

# 블라인드 등화를 위한 정보 포텐셜 분배 방법에 대한 BER 성능 분석

김남용\* , 권기현\*\*

## BER Performance Evaluation on the Method of Balancing Information Potentials for Blind Equalization

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### 요 약

블라인드 등화기법은 무선 통신 시스템에 널리 쓰이고 있는 기술이다. 이 논문에서는, 블라인드 등화를 위한 기준으로서, 두 확률밀도함수의 유클리드 거리를 최소화하는 기준에서 나타나는 정보 포텐셜에 대해 연구하였고 두 정보 포텐셜 사이의 적절한 균형을 활용한 방법의 BER 성능을 평가하였다. 정보 포텐셜에는 두 가지가 있는데 송신 심볼의 확률밀도함수와 맞도록 수신단에서 무작위로 발생시킨 심볼과 등화기 출력 샘플 사이에서 두 입자간 위치에너지처럼 나타나는 정보 포텐셜과 출력 샘플간에 나타나는 정보 포텐셜로 표현된다. 이 두 포텐셜을 적절히 균형 잡아 줄 때, 큰 성능향상을 보이는데 그 성능지표로 BER 성능 평가가 가장 신뢰성이 있으므로 BER 성능 평가를 시행하였다. 그 시뮬레이션 결과에서 정보 포텐셜 균형 방식은 블라인드 등화 환경에서 탁월한 BER 성능을 나타내었다.

### ABSTRACT

Blind equalization techniques have been widely used in wireless communication systems. In this paper, we investigate the information potentials in the criterion of minimizing Euclidian distance between two PDFs criterion for adaptive blind equalizers and evaluate BER performance of the method that has utilized an appropriate balance between the two information potentials, one from output samples and randomly generated desired samples at the receiver and another from the interactions among output samples. The balanced information potential method has shown in the BER performance results that it can produce significantly enhanced BER performance in blind equalization applications.

Key Word : BER, Blind equalization, ITL, Euclidian distance, PDF,  
balancing effect on Information potentials.

### I. Introduction

Wireless communication has been an increasingly focused topic in multipoint communication networks, the Internet, the

ATM, and the mobile networks [1]. In applications such as broadcast and multipoint networks, blind equalizers to counteract multipath effects are very useful since they do not require a training sequence to start

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up or to restart after a communications breakdown [2][3]. Problems involving the training of adaptive equalizers have been developed through the use of information theoretic optimization criteria. As a way for solving these problems, information-theoretic learning (ITL) has been introduced by Principe [4]. This approach is to choose the parameters  $W$  of the mapping  $g(\cdot)$  such that a figure of merit based on information theory is optimized at the output space of the mapper. ITL algorithms are based on a combination of a nonparametric probability density function (PDF) estimator and a procedure to compute entropy or information potential (IP). The difficulty in approximating Shannon's entropy is overcome by utilizing Renyi's generalized entropy. Estimating the data PDF nonparametrically is based on the Parzen window method using a Gaussian kernel. The combination of Renyi's quadratic entropy with the Parzen window leads to an estimation of entropy or information potential by computing interactions among pairs of output samples which is a practical cost function for ITL. This approach has been applied to Blind deconvolution of a linear channel by maximizing or minimizing the output entropy of an adaptive equalizer. Maximum entropy deconvolution is based on the idea that when the nonlinear device output PDF is forced to uniform by maximizing its output entropy, the pdf of equalizer output matches the pdf that is the derivative of the nonlinearity, where the nonlinearity is selected to be the cdf of the source signal. On the other hand, minimum entropy deconvolution method is also available. Erdogmus and his coworkers show that the minimization of Renyi's

entropy at the output of the equalizer can achieve Blind deconvolution [5]. But the global minimum of a minimum entropy Blind deconvolution criterion occurs for zero equalizer weights, corresponding to a zero equalizer output. This unwanted situation should be avoided by some measures such as modifying the cost function with the equalizer output variance.

Instead of using entropy minimization in blind equalization, a new method in which Euclidian distance between two PDFs is minimized has been introduced [8]. The authors investigated the interactions among not only output samples but also randomly generated desired samples at the receiver by utilizing Euclidian distance (ED) in their previous works [9]. But the method has not been thoroughly investigated on its BER performance which is considered a generally accepted performance evaluation criterion. It will be shown in this paper that BER performance of the method which places an appropriate balancing factor on the interactions among output samples and randomly generated desired samples at the receiver in blind equalization environments.

## II. Euclidian Distance of PDFs

Recently, Erdogmus introduced an information theoretic framework based on Kullback-Leibler (KL) divergence [7] minimization for training adaptive systems in supervised learning settings using both labeled and unlabeled data [8]. The KL divergence is a way to estimate mutual information which is capable of quantifying the entropy between pairs of random

variables. The KL divergence between two PDFs,  $f_x$  and  $f_y$  is:

$$KL[f_x, f_y] = \int f_x(\xi) \log[f_x(\xi)/f_y(\xi)] d\xi. \quad (1)$$

Since it is not quadratic in the PDFs, it can not be easily integrated with the information potential [4]. Based on the Euclidian distance, a new divergence measure between two PDFs [4] has been introduced which contains only quadratic terms to utilize the tools of information potential as

$$ED[f_x, f_y] = \int f_x^2(\xi) d\xi + \int f_y^2(\xi) d\xi - 2 \int f_x(\xi) f_y(\xi) d\xi. \quad (2)$$

For equalization application, the Euclidian distance between  $f_d$  the transmitted symbols PDF and  $f_y$  the equalizer outputs PDF, can be minimized with respect to equalizer weight  $W$  as

$$\begin{aligned} \text{Min}_W(ED[f_d, f_y]) &= \text{Min}_W \left( \int f_d^2(\xi) d\xi \right. \\ &\quad \left. + \int f_y^2(\xi) d\xi - 2 \int f_d(\xi) f_y(\xi) d\xi \right). \quad (3) \end{aligned}$$

In other words, we create desired symbols for the input signal during training by utilizing the equalizer outputs PDF and the previously known PDF information of the transmitted symbols.

In this paper, we propose a method of minimizing the Euclidean distance based on Parzen PDFs which are computed directly from data samples. Computing ED directly from data samples requires also a continuous and differentiable estimator for the two probability density functions  $f_d$

and  $f_y$ . Parzen windowing is a suitable method, which is in general biased but the bias can be asymptotically reduced to zero by selecting an unimodal symmetric kernel function such as the Gaussian and reducing the kernel size monotonically with increasing the number of samples. Selection of an optimal kernel size is one of the important steps in the Parzen windowing method but in this paper it will be left for future work.

For blind channel equalization, we assume here that the a priori probability  $f_d$  of transmitted symbols is known to the receiver but the exact training symbols are not available to the receiver. This assumption can be considered reasonable in most cases since the transmitter has a particular modulation scheme and the symbols are generally independent and identically distributed (i.i.d) as that of the transmitted data.

### III. Minimum ED Algorithm for Blind Equalization

Given the randomly generated  $N$  independent and identically distributed (i.i.d.) symbols  $\{d_1, d_2, d_3, \dots, d_N\}$ , the pdf can be approximated by

$$f_d(\xi) \cong \frac{1}{N} \sum_{i=1}^N G_\sigma(\xi - d_i) \quad (4)$$

where  $G_\sigma(\cdot)$  is typically a zero-mean Gaussian kernel with standard deviation  $\sigma$ . If the symbols are generated randomly so as to match with the PDF of the transmitted

symbols, the  $f_d(\xi)$  in (4) can be considered the same as the PDF of the desired symbols. The point noticeable here is that for Parzen PDF calculation, instead of the exact training symbols, the randomly generated symbols are used at the receiver. For example, in case of bipolar transmission with equal probability, random numbers +1, -1 are generated equiprobably for Parzen PDF calculation but no exact desired symbols are used for it. The number of generated random numbers is the same as that of the output symbols to be used in cost function calculation. This makes blind equalization possible because exact desired symbols are not used.

Since integral of the two Gaussian kernels generates another Gaussian kernel with different double standard deviation, we have

$$\int f_d^2(\xi) d\xi = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(d_j - d_i) \quad (5)$$

$$\int f_y^2(\xi) d\xi = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(y_j - y_i) \quad (6)$$

$$\int f_d(\xi) f_y(\xi) d\xi = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N G_{\sigma\sqrt{2}}(d_j - y_i) \quad (7)$$

Equation (5) is the sum of interactions of all pairs of randomly generated symbols. This can be called IP of the set of randomly generated symbols, or  $IP(d, d)$  in this paper. By summing the interactions among pairs of output samples we can obtain the  $IP(y, y)$  of output samples as in (6). Equation (7),  $IP(d, y)$ , indicates the interactions between the two different variables. Equations (6) and (7) which contain system output are a function of

weight but (5) is not a function of weight since  $d_j$  is the desired sample randomly generated

Now we derive a gradient descent method for the minimization of the cost function (3) with respect to equalizer weight  $W$ .

$$W_{new} = W_{old} - \mu \frac{\partial P}{\partial W}, \quad (8)$$

where  $P = IP(y, y) - 2 \cdot IP(d, y)$ .

In case of on-line linear equalization, a tapped delay line (TDL) can be used for input  $X_k = [x_k, x_{k-1}, x_{k-2}, \dots, x_{k-N+1}]^T$  and

output  $y_k = W_k^T X_k$  at time  $k$  (Fig. 1). The

randomly generated desired symbols  $D_N = \{d_1, d_2, \dots, d_j, \dots, d_N\}$  are used in

the equalization process regardless of time  $k$ . Then the gradient is evaluated from

$$\begin{aligned} \frac{\partial P}{\partial W_k} &= \frac{1}{2N^2\sigma^2} \sum_{i=k-N+1}^k \sum_{j=k-N+1}^k (y_j - y_i) \\ &\cdot G_{\sigma\sqrt{2}}(y_j - y_i) \cdot (X_i - X_j) \\ &- \frac{1}{N^2\sigma^2} \sum_{i=k-N+1}^k \sum_{j=1}^N (d_j - y_i) \cdot G_{\sigma\sqrt{2}}(d_j - y_i) \cdot X_i \end{aligned} \quad (9)$$

This method is referred to as minimum ED (MED) algorithm [6].

#### IV. Balancing Effects on Information Potentials

Equation (6), i.e.,  $IP(y, y)$  is the information potential of the set of randomly

generated symbols and (6), i.e.,  $IP(d, y)$  the information potential induced from the interaction between the set of randomly generated symbols and output samples. The two information potentials contribute to minimizing ED in (8) and (9). An important aspect we should notice is that when  $IP(y, y)$  decreases, it contributes positively to minimization of ED but  $IP(d, y)$  has to decrease in order to contribute to minimization of ED. This leads us to believe that the two information potentials are engaging in a struggle for influence on minimization of ED. If we rearrange the balance of the two information potentials, though it can play a little bit of negative role in minimization of ED, optimal balance of power between the two information potentials might be obtained. Furthermore, minimizing the information potential  $IP(y, y)$  can induce spreading output samples rather than concentrating them on one point as revealed in the research [5]. From this motive, we propose to put a weighting factor  $\alpha$  on  $IP(d, y)$  and  $IP(y, y)$  as follows.

$$P_{proposed} = IP(y, y) \cdot \alpha - 2 \cdot IP(d, y) \cdot (1 - \alpha) \quad (10)$$

This method has shown a significantly enhanced PDF performance in our previous works [9]. As an extension of this research for its performance evaluation, in the following section, we will give its BER performance results which is considered as a vital evaluation criterion.

## V. BER Results and Discussion

For BER performance evaluation, in this section, we present and discuss simulation results that illustrate the comparative performance of the proposed algorithm versus MED and constant modulus algorithm (CMA) [10] in blind equalization for two linear channels. The 4 level random signal  $\{\pm 3, \pm 1\}$  is transmitted to the channel and the impulse response,  $h_i$  of the channel model in [11] is

$$h_i = \frac{1}{2} \{1 + \cos[2\pi(i-2)/BW]\}, i = 1, 2, 3. \quad (11)$$

The parameter  $BW$  determines the channel bandwidth and controls the eigenvalue spread ratio of the correlation matrix of the inputs in the equalizer. In this paper,  $BW = 3.1$  (ESR=11.12) is used. The number of weights in the linear TDL equalizer structure is set to 11. As a measure of equalizer performance, we use BER of CMA and MED. The convergence parameter for CMA which have shown the lowest steady-state MSE is 0.00001. For MED we used a data-block size  $N = 20$ , a fixed kernel size  $\sigma = 0.5$  and the convergence parameter  $\mu = 0.006$ . The information potential weighting factor  $\alpha$  is set to 4. In  $D_N = \{d_1, d_2, \dots, d_j, \dots, d_N\}$ ,  $d_j =$  Randomly ordered  $\{\pm 3, \pm 1\}$ . We have studied the bit error rate performance of the proposed, MED and CMA as a figure of merit. Their results are illustrated in Fig. 1. CMA has shown unsatisfying performance in BER performance. On the other hand, the

proposed has revealed superior results. In BER performance comparison at BER =  $10^{-4}$ , the proposed has shown enhanced performance by about 1 dB compared to MED, and by about 3 dB compared to CMA.

### VI. Conclusion

The BER performance comparison is considered a generally accepted performance evaluation criterion. In this paper, BER performance of the balancing method was investigated that places an appropriate balancing factor on the interactions among output samples and randomly generated desired samples at the receiver in blind

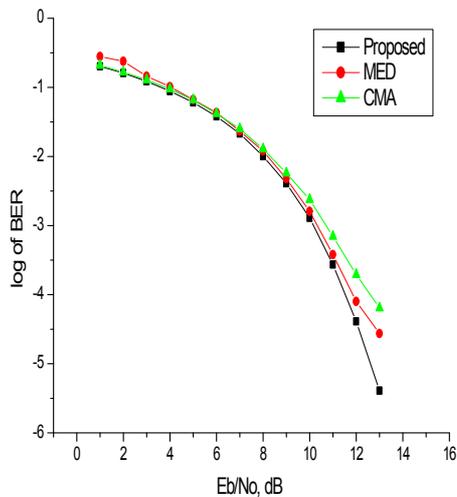


Fig. 1. BER performance comparison

equalization environments. The balanced information potential method has shown in

the BER performance results that it can produce significantly enhanced BER performance in blind equalization applications.

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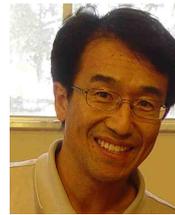
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