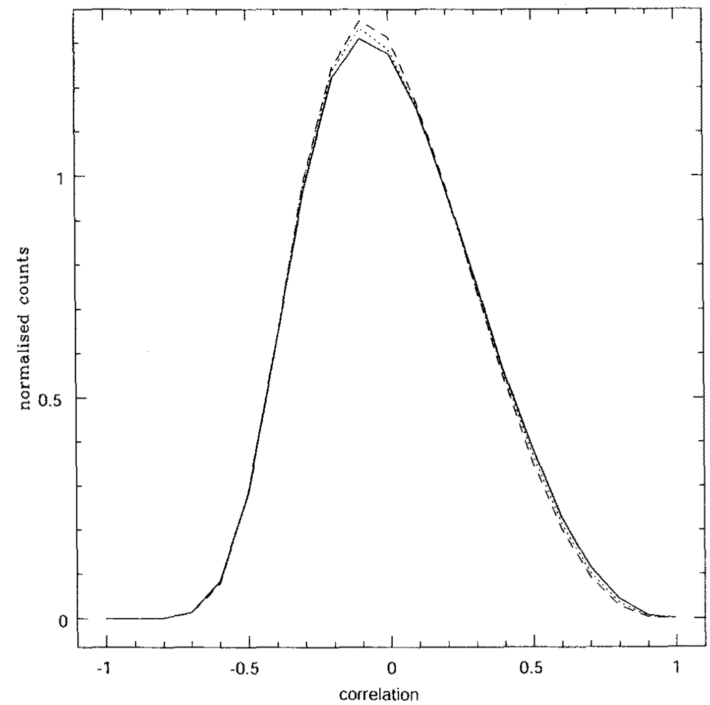

Figure 1. Distribution of correlations between selected mode pairs. Solid, dotted, and dashed curves show results from mode pairs whose frequency difference $\delta\nu$ is $\leq$ 1.0 μHz, $\leq$ 3.0 μHz, 395 $\leq \delta\nu \leq$ 400 μHz, respectively.

the correlations between pairs of modes than a short window. More importantly, it is shown that it is insensitive to the mode selection criterion. The apparent distribution seems unchanged even if we choose mode pairs separated by some arbitrary amounts, or even choose them randomly. Hence, we conclude that the origin of this excess of anticorrelations can be explained by a physical association with the proximity of mode pairs alone even if it plays any role.

In Figure 2, we show the distribution of correlations for various values for s and T to see how our findings depend on other analysis parameters. We fix $\delta\nu$ to 1.6 μHz and W to 3 days, in this example. The solid curve in all panels shows the distribution of correlations resulting from values of parameters, s and T, indicated at the upper right corner in the panel, and as a reference for comparison the result with $s = 8$ hours and $T = 36$ days is given with the dotted curve. Apparently, for a given set of analysis parameters T, $\delta\nu$, and W, when s becomes large the distribution becomes broader and more asymmetric. However, the reason why an appearance of the distribution looks different is due to the number of data points in a data string used in calculating correlation integration, that is, the ratio of s to W. For instance, one may find that two cases where $s = 3$ days and $T = 36$ days, and $s = 7$ days and $T = 72$ days look similar to each other. Furthermore, we find that whether s is long or short compared with the mode life time is not so crucial a factor in resulting distributions, as some of distributions look similar to that with an much shorter s than a widely accepted mode life

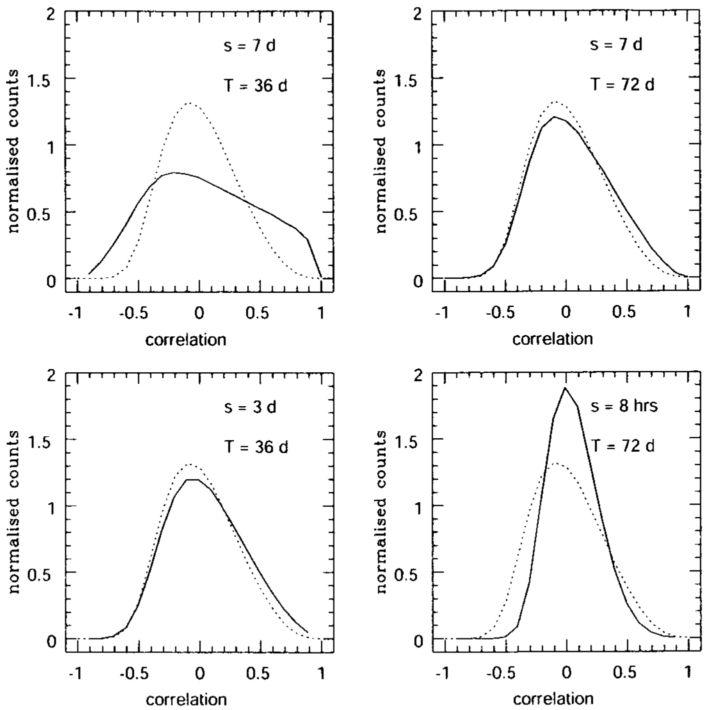

Figure 2. Similar plots to Fig. 1, but showing results for fixed $\delta\nu$ to 1.6 μHz and W to 3 days. Values for s and T are indicated at the upper right corners in each panel. The solid curve in all panels shows the distribution of correlations resulting from parameter values indicated, and the result with s = 8 hours and T = 36 days is given with the dotted curve for comparison.

time.

Interestingly enough, according to what is implied in Figure 2, one may consider variations of the mode power as a function of time as a sequence of pure random variables. To test such an idea we shuffle the order of temporal power variations of the observational data before calculating the correlation. If an element in the sequence of variables depends on previous ones, a random shuffling may affect the distribution of correlation values. What we found is that this naive expectation agrees with what we observe when s is large. However, when s becomes small and thus the number of elements in a sequence is large shuffling tends to make the distribution narrow. We also divide data strings into smaller substrings and randomly shuffling the order of the substrings. In Figure 3, we show the distribution of correlation values with the solid curve after dividing and shuffling. We divide the data string into 5 substrings in this particular example. We adopt analysis parameters s = 8 hours, T = 36 days, $\delta\nu$ = 1.6 μHz, and W = 3 days, in this example. The dotted curve shows the original distribution for comparison. We find that two distributions share main features. We use the Kolmogorov-Smirnov (K-S) test to compare two distributions. These two distributions are consistent with the high K-S probabilities ($\approx$ 72 %). In Figure 4, we show the distribution of mode power as a function of the power in units of its average value. The solid line shows the result

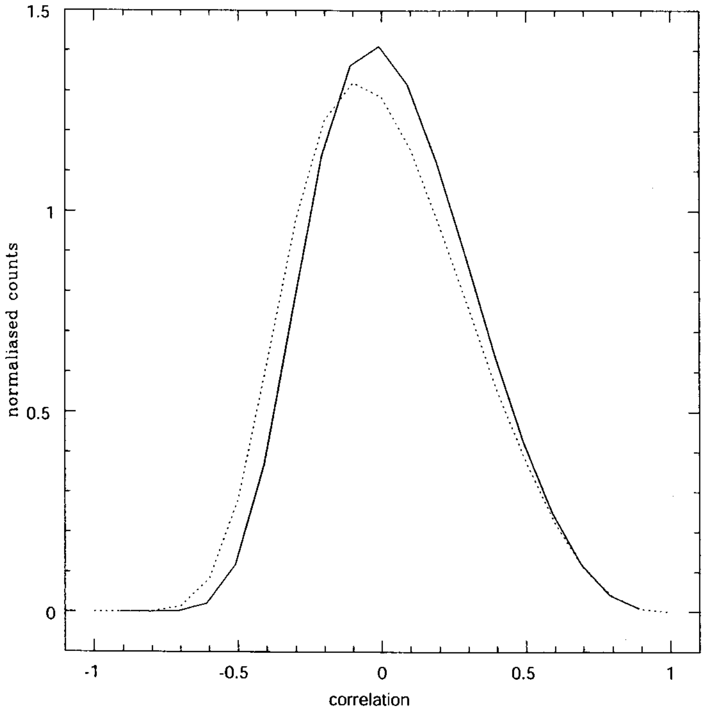

Figure 3. Similar plots to Fig. 1, but showing results for shuffled data strings with the solid curve. We adopt analysis parameters s = 8 hours, T = 36 days, $\delta\nu$ = 1.6 μHz, and W = 3 days. The dotted curve shows the original distribution.

and the dotted line represents the Boltzmann distribution. The distribution is close to the Boltzmann distribution, yet is not quite consistent with it. It is well known that the distribution function of stochastically excited oscillators is the Boltzmann distribution, and begins to deviate as averaging interval increases (Kumar, Franklin, & Goldreich 1988). We also note that there is the excess at high energy ends, which is reported earlier (Elsworth et al. 1995, Chang & Gough 1998).

3. Stochastically Excited Oscillators

We compare the distribution of the MDI data used in this analysis with that of stochastically excited oscillators. Solar oscillation modes are regarded as a damped simple harmonic oscillator, which is excited by random force. Such a force can be characterized by the numerous solar granulation such that a resulting forcing function is rather smooth than impulsive (Chang & Gough 1998). This model reproduces well enough the observational features, such as, mode energy variation in time and its distribution (Chang & Gough 1998). By employing the algorithm used by Chang & Gough (1998), we have generated artificial data sets, whose mode parameters are similar to those of MDI. In Figure 5, we show the distribution of correlations of our artificial data with the MDI results. The distributions derived from MDI data generally resemble the our model distribution which is

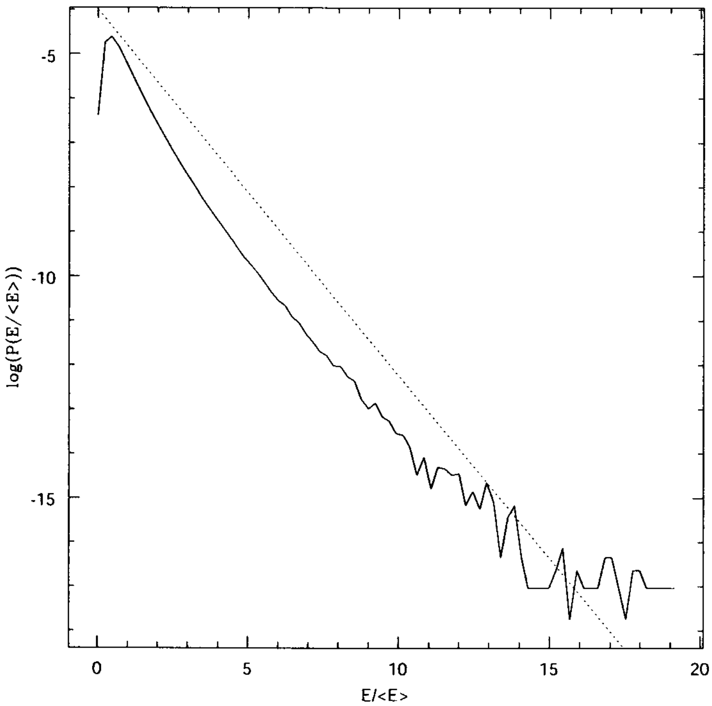

Figure 4. Distribution of mode power as a function of the power in units of its average value. The dotted line represents the Boltzmann distribution.

based on the Boltzmann-distributed power.

4. Discussions and Conclusion

A 72-day MDI Doppler spherical-harmonic time-series has been examined for a possible mode correlation. Temporal variation of mode power was extracted by running box-car windows in time domain. Correlation coefficients exhibit skewed distribution with usually more anti-correlated pairs, but frequency separation between a pair seems to have little effect on correlation coefficient. We have also found smaller excess of anti-correlated pairs than Roth's report (Roth 2001). A simple model of stochastically excited damped oscillator for intrinsically uncorrelated modes can reproduce the distribution very well. The observed skewness in the distribution function and the excess of anti-correlated pairs are not necessarily a manifestation of underlying correlation between modes. At the present accuracy a stochastically-excited damped-oscillator model for independent modes seems more than adequate to describe solar p-mode oscillations, including correlation between temporal power variation.

We cannot reject, based on what we have found, that the excess of anti-correlated pairs is no more than a result of natural statistical bias. However, we noted the discrepancy between Roth's previous work and the current work in the amount of the excess, which we so far could not resolve.

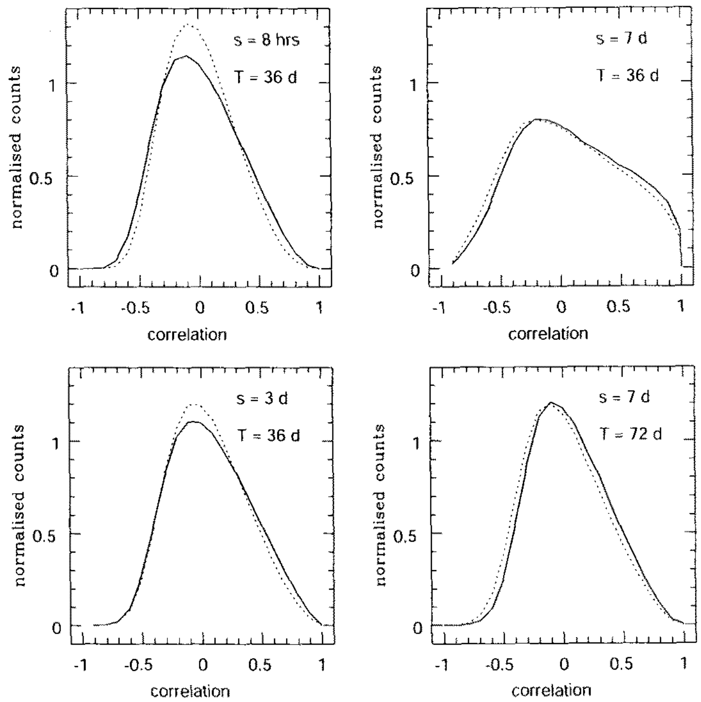

Figure 5. Similar plots to Fig. 1, but showing results from artificial data. Values for s and T are indicated at the upper right corners in each panel, and those for $\delta\nu$ and W are fixed to 1.6 μHz and 3 days, respectively. For comparison, the results from MDI data are plotted with the dotted curve.

If and when this is resolved, our conclusion may be a different one. In any case, it seems that looking at correlation coefficients in the present manner is not the best way to study intricate mode correlation.

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