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Intuitive Knowledge of Percentages Prior to Learning

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This research examined intuitive knowledge of 6th grade students in Israel prior to the formal learning of percentages in school. In other words, the research investigated knowledge of basic concepts and familiarity with the usage of percentages in daily life. Results have shown that students are familiar with the concept of percentages and that some students are able to handle simple problems composed of common percentages (50% and 25%). However, it was also found that many students had misconceptions that should be taken into account while the subject has been taught.

Keywords: percentages, intuitive knowledge
MESC Classification: F82
MSC2010 Classification: 97F80

INTRODUCTION

Percentages are found in various fields widely in daily life. Some examples are price discounts or increases, television viewing ratings, clothing tags, food packaging, and stock market information.

According to the educational program in Israel, we begin to teach percentages in the 6th grade. In the preceding two years, students have learned the subject of fractions and decimals, including solving word problems related to finding the partial quantity, the part, and the whole.

The students' knowledge of fractions and decimals constitutes an intuitive basis for

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learning 'percentages'. Educators state that intuition takes an essential role while learning mathematics. Researchers agree that in order to learn mathematics the didactic method should lead the learners into intuitive and analytic understanding (Fischbein, 1987). In addition, as Van den Heuvel-Panhuizen (1994; 2003) argues, in order for mathematics to be of human value, it must be connected to reality, be spoken to children in a way that they can understand, and be relevant to society.

All studies that we found regarding students' grasp of percentages examined students after the formal study of the subject. The research shows difficulties in learning and using percentages in spite of the broad usage of percentages in daily life and the students' previously established knowledge in fractions and decimal numbers. Some examples of these difficulties are: using an algorithm that is not suitable, problems with percentages that are greater than 100, ignoring the percent symbol, and looking at the algorithm as an algorithm of integers (Parker & Leinhardt, 1995; Chen & Rao, 2007). A research reports that the study of percentages is difficult because the notion itself is more complicated than it seems (Gay & Aichele, 1997).

All the findings above lead us to make this research in order to investigate the intuitive basis about percentages which student in the 6th grade in Israel has prior to formal study of this material.

METHOD

Research Questions

1. What is the meaning of the notion "percentage" for students prior to their formal study of percentages?
2. What is the students' conception of the whole (100%) before they learn percentages?
3. To what degree students are familiar with the percentages 25% and 50%, and how much do they use this knowledge for simple calculations before they study percentages?

Sample

The sample groups are consisted of 99 students of the 6th grade from two schools in different neighborhoods, prior to learning percentages.

- Group A: 47 students from a high-achieving school that received a mark of 96 on a national equivalency exam (the average mark was 73).
- Group B: 52 students from a school that received a mark of 50 on the same exam.

Research Tools

The questionnaire consisted of 11 items dealing with various aspects of knowledge about percentages, including

- The meaning of the term percentage in daily life (questions 1–2)
- The meaning of the term percentage in numeric expressions (questions 3–6)
- The understanding of simple procedures such as finding the partial amount (questions 7, 9 and 10), finding the part from the whole (question 11), and finding the whole (question 8).

Questions 7–11 required the students to conduct a series of mathematical calculations in order to find the solution. Prior to completing the questionnaire, the students had not studied how to carry out these calculations for solving percentage problems. However, the level was not difficult, so we assumed that the students would use intuitive knowledge in order to solve those problems. This intuitive knowledge is based on life experiences and formal education about fractions and decimals.

The questionnaire was validated by three experts in mathematics education. Each experts received his/her set of questions and were asked to analyze them to determine whether, in their opinion, the question check formal, procedural, or intuitive knowledge. In addition, the experts were asked to determine whether the questions could be solved by 6th grade students without formal knowledge in this subject. Before finalizing the questionnaire, a pilot study was conducted in which the 30 students completed the questionnaire. In response to the finding of the pilot study, we removed the questions in which most students were mistaken and rephrased some of the questions.

The questionnaire was given to the students three months after the start of the 6th grade school year. Items 1 and 2 were examined in Research Question 1; items 3–6 in Research Question 2; and items 7–11 in Research Question 3.

RESULTS

The meaning of percentage (Research Question 1)

In order to investigate this issue, we asked the students to respond to two items:

1. What is a percentage, in your opinion?
2. Provide a few examples of the usage of percentages

71% of the students responded to item 1. The common response was: "a percentage is a part of a whole or amount, or a kind of fraction." Some students answered: "a percentage is a unit of measurement." Others wrote: "a percentage is a kind of operation." There was no significant difference between the responses given to this item by Group A and Group B. 81% of the students gave examples of the usage of percentages. The common examples were related to reductions and discounts. Some students also mentioned grades, dollar devaluation, polls, and composition of products (*e.g.* "9% fat or 100% cotton"). A few students mentioned usage of percentages in common slang (*e.g.* "I am 100% sure"). There was a slight variation between Group A and B in relation to the connection among the responses to the two items. In Group B, only 17% of the students gave examples that were related to their definition of a percentage. In Group A, 32% of the students gave examples related to the definition. For example, one student that defined the percentage as a part of a whole gave the example: "50% of the students got 100 in the examination." As an example, a response was given in which there was a lack of connection between the definition and example: a student who defined the percentage as "a part of a whole but 100% of itself" gave the example as "100% sure of it". The results demonstrate that a large part of the students have heard about percentages in daily life prior to formal study.

Conception of the whole (Research Question 2)

This study examined the students' conception of the whole (100%) prior to formal study of percentages. This was examined through four items, each of which dealt with another aspect of this issue. The items examined the students' understanding of 100%, a number greater than 100%, a number smaller than 100%, and percentages related to different wholes.

Item 3 examined the students' understanding of 100% in relation to a given quantity.

3. Choose the right answer and explain:
100% (100 percent) of 500 is:

a. greater than 500
b. less than 500
c. exactly 500
d. unknown

54% of the students answered correctly, while 46% answered incorrectly. More than half of those who answered correctly also provided a correct explanation. For example, "100% is the whole, and therefore 100% is 500." A quarter of those who answered correctly did not provide an explanation. Six students who answered correctly provided an explanation that showed that they did not understand the question, for example: "500 is divisible by 100." 70% of the students who answered incorrectly chose option b (less than 500). The common explanation for this choice was "100 is less than 500." Another explanation given by a few students was: "when you divide 500 by 100, the answer is less than 500." These students do not distinguish 100% from the number 100, and they do not grasp that 100% represents the whole. In the answers to this item, significant differences are apparent between Groups A and B. 70% of the students in Group A answered correctly, while only 40% of the students in Group B answered correctly. These results indicate that in Group A a large part of the students understand the mathematic notion of 100%, while only a small portion of the students in Group B have this understanding.

Item 4 examined the understanding of a percentage "greater than 100%."

> 4. Choose the right answer and explain:
> 122% (122 percent) of 1690 is:
>
> a. more than 1690
> b. less than 1690
> c. exactly 1690
> d. unknown

39% answered to this item correctly, 47% incorrectly, and 14% did not answer. 65% of students that answered correctly also provided a correct explanation. For example, "100% is the whole and when you have 122% it is 22% more than the whole." 20% of students that answered correctly provided an incorrect explanation. For example, "I added 122 to 1690 and I got a larger number." The rest did not explain their answer.

90% of the students who answered incorrectly chose option b (less than 1690). Examples of explanations: "percentages are counted only under 100%", "when you divide 1690 by 122, you get a smaller number" and "it has to be less because you subtract 122 from 1690." In the first example, we can see that students think that the percent must be less than 100. In the other two examples, students conduct operations like division or subtraction in order to find the answer.

The answers to this item again reveal differences between the two groups. In Group A, 45% of the students answered correctly, 32% incorrectly, and 23% did not answer. In Group B, only 35% answered correctly, 60% incorrectly, and 5% did not answer. The results indicate that a large portion of the students do not understand that percentages may refer to an amount greater than the whole. This finding is more pronounced in Group B.

5. Choose the right answer and explain:
20% (20 percent) of 3,580 is

a. more than 3,580
b. less than 3,580
c. exactly 3,580
d. unknown

72% answered to this item correctly, 13% incorrectly, and 15% did not answer. There was no significant difference between the responses given to this question by Group A and Group B.

42% of students who answered correctly also provided a correct explanation. For example, "if it is less than 100, then it is a smaller number." 38% of the students who answered correctly provided an explanation which showed that they did not understand this item. For example, a common explanation was: "because 20 is less than 3,580." This answer shows that the students look at the percentage as a number and make the comparison merely between two numbers. A few students wrote: "because 3580 is divisible by 20." This answer shows that the students used the operation of division in order to answer the question. Other students explained: "percentages always reduce prices." This answer indicates that the students understood percentages as related to sales, or to the real world. 20% of the students that answered correctly did not provide an explanation. The common incorrect answer was c (exactly 3,580).

The results show that a high percentage of the students answered correctly, but the explanation show that only 30% understand the meaning of 20% of a given amount.

Item 6 addressed percentages in relation to different wholes.

6. Neta claims that sometimes 50% is greater than 100%. Is she right? Explain.

The expected answer for this item is: Neta is wrong if we refer to 50% and 100% of the same whole, but she is right if we refer to percentages of different wholes.

22% of the students answered correctly, 67% answered incorrectly, and 11% did not answer. There was no significant difference between the responses given to this question by Group A and Group B.

The students whose answers were accepted as correct referred only to the possibility that this item referred to different wholes. Half of them provided a correct explanation, *e.g.* "because 50% of 200 is 100, while 100% of 50 is 50." The others did not provide any explanation. Among the students that answered that Neta is wrong, the common explanation was "because 50% is half, and 100% is whole." Another explanation was "100% is the greatest." These explanations demonstrate that students assumed that this item referred to percentages of the same whole. They didn't consider that it may refer to different wholes. Some students wrote "50 is always smaller than 100". This answer indicates that they think of the percentage as a number.

We can see that many students referred only to one of the possibilities in the item, and this indicates that many students do not understand the notion of the whole.

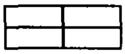
Familiar percentages such as 25% and 50% (Research Question 3)

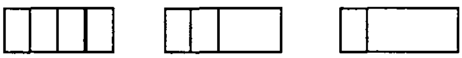
Item 7 examines the understanding of 25% through filling in an area of a shape

7. Fill in 25% (25 percent) of this rectangle 

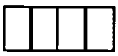
74% answered the item correctly, 19% incorrectly, and 7% did not answer.

There was no significant difference between the responses given to this item by Group A and Group B.

The common correct answer was: 

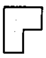
Other correct answers were: 

The common incorrect answer was to fill in half of the rectangle. Another incorrect answer was to divide the rectangle to four parts, without filling in any area:



The results indicate that most of the students are familiar with the notion of 25%. They connect it to a quarter and they know how to use it when they are requested to fill in a part of a rectangle.

Item 8 examines the understanding of 25% through finding a whole by referring to a shape that is part of the whole

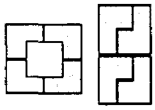
8. This shape  is 25% (25 percent) of the whole.

a. What is the shape of the whole?

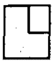
b. Is there more than one option? If so, give examples.

44% answered to the question correctly, 52% incorrectly, and 4% did not answer.

Among the students that answered correctly, the common answers were:



Only one student referred to the given shape as a member of a group. A possible reason for this is that this meaning of a fraction is rarely taught, even though it is part of the educational program. Common incorrect answers were: Completing the shape as a square:

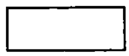


and referring to the shape as 50% of the whole: 

For this item, there were significant differences between the responses given by the two groups. In Group A, 62% of the students answered correctly while in Group B only 30% did.

Items 7 and 8 both dealt with the concept of 25%. The results of item 8 show that many students in both groups did not answer correctly. This finding was more pronounced in Group B. This finding apparently contradicts the findings of item 7, through which we saw that most of the students are familiar with the concept of 25%. This contradiction can be explained by the fact that item 8 deals with finding the whole from the part, which is very difficult for the students, leading to a lower success rate.

Item 9 examines the students' understanding of 50% through filling in an area of a shape.

9. Fill in 50% (50 percent) of this rectangle 

In both groups, all the students answered correctly and filled in half of the rectangle. The finding indicates that all of the students are familiar with the notion of 50%; they connect it with the notion of a half, and they know how to use it when they are requested to fill in an area of a rectangle.

Items 10 & 11 examined the students' understanding of 50% through calculation.

10. There are 16 flowers in the vase. 50% are red and the rest are yellow.
How many yellow flowers are in the vase? Explain

76% answered correctly, 17% incorrectly, and 7% did not answer. Most of the students that answered correctly provided a correct explanation, *e.g.*: "50% is 8. 16 minus 8 are 8". An example of an incorrect explanation: "50 flowers." The incorrect explanation indicates that the student does not distinguish the number 50 from 50%.

In this item there were marked differences between the two groups. In Group A, 85% answered correctly, while in Group B 70% answered correctly. The findings indicate that a large part of the students know how to use 50% in a word problem, and it is more visible in Group A.

11. Choose the correct answer and explain.
    Dana had $40. She bought a book for $25. Dana used:
    a. 50% of her money
    b. less than 50% of her money
    c. more than 50% of her money
    d. unknown

71% answered correctly, 27% incorrectly, and 2% did not answer. There was no significant difference between the responses given to this item by Groups A and B. 70% of the students who answered correctly added a correct explanation. For example: "40 minus 25 is 15. 25 is more than half of 40." Another example: "If 40 is 100% then 20 is 50%. Dana used $25, which is more than $20." The common incorrect answer was option b (less than 50%). The common explanation for this answer was: "25 is less than 50."

The findings of this item support the findings of items 9 and 10 and indicate that most students are familiar with 50% and can use it in word problems. Most of the students knew how to calculate 50% and 25% of the whole, both in word problems and in problems requiring them to fill in an area. However, they had difficulties when the problem provided the part and they had to find the whole.

CONCLUSION

Research has indicated that 'percentages' is a difficult topic in the middle grades' mathematics curriculum (Hart, 1981; etc.). In Israel, we begin to teach this subject in the 6th grade. The purpose of our research was to ascertain what degree of intuitive knowledge of percentages students have, prior to their learning the subject. Our research examined on the understanding of the meaning of percentage, the perception of the whole, and the familiar percentages (50% and 25%). The findings clearly illustrate that most students were exposed to percentages in daily life and could present various examples of percentages. Furthermore, 6th grade students have intuitive "knowledge" of percentages. The part of the "knowledge" that they have is correct and can promote the process of instruction of percentages. Other part of their intuitive "knowledge" is incorrect and it can hinder the formal educational process. Fischbein (1987) also concluded that intuition may promote or hinder educational processes.

We found that most students understand that the percentage is a part of the whole and that the whole is 100%. The relationship between 50% and half, or between 25% and a quarter is also clear to most of the students that were able to find 50% and 25% of the whole, in either word problems or problems requiring them to fill in an area. However, they had difficulties when the problem provided the part and they had to find the whole. Parker & Leinhardt (1995) have got a similar finding. They report that students in seventh and eighth grade, after learning percentages, understand that the whole is 100% and they are able to solve problems with familiar percents like 50% and 25%.

In our study, we exposed difficulties and misconceptions that must be considered in the process of instruction. One of the predominant findings was that some students regarded percentage as a number and not as an operator in word problems. A similar finding was accomplished by researchers that examined difficulties of students in seventh and eight grades after their learning percentages. Gay & Aichele (1997) reported that many students demonstrated an understanding of 100% as the whole and used 100% as a reference point, noting that the percent in a particular problem was greater than, less than, or equal to 100%. However, for some students, the percent symbol appeared to be no meaning. These students responded to questions as if the symbol was not present or simply attached it onto another number in the question. It seemed that since these students did not know what the percent symbol meant, it was ignored, allowing them to focus on the two numbers present in the problem.

Another difficulty disclosed in our research was students' usage of an incorrect algorithm. For example, students subtracted or divided two numbers: the percentage and the whole quantity. Similar findings are presented by Gay & Aichele (1997).

In the present research, we found that students are familiar with the notion of 100% but do not fully understand its meaning. Parker & Leinhardt (1995) and Gay & Aichele (1997) are reporting similar findings. They claim that students think the number 100 as the whole and therefore they have difficulties with word problems in which a number other than 100 is provided as the whole.

The findings of our study also indicate that students correctly answer the questions related to a part of the whole which is smaller than 100%. Though at first this seems to indicate that they understand such questions, their explanations show that they do not, in fact, understand. In item 5, which asked about 25% of 3,580, most of the students correctly answered that 20% of 3,580 is less than 3,580, but they explained their answer with the statement that 20 is less than 3,580. This again shows that students regard percentage as a number. Another explanation was that they divided the whole by 20 and they thought that division always makes smaller quantities. This example exposes a misconception of the notion of percentage. Similar findings were gotten when the students were requested to deal with percentages greater than 100%. For some of the questions in our study, particularly items 2, 3, 4 & 10, there were significant differences between the responses of the two participant groups. Group A, from the higher-achieving school, demonstrated a better intuitive knowledge about percentages and is more prepared to learn percentages.

In conclusion, the findings of our study are similar to the findings in other studies. The difference between this study and others is that this study examined intuitive knowledge prior to formal education regarding percentages. The findings show that students do have intuitive knowledge of percentages before formal study of the subject. An important finding was that the same mistakes made by students after formal study of percentages were also made before formal study. This finding raises questions about the instruction process: In what way can teachers rely on the students' intuitive knowledge when they plan their instruction program? How can they take advantage of the instruction effectively so that they can address areas that students do not intuitively understand? Our research clearly indicates that teachers must consider their students' intuitive knowledge when they prepare their instruction plan.

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