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A Multi-Resolution Approach to Non-Stationary Financial Time Series Using the Hilbert-Huang Transform

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Abstract

An economic signal in the real world usually reflects complex phenomena. One may have difficulty both extracting and interpreting information embedded in such a signal. A natural way to reduce complexity is to decompose the original signal into several simple components, and then analyze each component. Spectral analysis (Priestley, 1981) provides a tool to analyze such signals under the assumption that the time series is stationary. However when the signal is subject to non-stationary and nonlinear characteristics such as amplitude and frequency modulation along time scale, spectral analysis is not suitable. Huang *et al.* (1998b, 1999) proposed a data-adaptive decomposition method called empirical mode decomposition and then applied Hilbert spectral analysis to decomposed signals called intrinsic mode function. Huang *et al.* (1998b, 1999) named this two step procedure the Hilbert-Huang transform(HHT). Because of its robustness in the presence of nonlinearity and non-stationarity, HHT has been used in various fields. In this paper, we discuss the applications of the HHT and demonstrate its promising potential for non-stationary financial time series data provided through a Korean stock price index.

Keywords: Empirical mode decomposition, Hilbert-Huang transform, Hilbert spectral analysis, multi-resolution analysis, non-stationarity.

1. Introduction

In financial time series analysis, one of the main issues is modeling and forecasting price or the index for a financial instrument. Usually, the transformation of a financial time series, rather than its original scale, is taken for describing its dynamics. Proper transformation is necessary to convert non-stationary processes to stationary processes and subsequently to utilize favorable mathematical and statistical properties for stationary processes. However, the assumptions of stationarity and linearity are not always true for some financial time series. Figure 1.1 shows the weekly KOSPI 200

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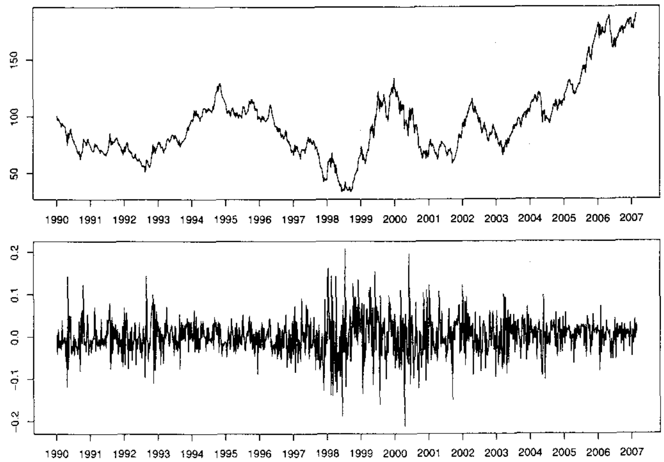

Figure 1.1. Weekly KOSPI 200 index and its log return

index. The KOSPI 200 Index is a capitalization-weighted index of 200 Korean stocks. It is clear from Figure 1.1 that neither KOSPI 200 index nor its log return can be regarded plausibly as the products of a stationary stochastic process.

The empirical mode decomposition(EMD) and Hilbert spectrum invented by Huang *et al.* (1998b, 1999) provides a new approach for analyzing non-stationary and nonlinear signals. This new procedure is termed the Hilbert-Huang transform(HHT). Some modifications have also been proposed by several authors (Deering and Kaiser, 2005; Huang *et al.*, 2003a; Rilling *et al.*, 2003; Zeng and He, 2004) to improve the algorithmic aspects. HHT has been used in various areas because it can analyze nonlinear and non-stationary signals that other methodologies cannot. This includes various areas such as speech analysis (Liu *et al.*, 2005), biological data analysis (Huang *et al.*, 1998a; Huang *et al.*, 2002), earthquake (Zhang *et al.*, 2003), climate (Coughlin and Tung, 2004), and finance time series analysis (Huang *et al.*, 2003b). In this paper, we introduce a novel method for the analysis of financial time series data with empirical mode decomposition and the Hilbert spectrum which generalize the Fourier analysis. We then apply the HHT procedure to the KOSPI 200 index and make some inferences on its multi-resolution property of enhancing the understanding of the dynamics of the KOSPI 200.

In this paper, we implement EMD and its Hilbert spectral analysis with the package `EMD` written by `R`. The source code, windows binary and reference manual are available from the Comprehensive R Archive Network(CRAN) (Kim and Oh, 2008). In addition, we provide `R` codes for details of the procedure used in this paper from `http://dasan.sejong.ac.kr/~dhkim/work_index.html`. This paper is organized as follows. Section 2 introduces the EMD, Hilbert spectrum and related concepts. Section 3 analyzes the KOSPI 200 index showing the capability of HHT. Finally, in Section 4 some concluding remarks are addressed.

2. Methodology

HHT consists of two procedures: EMD for time domain analysis and Hilbert spectral analysis for frequency domain analysis. EMD decomposes a signal into intrinsic mode functions(IMF) according

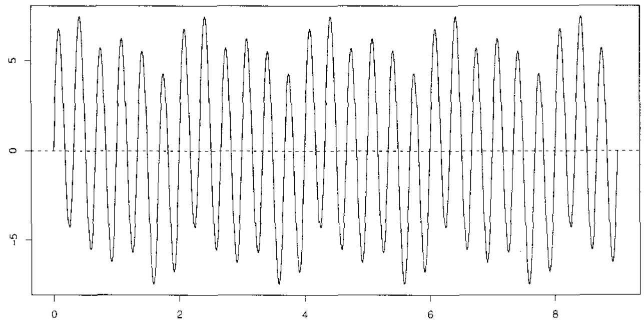

Figure 2.1. A sinusoidal function with 3 components

to the levels of its local oscillation or frequency. EMD has several advantages over the Fourier analysis in that the oscillations embedded in a signal are automatically and adaptively extracted from the signal, it is relatively easy to implement and especially that it is robust for nonlinear and non-stationary signals. EMD efficiently captures nonlinear characteristics with respect to amplitude and frequency modulation at local time scale. Once IMFs are obtained, Hilbert spectral analysis provides frequency information varying over time. This is the key component of time-frequency analysis for non-stationary financial time series.

2.1. Local oscillation

The fundamental idea of EMD is identifying an oscillation embedded in a signal from the local time scale. EMD extracts posteriori (data-adaptive) wave forms from given data while Fourier expansion uses predefined wave functions, sine or cosine functions, for identifying frequency. This aspect is the crucial difference between EMD and Fourier analysis. We may perceive properties of oscillation or frequency as 1) oscillating and periodic patterns are repeated, 2) the local mean is zero and the signal is symmetric to its local mean and 3) one cycle of oscillation can be regarded as a sinusoidal wave function. Let e denote either the local maxima or the local minima of a signal, and let 0 denote its zero crossings. Then, one cycle of an oscillation is denoted by the sequence $\{e, 0, e, 0, e\}$, which has the characteristics of a cosine cycle, or by the sequence $\{0, e, 0, e, 0\}$, which has the characteristics of a sine cycle. Observe that the number of extrema and the number of zero crossings in either sequence differ by one. See the artificial example in Figure 2.1 generated by Model (2.1).

Fourier analysis separates each frequency based on predefined basis functions, sine and cosine, and EMD decomposes each component with the intrinsic mode function through a *sifting algorithm*.

2.2. Sifting and empirical mode decomposition

Suppose we observe the signal $X(t)$, which consists of several low and high frequencies,

$$X(t) = \sin(\pi t) + \sin(2\pi t) + 6\sin(6\pi t) + \epsilon, \tag{2.1}$$

where ϵ is noise. For illustrative purposes, we ignore the noise. The signal in Figure 2.1 consists of 3 components from the highest frequency wave $6\sin(6\pi t)$ to the lowest frequency wave $\sin(\pi t)$. Recall that for a real-valued function f, the period P is a value such that $f(t) = f(t + P)$ for all

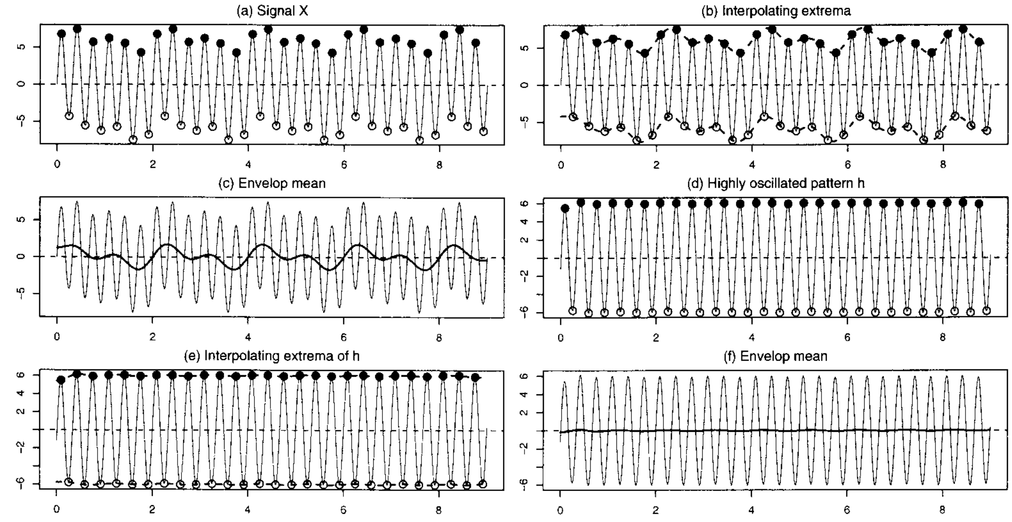

Figure 2.2. The sifting procedure

$t \in \mathbb{R}$. Thus the periods are $1/3, 1$ and 2. The frequencies, the reciprocal of the period, are $3, 1$ and $1/2$ from the highest to the lowest.

Huang *et al.* (1998b) suggested an algorithm for extracting each component. First, identify local extrema (Figure 2.2(a)). Consider the two functions interpolated with a spline function passing through the local maximum and local minimum, called the upper envelope and the lower envelope, respectively (Figure 2.2(b)). It is known that a cubic spline is optimal for interpolating local extrema. Then, the overall pattern of the signal will be seized between two envelopes, so that their average will yield a lower frequency component than the original signal (Figure 2.2(c)). By subtracting the envelope mean, the lower frequency component from the original signal $X(t)$, the highly oscillating pattern h is separated. See Figure 2.2(d). Huang *et al.* (1998b) defined an oscillating wave as an intrinsic mode function(IMF) if it satisfies two conditions: 1) the number of extrema and the number of zero crossings differ only by one and 2) the local average is zero. A single iteration of the aforementioned procedure does not guarantee that the resulting signal h is an IMF. For real world signals, there might be overshoots and undershoots. The same procedure is applied to the signal h until the properties for IMF are satisfied. See Figures 2.2(d), (e) and (f).

Above iterative algorithm is called *sifting*. Sifting makes the remaining signal more symmetric by pushing the local mean towards zero and by making the maxima positive and minima negative. In other words, sifting causes the envelope mean to bisect the signal evenly so that there are neither overshoots nor undershoots. When the signal h satisfies the two conditions stated above, the sifting process stops and recognizes h as the first IMF imf_1. The first IMF imf_1 produced by sifting is the highest frequency by its construction and the remaining signal $r_1 = X - \text{imf}_1$ may still be a compound of several frequencies. See the left panel of Figure 2.3. Figure 2.3 illustrates the process by which the first two IMFs are separated from the signal. Note that the only difference between the original signal and the remaining signal r_1 is the highest frequency imf_1 that the original signal contains. In other words, sifting decomposes the original signal X into the highest frequency imf_1 and residue signal r_1 less oscillated than the original signal. Once the highest frequency is removed

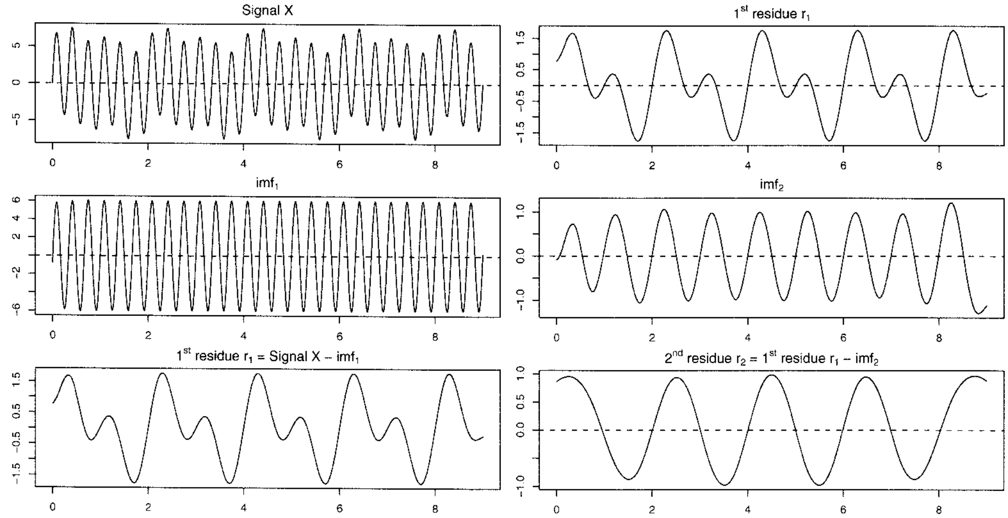

Figure 2.3. The sifting algorithm for two IMFs

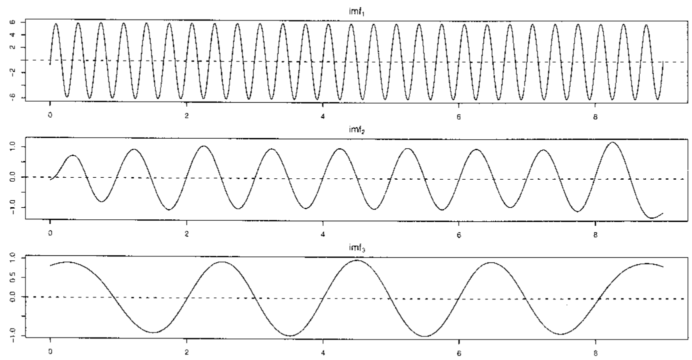

Figure 2.4. Decomposed components by EMD

from the signal, the next highest frequency is identified by the same process. The residue r_1 is now regarded as a new signal to decompose. See the right panel of Figure 2.3. By construction, the number of extrema will eventually decrease as the procedure continues so that a signal is sequentially decomposed into constituent frequencies from the highest frequency component imf_1 to the lowest frequency component imf_n, for a finite n and residue signal r. Finally, we have n IMFs and the residue signal as

$$X(t) = \sum_{i=1}^{n} \text{imf}_i(t) + r(t). \tag{2.2}$$

EMD decomposes the signal generated by Model (2.1) into three components with frequency $3, 1$ and $1/2$, respectively (See Figure 2.4). Each IMF corresponds to $6\sin(6\pi), \sin(2\pi)$ and $\sin(\pi)$. Finally, the EMD algorithm can be stated as follows:

1. Take input signal r_{k-1} to decompose. r_0 is the original signal X.

   1.1. Identify the local extrema of the signal r_{k-1}.

1.2. Construct the upper envelope emax_k and lower envelope emin_k interpolating the maximum and minimum, respectively. Use a cubic spline for interpolation.

1.3. Approximate the local average using the envelope mean em_k, by taking the average of two envelopes, emax_k and emin_k. That is $\text{em}_k = (\text{emax}_k + \text{emin}_k)/2$.

1.4. Compute the candidate intrinsic mode function $h = r_{k-1} - \text{em}_k$.

1.5. If h is IMF, decompose the signal r_{k-1} as IMF $\text{imf}_k = h$ and the residue signal $r_k = r_{k-1} - \text{imf}_k$. Otherwise repeat steps 1.1 through 1.5.

2. If r_k has an implicit oscillation mode, set r_k as the input signal and repeat step 1.

2.3. Hilbert transform and instantaneous frequency

In the Fourier analysis, spectral analysis is based on predefined sine or cosine functions covering the whole time scale. Thus, frequency and amplitude are constant over the entire time scale. Such an analysis would be meaningless for a non-stationary time series in which the frequency and amplitude change with time. For non-stationary time series, it is necessary to have a more flexible and extended notion of frequency which reflects time-varying properties. Huang *et al.* (1998b) employed instantaneous frequency through the Hilbert transform. For a comprehensive explanation of the Hilbert transform, refer to Cohen (1995).

For a real signal $X(t)$, the analytic signal $Z(t)$ is defined as $Z(t) = X(t) + i\,Y(t)$, where $Y(t)$ is the Hilbert transform for $X(t)$,

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(s)}{t-s}\, ds, \tag{2.3}$$

where P is the Cauchy principal value. The analytic signal $Z(t)$ can be represented by a polar coordinate form as $Z(t) = a(t)\exp(i\theta(t))$, where amplitude $a(t) = \|Z(t)\| = \sqrt{X(t)^2 + Y(t)^2}$ and the phase $\theta(t) = \arctan(Y(t)/X(t))$. The analytic signal can capture the local characteristics of a signal $X(t)$ since 1) the Hilbert transform is the convolution of $X(t)$ with $1/t$ using Equation (2.3), and 2) the polar coordinate form provides time-varying amplitude and phase. Define instantaneous frequency as the time-varying phase,

$$\frac{d\theta(t)}{dt}.$$

Then, we can extract localized information in the frequency domain. Figure 2.5 describes the Hilbert spectrum for the signal of the form

$$X(t) = \exp(0.2t)\sin(4\pi(1+t)t), \quad 0 < t < 5.$$

The top panel of Figure 2.5 shows increasing variations in the frequency contents with the increasing amplitude of the above signal. The traditional periodogram analysis have restrictions to capture this kind of features. Hilbert spectrum of the bottom panel of Figure 2.5 reveals the varying frequency as well as amplitude according to time. The x-y axis represents time and instantaneous frequency, and the grey intensity of the image depicts the instantaneous amplitude.

Once EMD decomposes a signal into IMFs, apply the Hilbert transform to the decomposed IMFs and construct the Hilbert spectrum, which is the representation of amplitude and instantaneous frequency with respect to time. There are several ways to obtain instantaneous frequencies, including a wavelet transform and Hilbert transform (Boashash, 1992; Mallat, 1998). It is well known that the wavelet transform does not provide good resolution of frequency changes over time, while the IMF

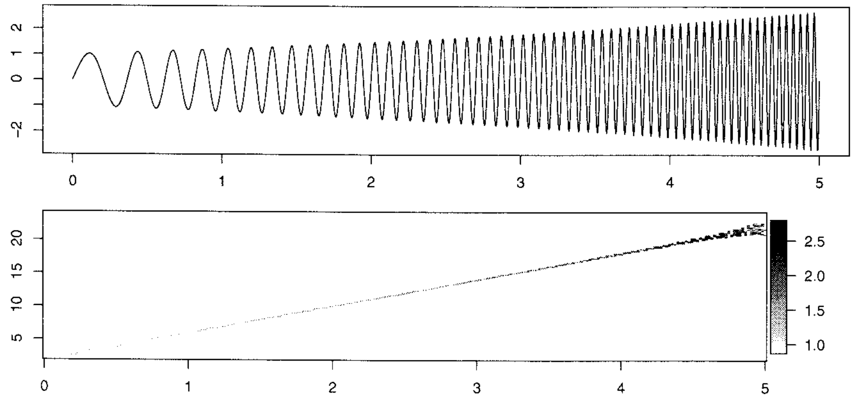

Figure 2.5. Amplitude - Frequency - Time Analysis

admits well-behaved Hilbert transform (Huang *et al.*, 1998b). Through the Hilbert transform, the IMFs yield instantaneous frequencies as a function of time. This identifies hidden local structures embedded in the original signal. HHT provides us with a useful tool in that any local property can be preserved in the time domain with EMD as well as in the frequency domain with the Hilbert transform.

3. Analysis of KOSPI 200

The ultimate goal of this paper is to investigate the weekly KOSPI 200 index from January, 1990 to February, 2007 and to demonstrate the applicability of the Hilbert-Huang transform. Figure 1.1 displays the dynamics of the weekly KOSPI 200 index. The behavior of the index illustrates non-stationary features. For example, we can observe two major waves from mid 1992 to mid 1998, from mid 1998 to 2002, and two waves from 2002 to mid 2004. The periods of those waves shorten from 6 years to 1 year. The overall trend shows decreasing patterns up to mid 1998 and then an increasing pattern thereafter with a steep rise after 2003. Volatility increased between 1997 and 2001, a period marked by the asian financial crisis.

3.1. Decomposition

For the KOSPI 200 index, small waves were repeatedly detected on the fine local time scale, while waves with low frequencies spanned the entire time scale. EMD decomposed this index into eight IMFs and also produced a global trend, shown in Figure 3.1. IMFs imf_1 and imf_2 represent the high frequency characteristics of the index, while IMFs imf_3 through imf_6 extract mid-range frequency signals. In particular, imf_4 through imf_6 correspond to components with periods of 6 month, 1 year and 2 years, respectively. Long-term behaviors are well described by the behaviour of imf_7 and imf_8. The component imf_8 captures the dynamics of the 7–8 year cycle. The imf_7 indicates about 6 year cycle from mid 1992 to mid 1998 and two waves from mid 1998 to mid 2004. The waves imf_7 and imf_8 reflect large-scale characteristics of the index with one oscillating wave, riding on top of another oscillating wave. These riding patterns describe the non-stationary process effectively. Note that the residue signal is the signal remaining after all the oscillatory components have been

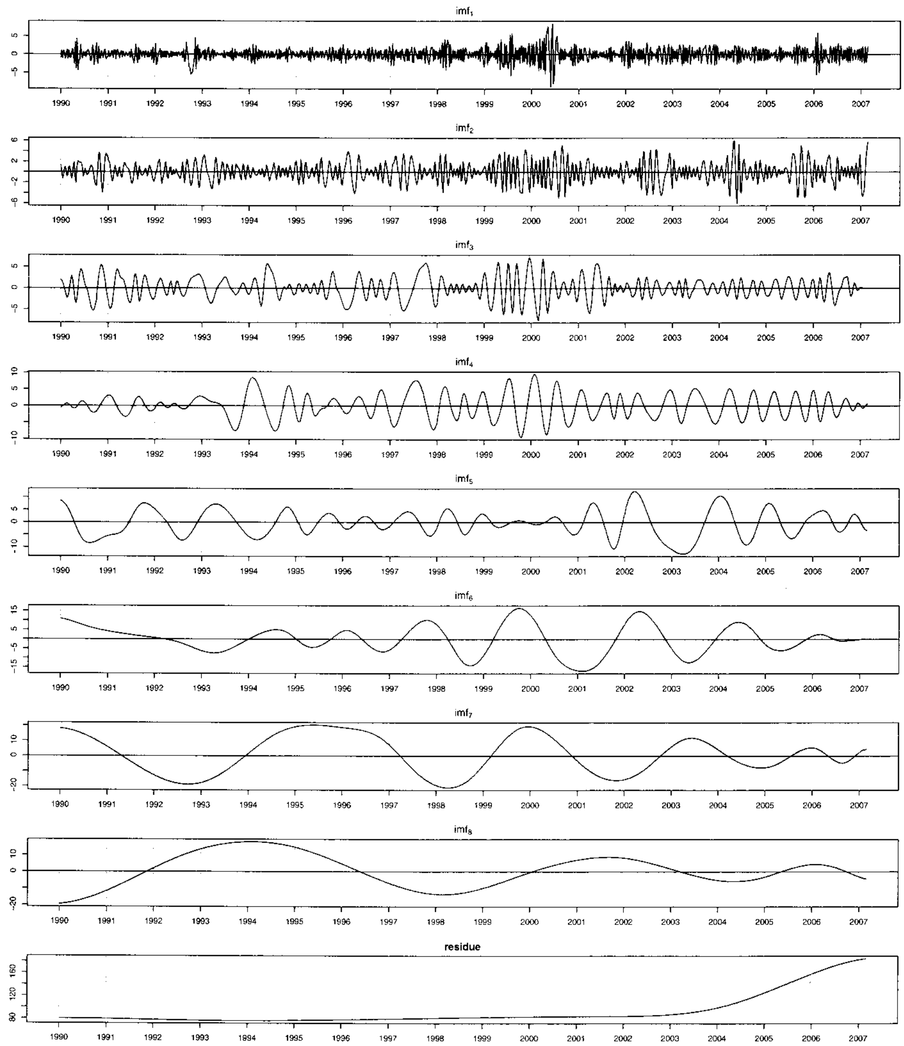

Figure 3.1. Decomposition of the KOSPI 200 index by EMD

removed from the original signal. Thus, the residue signal might be physically interpreted as a trend. The residue signal in Figure 3.1 effectively represents steep rise pattern after 2003. EMD gives us a different approach for extracting a trend, while most statistical methods utilize local averaging process. Based on this observation, we propose a smoothing technique based on EMD in the next subsection.

An alternative to decompose a signal according to time scale is singular spectral analysis(SSA) utilizing principal component analysis. SSA is based on covariance structure between lagged copies of a signal and a signal itself. Singular value decomposition of covariance matrix provides frequency information according to lagged time. We can identify components with dominant frequency through component's contribution to the overall variance. Alexandrov (2008) proposed the parametric

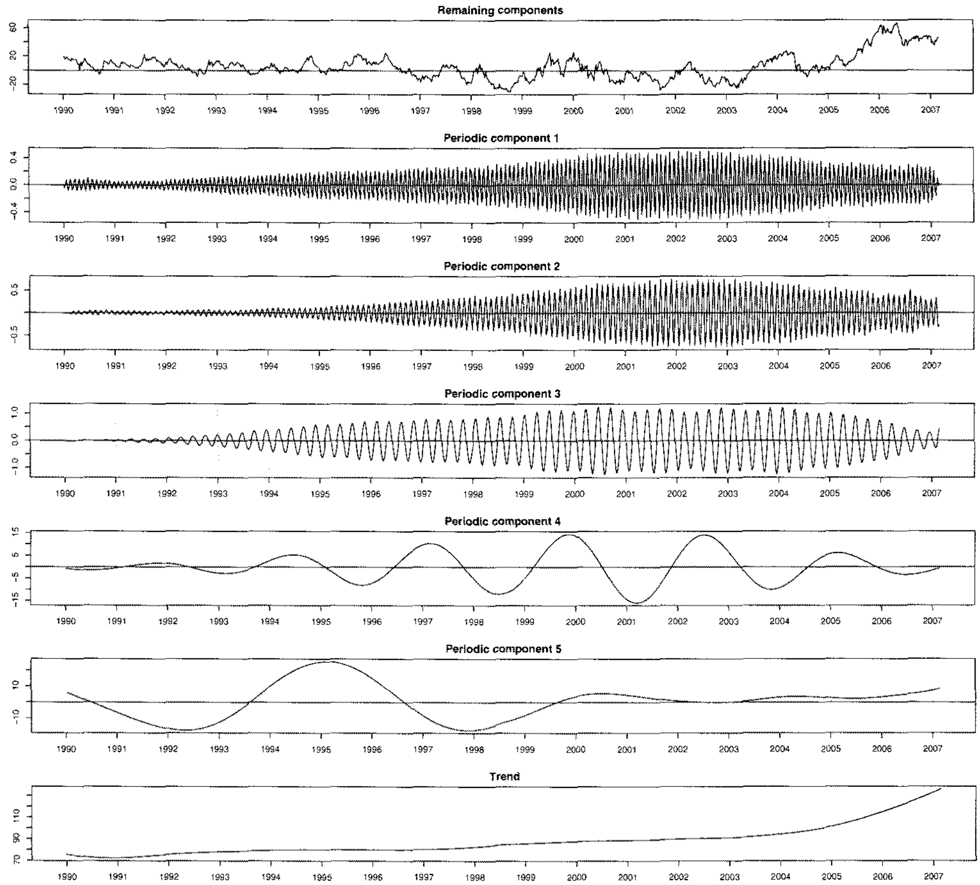

Figure 3.2. Decomposition of the KOSPI 200 index by SSA

method for identifying trends and periodic components. This method provides the five periodic components and trend explaining 96.5% of overall variance. See Figure 3.2.

Wavelet analysis decomposes a signal into several components having increasing (or decreasing) hierarchy of frequency contents through multi resolution analysis. Although wavelet analysis is well known for analyzing non-stationary signals, it also suffers from its non-adaptive nature because it applies the same basis function to the whole range of data. See Figure 3.3 for wavelet analysis result.

Figure 3.4 illustrates data-adaptive nature of EMD for non-stationary signals compared to singular spectral analysis and wavelet analysis. We can observe two distinct behaviors of KOSPI 200 index in Figure 1.1: (1) major waves from the mid 1992 are repeated and periods of those waves shorten. (2) increasing pattern of the overall trend starts at 2003 with steep increment. The top panel of Figure 3.4 describes 7^{th} IMF, 5^{th} component of SSA and 8^{th} component of wavelet, which are best describing decreasing periodic feature of the KOSPI 200 index by each method. The bottom panel shows the trend of each method. We conclude that EMD effectively reveals two distinct non-stationary behaviors embedded in the KOSPI 200 index.

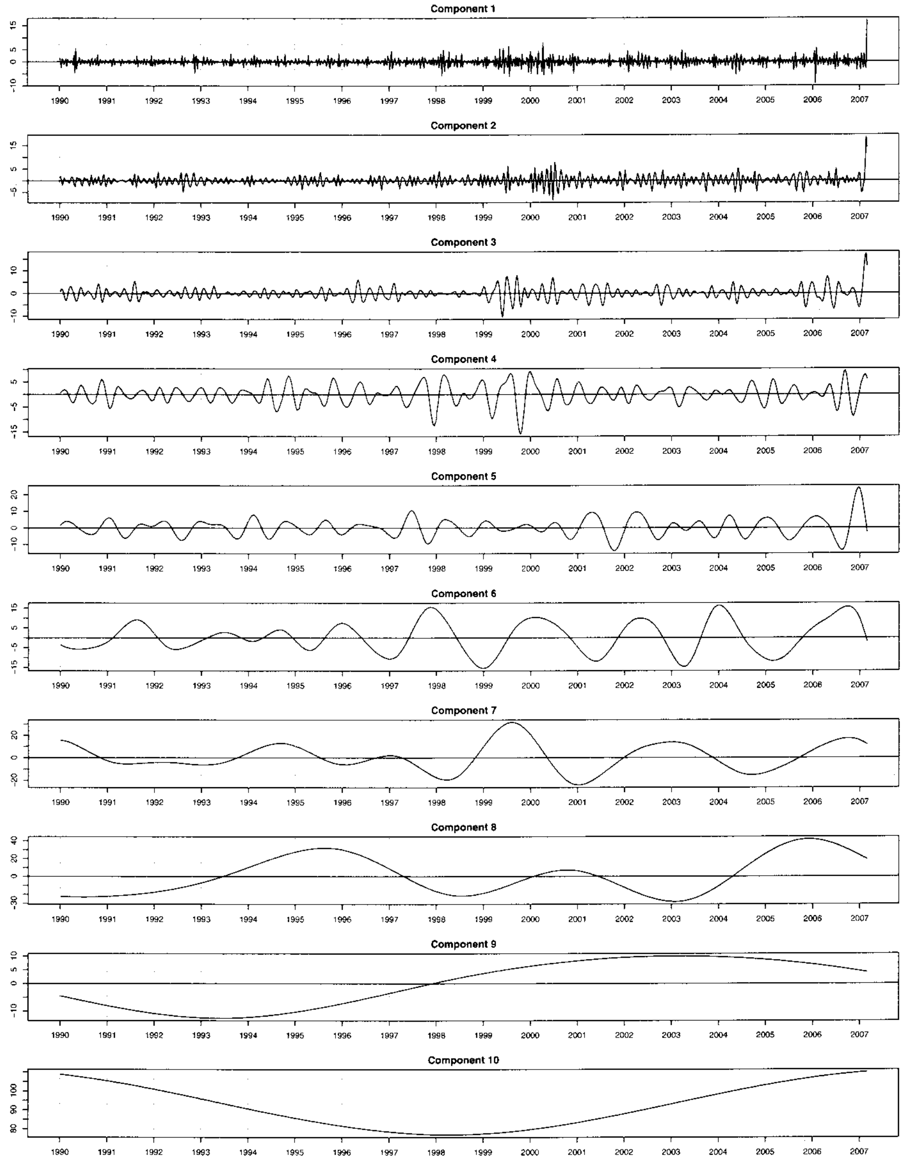

Figure 3.3. Decomposition of the KOSPI 200 index by wavelet analysis

3.2. EMD and filter

Huang *et al.* (2003a) and Flandrin *et al.* (2004) pointed out the role of EMD as a low or high pass filter. IMFs of high frequency contain localized information at a specific time and IMFs of low frequency describe a trend over the whole time span. Thus, EMD can play a role as a filter by properly choosing the resolution level of the IMFs.

In practice we observe the signal contaminated by noise. When the noise is not the component of

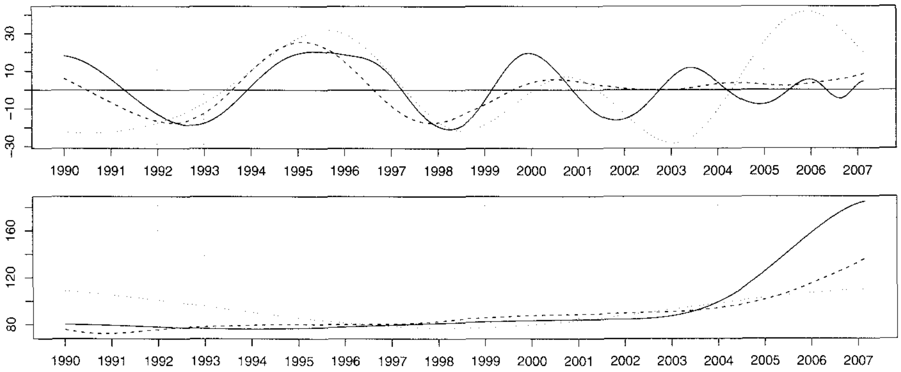

Figure 3.4. Major periodic components and trend of the KOSPI 200 index: Solid line by EMD, Dashed line by SSA, Dotted line by wavelet

interest, pre-processing is required to smooth out the noise from the observations. Otherwise the first several IMFs are meaningless components reflecting noise. Statistical research has sought to smooth noisy signal based on additional information such as the distribution of error or shape of the signal. These methods include the simple moving average, kernel smoothing, smoothing splines and wavelet shrinkage. However, imposing a restriction on the signal contradicts the philosophy of the EMD because its strength is the ability to deal with non-stationary signals. Also due to the nonlinear decomposition procedure of the EMD, it is hard to characterize the error term and to extract information on noise. Based on Equation (2.2), EMD can be utilized as a low pass filter. Let us define a low pass filter L based on the decomposition (2.2) as

$$L_k(t) = \sum_{i=k}^{n} \text{imf}_i(t) + r(t), \qquad \text{for some } k. \tag{3.1}$$

By controlling the amount of local information for each IMF, *i.e.* choosing the proper level k in Equation (3.1), EMD smooths out the noise in the signal. Figure 3.5 shows that by deleting IMFs of high frequency sequentially, smoothing is more effective on the wider time span.

Kim and Oh (2006) proposed an efficient method, termed the hierarchical smoothing technique, by combining cross-validation and thresholding. Once EMD is applied to the original noisy signal and extracts several IMFs $\text{imf}_1, \ldots, \text{imf}_n$, we can obtain new thresholded IMFs $d_1, \ldots, d_k$ for some $k < n$ by thresholding the original IMFs. The procedure can be summarized as follows: for properly selected threshold values $\lambda_i, i = 1, \ldots, k$, the thresholded IMFs $d_1, \ldots, d_k$ are defined as

$$d_i(t) = \begin{cases} 0, & \text{if } |\text{imf}_i(t)| < \lambda_i, \\ \text{imf}_i(t), & \text{otherwise.} \end{cases} \tag{3.2}$$

Equation (3.2) implies that when the signal at a specific time is negligible, the signal is removed. By recombining the thresholded IMFs and residue signal, a denoised signal $\hat{X}$ is constructed as

$$\hat{X}(t) = d_1(t) + \cdots + d_k(t) + \text{imf}_{k+1}(t) + \cdots + \text{imf}_n(t) + r(t). \tag{3.3}$$

Equation (3.3) can be regard as a generalization of the low pass filter (3.1). Following the hierarchical smoothing technique of Kim and Oh (2006), we applied 2-fold cross validation for thresholding the

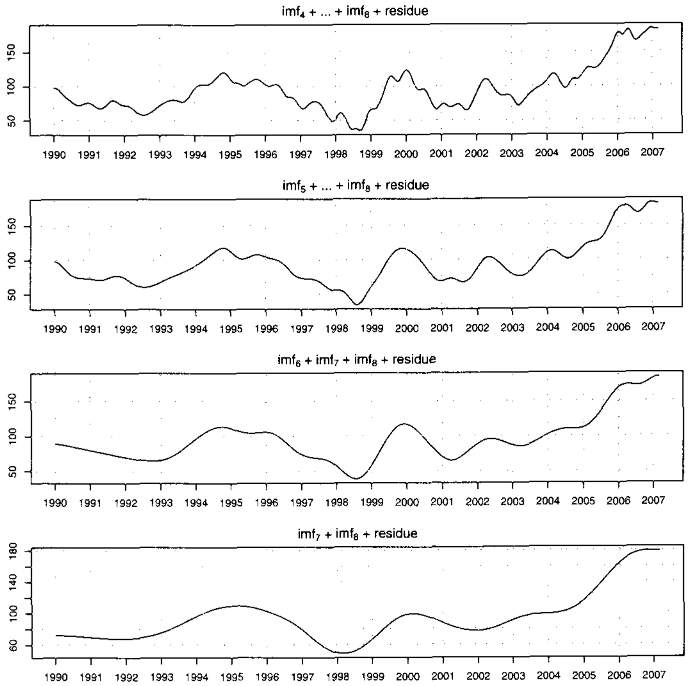

Figure 3.5. EMD used as a low pass filter

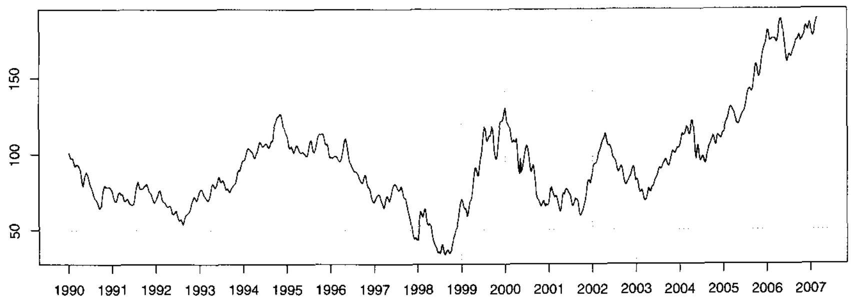

Figure 3.6. Smoothed KOSPI 200 index employing cross validation

first four IMFs of the KOSPI 200 index, which produces $\lambda_1 = 10.273, \lambda_2 = 0.303, \lambda_3 = 0.296, \lambda_4 = 1.009$ and the corresponding result is depicted in Figure 3.6. As expected, the results detect local characteristics of the data when compared with the result obtained by the low pass filter.

Figure 3.1 implies that IMFs with high frequency are the source of the volatility of the KOSPI 200 index and that the volatility depends on time. To measure the volatility of the signal according to time, EMD can be utilized as a high pass filter. The high pass filter H is

$$H_k(t) = \sum_{i=1}^{k} \mathrm{imf}_i(t), \qquad \text{for some } k. \tag{3.4}$$

Equation (3.4) captures local characteristics of the local time t, and the high pass filter H can be

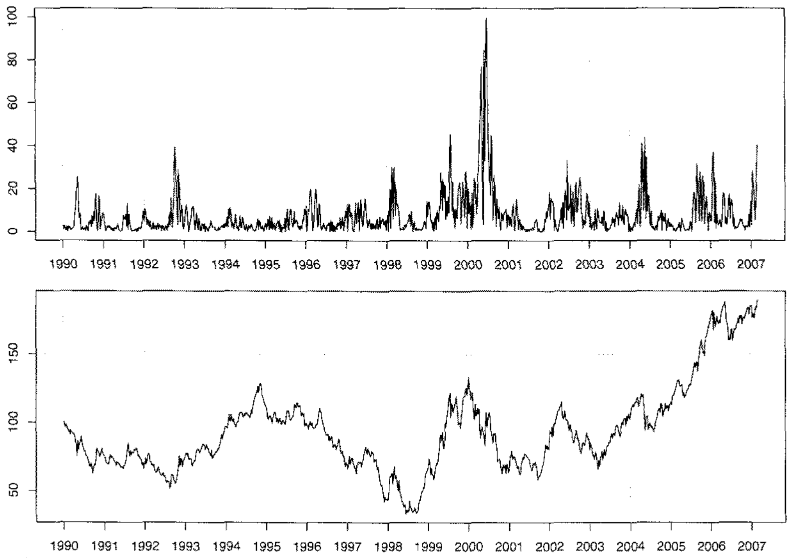

Figure 3.7. Volatility measure by EMD

appropriately employed as a measure of volatility. Note that our intention is not to estimate the volatility as a model parameter, but to build some indicator for fluctuation according to time. The squared version $V(H_k)$ for the high pass filter H induced by an energy concept is appropriate for identifying volatile signal at a particular time;

$$V(H_k)(t) = \sum_{i=1}^{k} \text{imf}_i^2(t). \tag{3.5}$$

Figure 3.7 comparing the $V(H_k)$ and the KOSPI 200 index visually allows for the efficient detection of the volatile source of the index. Volatile patterns between 1997 and 2001, during the asian financial crisis, can be detected easily. The mid 1992, mid 2002, mid 2004 and 2006 are also detected as a volatile time spot.

3.3. The Hilbert spectrum

Huang *et al.* (2003a) pointed out that the Hilbert spectrum is a general version of the Fourier spectrum. While the Fourier spectrum measures the constant amplitude and frequency over the entire time domain, the Hilbert spectrum captures locally variable amplitude and frequency. Figure 3.8 shows instantaneous frequency of three low frequency components by EMD. EMD captures non-stationary feature of the KOSPI 200 index through time-varying instantaneous frequency and amplitude.

Through the Hilbert spectrum, volatility can also be measured. The Hilbert spectrum HS measures the local energy, *i.e.* instantaneous amplitude and frequency in the (t, ω) dimension, the time-frequency dimension. Thus, the squared energy E_k up to IMF k might be an alternative indicator of volatility;

$$E_k = \sum_{\text{imf}_1,\ldots,\text{imf}_k} HS^2(t,\omega). \tag{3.6}$$

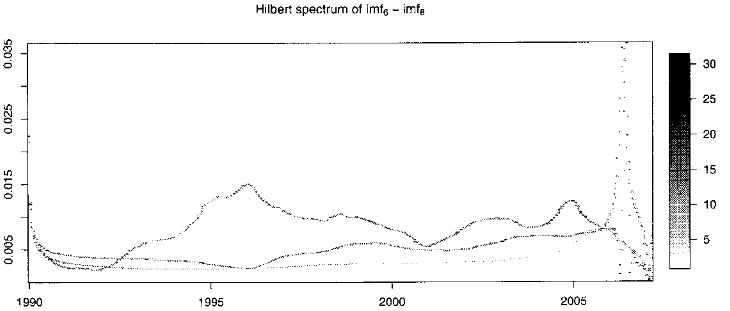

Figure 3.8. Hilbert spectrum of low frequency components by EMD

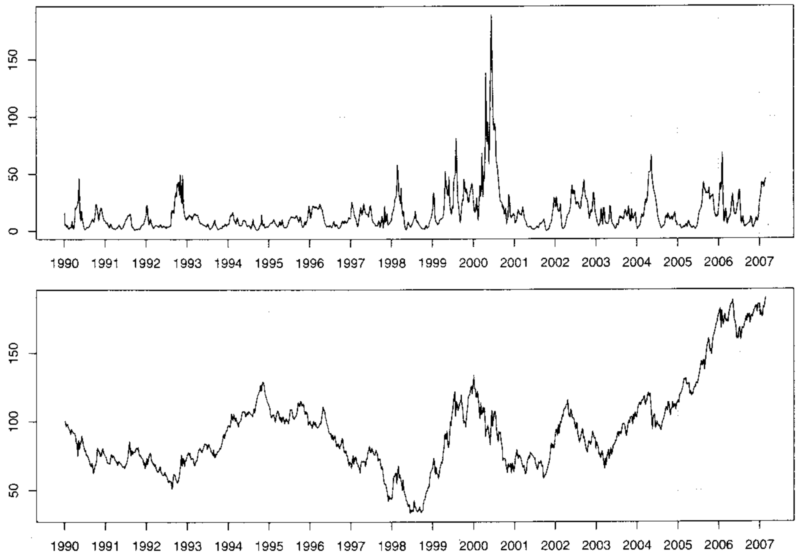

Figure 3.9. Squared energy of the KOSPI 200 index

Note that Figure 3.7, based on Equation (3.5), and Figure 3.9, based on the squared energy of Equation (3.6), contain almost the same information.

Other usage of filter-bank-based approach of HHT is forecasting of time series. To achieve a stable forecasting, important persistent features of a signal must be obtained. Hilbert spectrum provides a proper way to extract such features since HHT decomposes a signal according to its scales. Kim *et al.* (2008) proposed a filter-bank-based forecasting method coupling HHT with the existing forecasting methods. They extract dominant IMFs of a signal using Hilbert spectrum and then forecast patterns based on those IMFs. See Kim *et al.* (2008) for details.

4. Concluding Remarks

This paper discusses the applications of HHT, a two-step procedure combining EMD and the Hilbert spectrum. HHT is a data-adaptive method that captures local properties. It is easy to implement and robust to the presence of nonlinearity and non-stationarity. We have demonstrated the promising capability of HHT for non-stationary financial time series data through the KOSPI 200 index. Applications of HHT include the decomposition of complicated signals, denoising, detecting volatility and forecasting. We hope that HHT provides a new means for dealing with non-stationary financial time series as well as a variety of other applications.

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