情韓國國防經營分析學會誌
第35卷 第1號, 2009. 4. 30.

Development and Testing of a New Area Search Model with Partially Overlapping Target and Searcher Patrol Area

†Giyoung Kim*, James N. Eagle**, Sungjin Kang***

ABSTRACT

In this study, the author uses a MATLAB simulation to develop and test a generalization of the traditional Random Search model which allows both the searcher and target to move and to be in different, but overlapping, areas. Also the best evasion speed for a randomly moving target against a Systematic Search is studied.

Keywords : Random Search, Exhaustive Search, Systematic Search, Search and Detection, MATLAB Simulation

---

논문접수일 : 2009년 3월 4일 논문게재확정일 : 2009년 4월 22일

* ROKAF 121 Squadron KF-16 Pilot

** US Naval Postgraduate School Operations Research Professor

*** Korea National Defense Univ. Operations Research Professor

† 교신저자

1. Introduction

Random Search is a popular model for area search because it is mathematically simple and it provides a conservative, lower bound on the probability of detecting a stationary target with Systematic Search. However, the Random Search model also has significant limitations. In particular, it assumes that the searcher and target are contained in the same area and that the target is stationary. In this study, we address these model limitations. The research questions of this study are as follows:

1. What is the proper mathematical generalization of the Random Search model for the situation where the searcher and target areas are not coincident and both the searcher and target are moving?
2. What is the best speed for a randomly moving target to evade a searcher conducting a Systematic Search?

In order to find answers these two questions, we need to use computer simulation because there is no close formed mathematical formula.

2. Overview of Random Search theory

2.1 Exhaustive Search

Exhaustive Search is an idealized search model which assumes a stationary target and a 'perfect search', meaning no search overlap, no search effort placed outside the target area, and all of target area is completely covered by the search sensor. The searcher has a 'cookie-cutter sensor' with range R, sometimes called a definite range law sensor. Such a sensor always detects a target within a specified range R, and never detects targets beyond that range. Because of its optimistic area coverage assumptions, Exhaustive Search is generally assumed to provide an upper bound on the performance of realistic search.

2.2 Continuous Search

2.2.1 Model assumption and definitions

☞ $\gamma(t)$=detection rate at time t (units: 1/time). Detection rate has two equivalent interpretations:

i. $P(\text{detection in } [t, t+\Delta t]) = \gamma(t)\Delta t + o(\Delta t)$
$\approx \gamma(t)\Delta t$, for small Δt .

Note: O(Δt) is a function of Δt which goes to 0 faster than linearly.

That is: $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$

ii. $E(\text{number of detections in } [t, t+\Delta t]) = \int_t^{t+\Delta t} \gamma(s)ds$
$\approx \gamma(t)\Delta t$, for small Δt .

☞ T is the random time of initial detection, and we define

$F_T(t) = P(T \le t)$, and $G_T(t) = P(T > t) = 1 - F_T(t)$.

☞ Detection events in non-overlapping time intervals are probabilistically independent. So,

$$P\{N(t+\Delta t)=0\} = P\{N(t)=0,\ N(t+\Delta t)-N(t)=0\}$$
$$= P\{N(t)=0\} \cdot P\{N(t+\Delta t)-N(t)=0\}$$

where, $N(t)$ be the randdomnumber of event occur duing (0, t].

2.2.2 Derivation of $G_T(t)$, $F_T(t)$ and $E(T)$

Since the events of detection (and non-detection)

are independent in non-overlapping time intervals,
$G_T(t+\Delta t) = \underbrace{G_T(t)}_{A}\underbrace{(1-[\gamma(t)\Delta t + o(\Delta t)])}_{B}$.

Where: A=P(non-detection from time to 0 to time t), B=P(non-detection from time t to time t+Δt). Re-arranging terms,

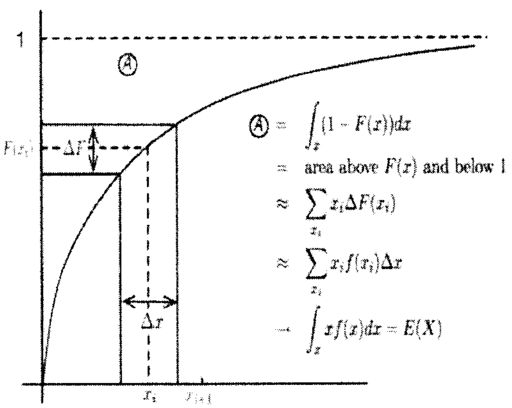
$$\frac{G_T(t+\Delta t)-G_T(t)}{\Delta t} = -G_T(t)\gamma(t) - \frac{G_T(t)o(\Delta t)}{\Delta t}$$

Letting Δt go to 0, $\frac{d}{dt}G_T(t) = -\gamma(t)G_T(t)$. The solution to this differential equation is : $G_T(t) = \exp(-\int_0^t \gamma(s)ds)$. Therefore:

$$F_T(t) = 1 - G_T(t) = 1 - \exp(-\int_0^t \gamma(s)ds)$$
$$f_T(t) = \frac{d}{dt}F_T(t) = \gamma(t)\exp(-\int_0^t \gamma(s)ds)$$
$$E(T) = \int_{t=0}^{t'} t\, f_T(t)dt = \int_{t=0}^{t'} t\ \gamma(t)\exp(-\int_0^t \gamma(s)ds)dt$$

If $\int_{t=0}^{t'} f_T(t)dt = 1$, then T is a proper random variable, eventually detection is certain, and a slightly simpler expression is possible. For any proper non-negative random variable T:
$E(T) = \int_{t=0}^{t'} (1 - F_T(t))dt = \int_{t=0}^{t'} \exp(-\int_0^t \gamma(s)ds)dt$.

The proof is sketched below in Figure 1.



〈Figure 1〉 $E(X)$ for proper, Non-negative Random variables X

2.3 Random Search

2.3.1 Characteristics of Random Search

Assume a search where:

a. Searcher has a cookie-cutter sensor.

b. Each small segment of the searcher's track is randomly and uniformly distributed over the search area.

c. No search effort falls outside the search area.

d. Target position is fixed in search area.

During any time interval Δt, the searched area is 2RVΔt. Since this area is uniformly distributed over the search area A, the probability of its covering the target is:

$$\frac{2RV\Delta t}{A} = P(\text{detection during } [t,\ t+\Delta t]).$$

Thus, we have a constant detection rate with $\gamma = (2RV)/A$. Therefore:

$$F_T(t) = 1 - \exp(-2RVt/A)$$
$$f_T(t) = (2RV/A)\exp(-2RVt/A).$$
$$E(T) = A/(2RV)$$

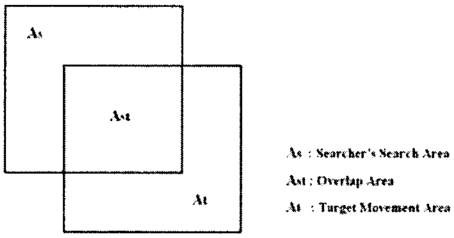
Now assume the target with speed U and the searcher with speed V move randomly and independently over area A. The searcher sweep width is 2R, and T is the random time of initial detection. As before, we wish to compute $F_T(t) = P(T \le t)$, and for $U << V$, we would expect that $F_T(t) \approx 1 - e^{-2RVt/A}$. And more generally, we will look for a speed $\tilde{V} > \max(V,U)$ such that $F_T(t) \approx 1 - e^{-2R\tilde{V}t/A}$. $\tilde{V}(\theta)$ is the relative speed between searcher and target when Θ is the difference in their courses. By the Law of Cosines, $\tilde{V}(\theta) = \sqrt{V^2 + U^2 - 2UV\cos(\theta)}$. We now assume that Θ is uniformly distributed between

0 and 2π, so the average $\tilde{V}(\theta)$ is:

$$\tilde{V} = \int_{\theta=0}^{2\pi} V(\theta) f(\theta) d\theta$$
$$= \int_{\theta=0}^{2\pi} \sqrt{V^2 + U^2 - 2UV\cos(\theta)} (\frac{1}{2\pi}) d\theta$$
$$= \frac{1}{\pi} \int_{\theta=0}^{\pi} \sqrt{V^2 + U^2 - 2UV\cos(\theta)} d\theta$$
$$\geq \max(V, U).$$

This equation suggests that in Random Search with dynamic enhancement, the searcher speed is effectively "enhanced" by the target speed.

2.4 Random Search when Searcher and Target Patrol Areas are not Identical



〈Figure 2〉 illustration of 'Searcher and Target Patrol Areas are not Identical'

As shown above in Figure 2, we now consider the situation where the search area is not completely overlapped with the target area. Two types of target behavior were assumed in order to get a simplified mathematical expression.

a. Slow target

Assumptions:

- Target initial position is uniformly distributed over A_t.
- A target starting inside (outside) A_s will remain so for the entire search time.

In this case $F_T(t)$ is:

$F_T(t)$ = P(detection by time t | target start in A_{st}) P(target starts in A_{st}) + P(detection by time t | target start outside A_{st})P(target starts outside A_{st})

= P(detection by time t | target start in A_{st}) P(target starts in A_{st})

$$= (\frac{A_{st}}{A_t})(1 - \exp(-2R\tilde{V}t/A_s)).$$

b. Fast target

Assumption:

- Target spends (A_{st}/A_t) of the search time inside A_s.

By time t, this target has been available for detection for $(A_{st}/A_t)t$ time units. Therefore $F_T(t) = 1 - \exp(-2R\tilde{V}(A_{st}/A_t)t/A_s)$. In light of these assumptions, eventual detection for the fast target is assured. In contrast, the upper bound of for the slow target is (A_{st}/A_t).

In this paper, we will attempt to generalize the fast and slow target models to allow $F_T(t)$ to be estimated for intermediate target speeds.

3. Simulation of Random Search

3.1 Description of Random Search Model

a. Characteristics of Searcher and Target

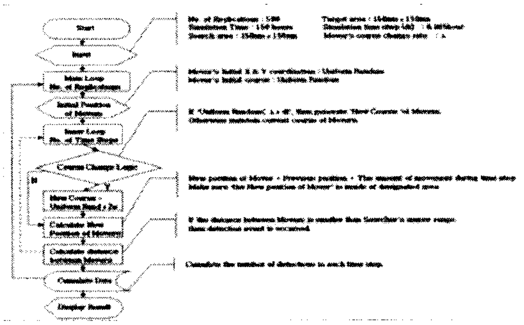
The searcher is assumed to have a "cookie-cutter" sensor of radius R. In addition, the searcher is not allowed to search for a target outside the search area. Therefore the searcher's allowable position is limited by sensor radius R. For example, if the search region's X-axis length is 50nm and the Y-axis length is also 50nm, then the overall search area (A_s) is $50 \times 50\text{nm}^2$. But the searcher's actual moving area (A') is $(50-R)\times(50-R)\text{nm}^2$.

The searcher's initial position is uniformly distributed inside of the search region $A'.\lambda_s$ After that, the searcher chooses his course randomly, independent of the target's movement. The course change event is determined by Poisson process with rate. It is assumed that the speed of the searcher is always faster than that of the target. This is allowed because the searcher and target roles can be reversed in the simulation, resulting in the same probability of detection. The target's initial position is uniformly distributed over the search region A. The logic of the target movement is λ_t the same as that of the searcher, and the target has its own, independent Poisson process with course change rate.

b. Functioning of Program

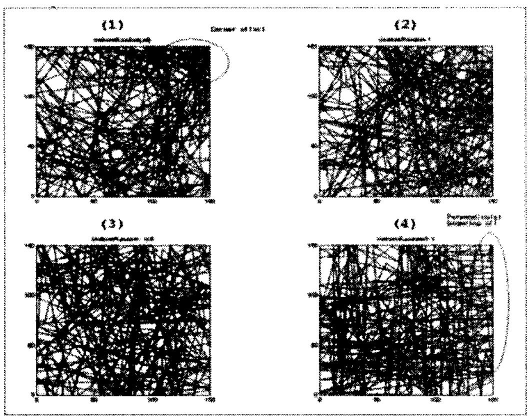
When a new replication begins, the initial positions of the searcher and the target are chosen from a Uniform Distribution over the search area. The only difference between position selection logic of the searcher and the target is caused by the searcher's sensor range (R). In order to prevent over-searching, the searcher's initial position should be limited to search area (A').

The initial course is also drawn from a Uniform Distribution between 0 and 2π. The subsequent course changes for searcher and target occur according to Poisson Processes with rate λ_s and λ_t respectively. In particular, at each time step Δt, if $Uniform\_Random(0,1) < \lambda_s \Delta t$, then a new random course $C_s = Uniform\_Random(0,1)\times 2\pi$ is selected for the searcher. The course changes for the target are determined in the same manner by using λ_t. (see Figure 3)



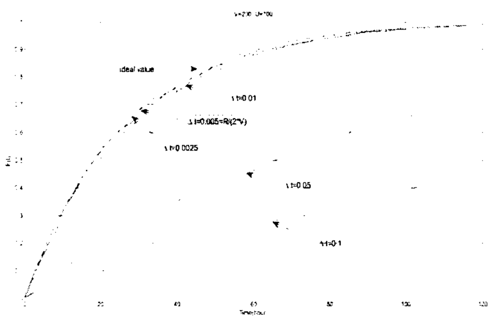
〈Figure 3〉 Flow chart of Random Search computer algorithm

When the searcher or target encounters an area boundary, then a random reflection occurs. After the reflection, the new course in radians is $Uniform\_Random(C_\perp -.5,\ C_\perp +.5)$, where $C_\perp$ is the perpendicular course from the reflection boundary. The parameter .5 was determined experimentally to prevent both "corner capture" and too many near "perpendicular reflections." (see Figure 4)



〈Figure 4〉 Random movement behavior at the edge depending on different parameters

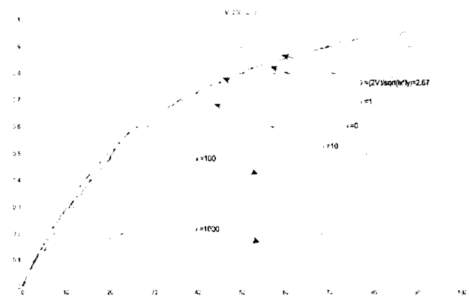
At each time step, we store the position of the searcher and the target, and then measure the distance between them. In order to closely approximate a continuous simulation, Δt should be small. However, too small a Δt means too many calculations, which in turn require an excessive time to simulate. How small a value of Δt is sufficient to produce an accurate result?



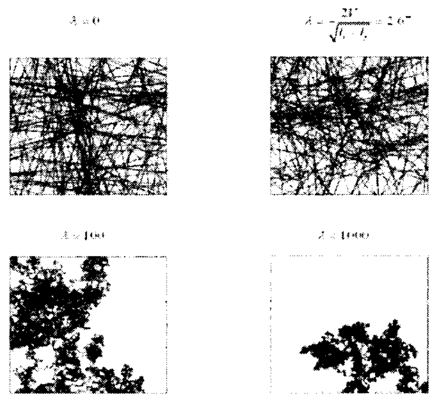
⟨Figure 5⟩ $F_T(t)$ for various value of Δt

Figure 5 above shows that $\Delta t \approx \frac{R}{2V}$ is an appropriate value to use. Thus the searcher moves half the distance of the cookie-cutter sensor's radius at each time step. The author also experimented with the Poisson course change rate λ to produce searcher and target motion most closely satisfying the Random Search assumptions. As indicated in Figure 6, a good value is $\lambda = \frac{2V}{\sqrt{l_x \times l_y}}$.

This formula implies that on the average, two course change events occur during the time required for the mover to go from edge to edge. If λ is too small, the mover has a very small chance to change course before bouncing off the edge. On the other hand, if λ is too large then the mover's position will be potentially limited to a small part of the searcher and target area.(see Figure 7)



⟨Figure 6⟩ $F_T(t)$ for various value of λ



⟨Figure 7⟩ Searcher's movement pattern for various λ

4. Simulation of the Extended Random Search Model Where Searcher and Target Areas Are Not Coincident

4.1 Description of Extended Random Search Model

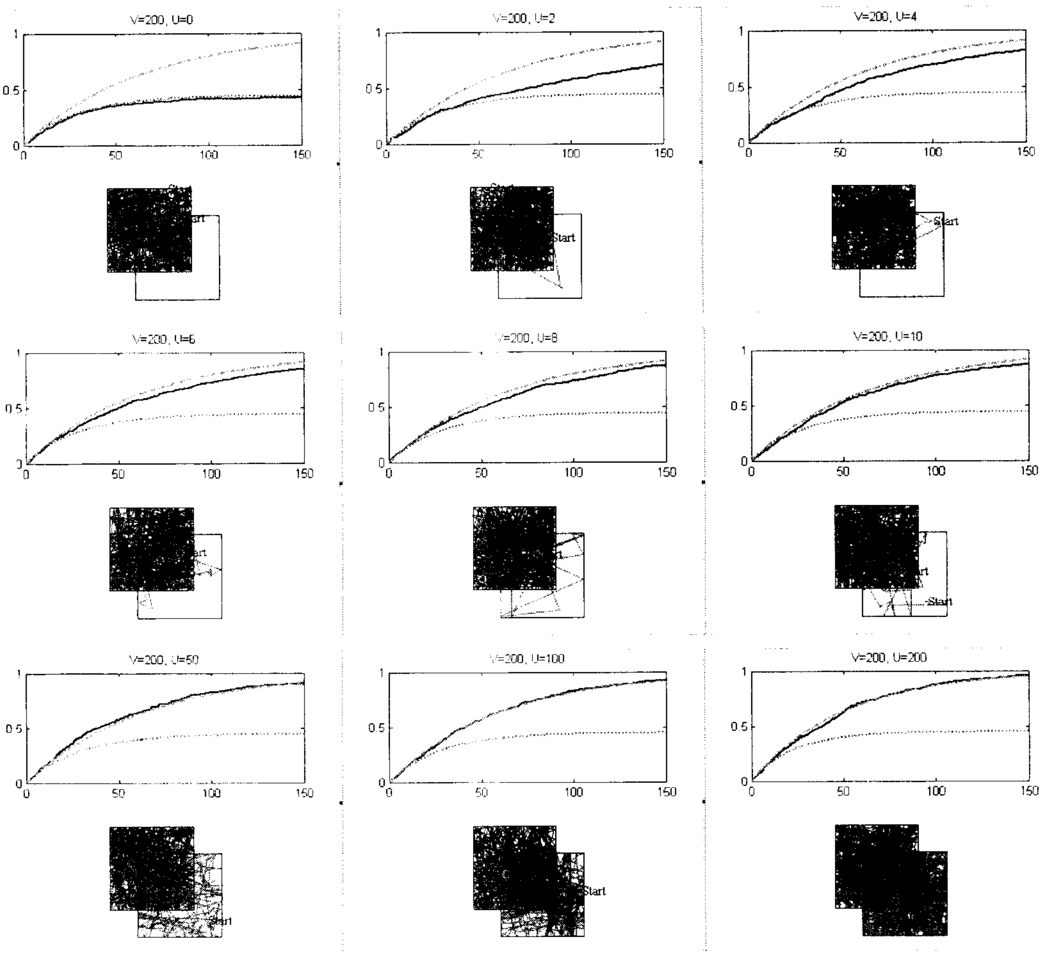
The only difference from the previous model is that the search area is now not completely overlapped with the target area.

a. Computer algorithms

- Number of simulation replications, $N_{reps} = 500$.
- Maximum simulation time, $t_{\max} = 150$ [hour].

- The length of search area in X direction, $l_x = 150$ [nm].
- The length of search area in Y direction, $l_y = 150$ [nm].
- The length of target area in X direction, $l_{xt} = 150$ [nm].
- The length of target area in Y direction, $l_{yt} = 150$ [nm].
- Searcher speed, V=200[nm/hour].
- Target speed, U[nm/hour].
- Searcher's detection range, R=2[nm].
- Searcher's course change rate, λ_s [1/hour].
- Target's course change rate, λ_t [1/hour].
- The unit time of simulation, $\Delta t = \frac{R}{2V} = 0.005$ [hour].
- The size of search area, $A_s = 150^2$ [nm^2].
- The size of target area, $A_t = 150^2$ [nm^2].
- The size of overlap area, $A_{st} = 100^2$ [nm^2].

Figure 8 shows simulation results for several target speeds plotted against the fast and slow target models:



⟨Figure 8⟩ $F_T(t)$ for Various Target Speed

1. Green line(-·-·-) : fast target assumption

   $F_{Tfast}(t) = 1 - \exp(-2R\tilde{V}(A_{st}/A_t)t/A_s)$

2. Pink line(·····) : slow target assumption

   $F_{Tslow}(t) = (A_{st}/A_t)(1 - \exp(-2R\tilde{V}t/A_s))$

3. Blue line : simulation results

The fast target and slow target equations can be combined as follows to approximate the simulation results for any target speeds:

$$F_{Tcombined}(t) = \underbrace{(\exp(-\xi\sqrt{t}))}_{\alpha}F_{Tslow}(t) + \underbrace{(1-\exp(-\xi\sqrt{t}))}_{1-\alpha}F_{Tfast}(t) \qquad (1)$$

where $\xi = \frac{A_{st}U}{\sqrt{A_s A_t R V}}$ $[1/\sqrt{\text{time}}]$.

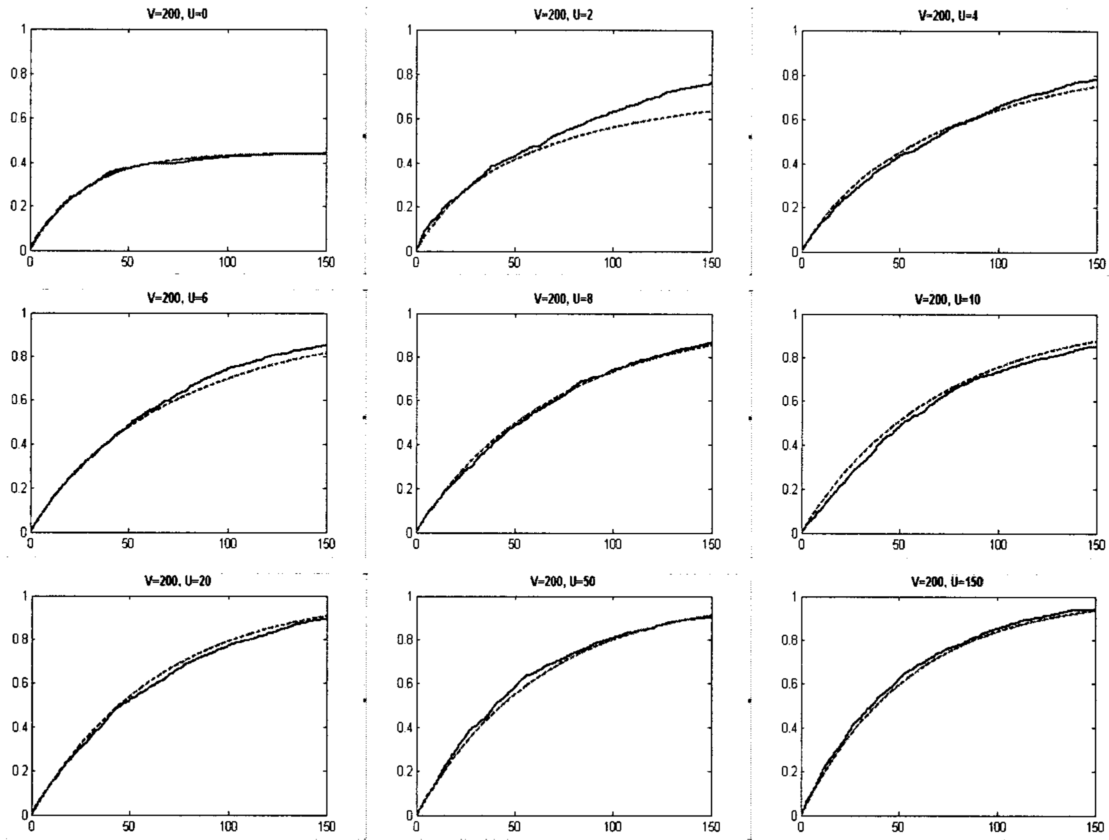
'α' decreases from 1 to 0 as the target speed U increases and as time goes to infinity, resulting in the fast target equation. Figure 9 shows plots of simulation results and $F_{Tcombined}(t)$. The red line (---) in the Figure 9 represents the equation (1).

Given these results, it appears that equation (1) can be used as a conservative estimate (that is, a lower bound) of $F_T(t)$ when searcher and target patrol areas are not identical.

5. Simulation of a Systematic Random Search Against a Randomly Moving Target

5.1 Description of Systematic Search Model

Actual searchers cannot perform Exhaustive

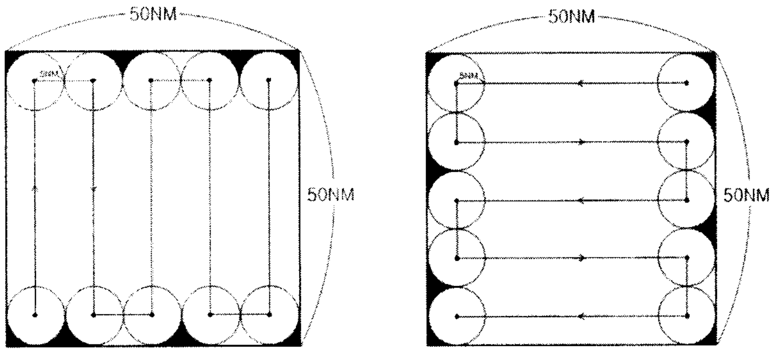


⟨Figure 9⟩ The comparison between simulation results and $F_{Tcombined}(t)$

Search, but it is often approximated by parallel sweeps, which is similar to mowing the lawn. This is called 'Systematic Search'. Other forms of Systematic Search include spiral-in and spiral-out tracks.

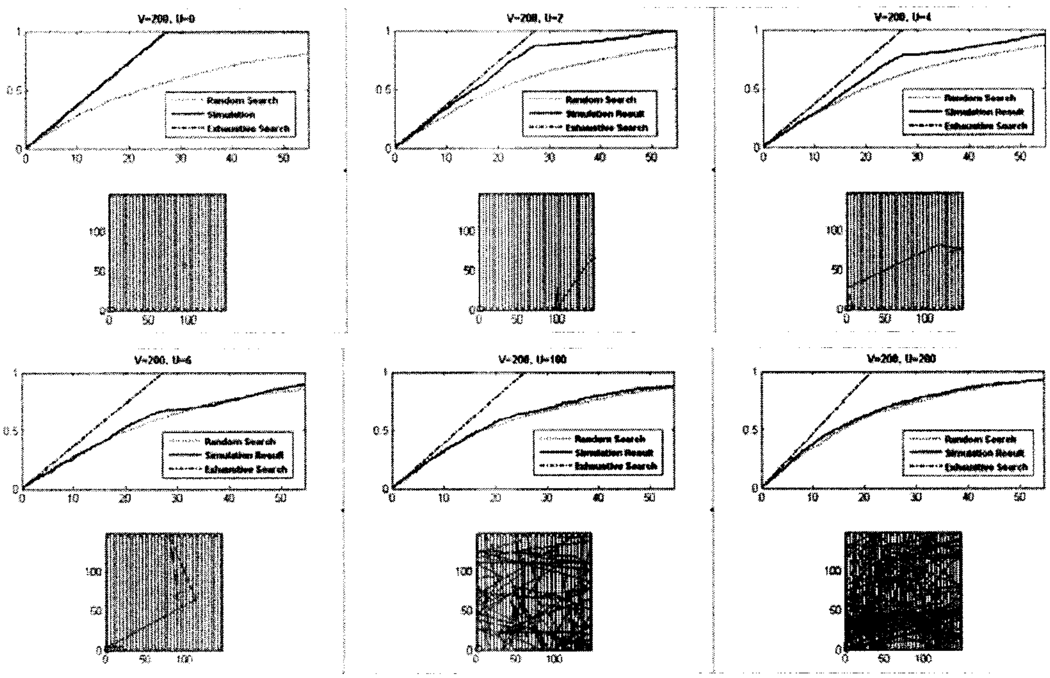
Figure 10 illustrates simplified examples of a 'Systematic Search' track. We examine the two types of going and returning 'Systematic Search' patterns. One is a 'Reverse Course Systematic Search' in which the searcher goes back via the exact same track to the starting point. The other is a 'Cross Course Systematic Search' in which the searcher follows the track shown in Figure 10(1) and then goes back via the Figure 10(2) track.

In general, it is thought that a simulation of



(1) (2)

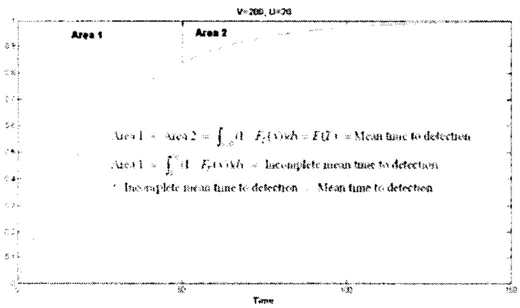
〈Figure 10〉 Simple examples of 'Systematic search' patterns



〈Figure 11〉 $F_T(t)$ for Various Target Speed (2 Times Sweep)

Systematic Search always performs better than Random Search. Figure 11 suggests that this is true.

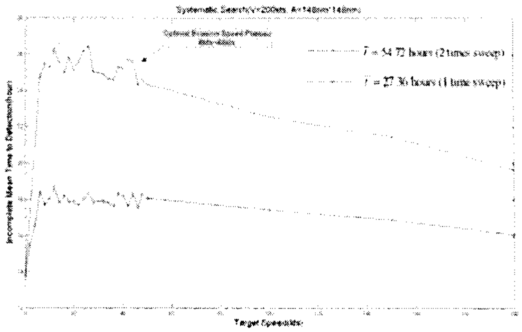
The measure of effectiveness used when evaluating search plans was "Incomplete Mean Time to Detection evaluated at time $\bar{t}$ (IMTD($\bar{t}$))," defined as, $\text{IMTD}(\bar{t}) = \int_0^{\bar{t}} (1 - F_T(x))dx$. IMTD($\bar{t}$) is used as a surrogate for Mean time to detection($E(T)$). In fact, $\lim_{\bar{t} \to \infty} \text{IMTD}(\bar{t}) = E(T)$. We use IMTD($\bar{t}$) because we often can not run the search simulation long enough to accurately estimate E(T). (see Figure 12).



〈Figure 12〉 Example of 'Incomplete Mean Time to Detection' where $\bar{t}$ =50hours

We assume that the target would like to select a speed maximizing IMTD($\bar{t}$). In Figure 13, IMTD($\bar{t}$) is dramatically increased when the target speed changed from 0kts to 8kts. And there are no significant changes between 8kts and 44kts. For target speeds greater than about 44kts, IMTD($\bar{t}$) steadily decreases.

Naval Postgraduate School Professor, Alan R. Washburn, suggested this when he wrote, "a target that wishes to avoid detection might actually choose to move around at U=0.2V, on



〈Figure 13〉 Incomplete Mean Time to Detection for various $\bar{t}$

the grounds that this is enough motion to prevent an Exhaustive Search, but nonetheless increases the equivalent searcher speed by only 1%."[1] After that speed, as Professor Washburn expected, the IMTD($\bar{t}$) decreases because the fast random movement of the target runs it into the searcher more often than away from the searcher. All things considered, the author suggests that the optimal evasion speed for target evading a Systematic Search is approximately 0.05 to 0.2 of the searcher speed.

6. Conclusion and Recommenda Tions

The main contributions of this paper can be narrowed down to two results. The first result is a generalization of the Random Search formula which allows both searcher and target to move and does not require the patrol areas to be identical. With this result, we can decide how long we should conduct search until we attain desired probability of detection. Which can be applied to 'Search and Rescue' missions.

The other result is that the best speed for a randomly moving target evading a Systematic

1) Alan R. Washburn, Search and detection, 4th ed. 6-3.

Search ranges from 0.05 to 0.2 of the searcher speed. For example, if our friendly submarines try to minimize the probability of detection from enemy's ASW aircraft then we should consider this result. Depending on enemy's search types, the author suggests these three recommended evading speeds. First, if enemy conducts Random Search then we should stay still. Second, if enemy conducts Systematic Search and we want to escape the search area then the recommended speed of submarine is 0.2 of enemy speed. Third, if we have no idea of enemy's search pattern we can minimize the probability of detection by maintaining 0.05 of enemy speed.

List of References

〔1〕 Chudnovsky, David V., and Gregory V. Chudnovsky, Search theory: Some recent developments, (New York : Marcel Dekker, 1989).

〔2〕 Iida, Koji. Studies on the optimal search plan, (Berlin, New York: Springer-Verlag, 1992).

〔3〕 Koopman, Bernard Osgood. Search and screening, (New York : Pergamon Press, 1980).

〔4〕 NATO Advanced Research Institute, Search theory and applications, (New York: Plenum press, 1980).

〔5〕 Stanoyevitch, Alexander. Introduction to MATLAB with numerical preliminaries, (Hoboken, N.J.: Wiley-Interscience, 2005).

〔6〕 Washburn, Alan R. Search and detection, 4th ed., (United States of America: Informs, 2002).

〔7〕 McNISH, MICHAEL J., Lieutenant, U.S. Navy, "Effects of Uniform Target Density on Random Search," MS thesis, September 1987.

〔8〕 COLAK, UMIT, Lieutenant Junior Grade, Turkish Navy, "A Model for Evaluating the Effectiveness of Systematic Search for a Moving Target," MS thesis, March 1987.

〔9〕 SANTOS, ALMIR GARNIER, Lieutenant Commander, Brazilian Navy, "Using Multiple Searchers to Locate a Randomly Moving Target," MS thesis, September 1993.

❙ 저자소개 ❙

Giyoung Kim(E-mail: alunbong@naver.com)

1998 BS-Korea Airforce Academy.

2008 MS-Naval Postgraduate School.

2009 MS-Korea National Defense Univ.

<Present / Research Interest>

- ROKAF 121 Squadron KF`-16 Pilot
- Search and Detection, Modeling and Simulation, System Optimization

James N. Eagle(E-mail: jeagle@nps.edu)

1969 BS-U.S. Naval Academy.

1972 MS-Stanford Univ.

1975 PhD-Stanford Univ.

<Present / Research Interest>

- Naval Postgraduate School Operations Research Professor
- Search and detection, Applied Probability, Military Modeling

Sungjin Kang(E-mail: sjkang@kndu.ac.kr)

1970 BS-Korea Military Academy.

1983 MS-Naval Postgraduate School.

1988 PhD-Taxas A&M.

<Present / Research Interest>

- Korea National Defense Univ. Operations Research Professor
- Cost Estimation, Military OR, EVM.