

# Computation of the Mutual Radiation Impedance in the Acoustic Transducer Array: A Literature Survey

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(Received June 6 2009; accepted June 9 2009)

## Abstract

Mutual radiation impedance becomes more important in the design and analysis of acoustic transducers for higher power, better beam pattern, and wider bandwidth at low frequency sonar systems. This review paper focused on literature survey about the researches of mutual radiation impedance in the acoustic transducer arrays over 60 years. The papers of mutual radiation impedance were summarized in terms of transducer array structures on various baffle geometries such as planar, cylindrical, spherical, conformal, spheroidal, and elliptic cylindrical arrays. Then the computation schemes of solving conventional quadruple integral in the definition of mutual radiation impedance were surveyed including spatial convolution method, which reduces the quadruple integral to a double integral for efficient computation.

**Keywords:** *Mutual radiation impedance, Transducer array structure, Baffle geometry, Quadruple integration, Spatial convolution*

## 1. Introduction

Recently arrays of transducers are more frequently used in underwater acoustics for higher power and better beam directivity. Underwater transducers usually pursue low frequency and wideband as well as high radiated power in order to achieve better performance of active sonar system. As frequency becomes lower, interaction between array elements in a transducer becomes more important because the distance between elements in an array is smaller compared with the acoustic wavelength. Then mutual radiation impedance is fundamental for the design and analysis of acoustical transducers because the assumption of independently acting elements becomes less accurate in array transducers [1].

Mutual radiation impedance is known to be dependent on the size and shape of the baffle and the size, shape, and separation of the sources [2]. The acoustic radiation impedance of different sources on baffles has been extensively investigated in the literature for various baffle geometries. [3] Therefore this current work briefly summarized researches of mutual radiation impedance in terms of transducer array structures on various baffle geometries such as planes, cylinders, spheres, oblate and prolate spheroids, elliptic cylinders, and conformal structures.

By definition, mutual radiation impedances of acoustic transducer array include a quadruple integral. Because of the difficulties of analytical solution and inefficiency of numerical computation of quadruple integration, lots of efforts have been put forth to decrease computation time of the radiation impedances using various numerical schemes. In this review paper, these computation schemes of solving conventional quadruple integration were briefly traced

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for practical applications. Recently the spatial convolution approach was proposed to reduce the quadruple integral to a double integral, so that the computational time of calculating the radiation impedance of an arbitrary shaped piston could be dramatically reduced. In the last section, these were briefly summarized.

## II. Mutual radiation impedance in various arrays

In general, the mutual radiation impedance depends on the elements in transducer array structure. This structure can be determined by the array sensors and baffles where the mutual radiation impedance was computed, designed and considered. In this chapter, the mutual radiation impedances were discussed depending on the array structures.

### 2.1. Planar array

Since Rayleigh studied the real acoustic power radiated by multiple sources for the first time, several studies were restricted only to resistive effects and did not include the reactive effects by 1930s [4]. Then the study of the mutual radiation impedance was started from two circular pistons vibrating in the same infinite plane as a simple case. When these two pistons radiate into a semi-infinite medium, there is an interaction between them, and the mutual radiation impedance needs to be calculated. Wolff and Malter obtained an approximation for the resistive part of this interaction when the pistons were in phase [5]. Later a direct integration procedure was used for both resistive and reactive parts of the mutual impedance [6], and a general and rigorous expression by Green's functions was introduced for the mutual radiation impedance of many sources in an array [4] [7]. However, the studies were accomplished by the complicated mathematical methods including a multiple integral. Hence, the different approach to calculate the mutual radiation impedance was introduced by Pritchard in order to

simplify the computation [4]. During the study of mutual radiation impedance from two pistons in the planar array, the shape of a radiator was considered for computing. The circular piston was used in the beginning, and later the rectangular piston was considered [8]. This rectangular piston required the more complicated calculation due to the quadruple integral. In addition to the piston shape, the physical properties of the piston were considered for computing the radiation impedance because the acoustic array transducer should be designed and manufactured [9]. The radiation type is also one of the factors for the acoustic transducer design, and some studies were reported in terms of the transient and steady-state acoustic loading on a baffled piston [10] [11]. The spacing of pistons was also studied as a compact planar array. This study was to investigate the maximum mutual interaction between a radiator and its nearest neighbors in a planar array [1]. In summary, two pistons in an infinite planar baffle were considered as a simple case to compute the mutual radiation impedance in the array transducer and the more complicated studies have been gradually carried out.

From 1990's, as the numerical methods have been rapidly developed, the mutual radiation impedance has been actively studied. Integral equations in a planar array were derived and numerically simulated in an infinite rigid baffle. Afterward, the acoustic radiation power was estimated to predict the performance and efficiency of the array transducer [12-15]. Even though the computation of the mutual radiation impedance in the planar array has been studied by many researchers, heavy computation was a problem. Therefore, studies to reduce the computation time have been followed. A square vibrating surface was divided into small elements and the duplicate calculations were eliminated in the process of computing the mutual effects of elements [16]. Furthermore, the shape of the vibrating surface could be adjusted by the distribution of these small elements [17]. The mutual radiation resistance from cross-model coupling was calculated for a simply

supported rectangular plate and its effects on the radiated sound power were investigated. It was shown that the mutual radiation resistance could be obtained easily in the whole frequency range by recasting the quadruple integrals into several double integrals [18]. A modal Pritchard approximation has been developed to compute the mutual radiation impedance for acoustically hard arrays [19]. Those formulations, however, were based on uniformly vibrating rectangular patches, which may not be extended to find acoustic interaction for a flexibly vibrating case. In order to use a finite baffle, the outside of the finite baffle was considered as an imaginary negative acoustic source [20]. The size and the material properties of the finite baffle as well as the characteristics of the radiators were investigated [21]. In recent years, both sides of the piston and the baffle were simultaneously studied for an almost realistic array transducer [22].

## 2.2. Cylindrical array

Cylindrical array transducers have been widely used in the underwater acoustic systems. Hence computation of the mutual radiation impedance has been actively studied in the cylindrical array. The early studies to compute the mutual radiation impedance in the cylindrical array were focused on an array of finite cylinders [7]. Finite cylindrical sources were assumed and these cylinders made up the active part of an infinitely long, rigid, cylindrical baffle. These studies, however, was nothing but the infinitely long linear array along the direction of the height of the finite cylinder. Because the slotted finite cylinder transducer has been found to be a useful source for low-frequency applications due to the small size, the computation of the radiation impedance of the finite cylinder has been constantly developed. A mathematical model for the radiation impedance of a finite cylinder with extended rigid ends was developed using a Fourier series approach [23]. Greenspon and Sherman studied the cylindrical array whose pistons were rectangular portions of the

surface of an infinitely long cylinder as shown in Fig. 1 [24]. It was known that the results for the cylinder were very similar to those for a plane in certain ranges (Fig. 2).

The mutual radiation impedance was not confined to the theoretical computation and was compared with the experimental results even if the cylindrical radiator was used not in the infinite baffle but in the finite baffle [25]. Like the planar array, the studies to decrease the computation time were followed in

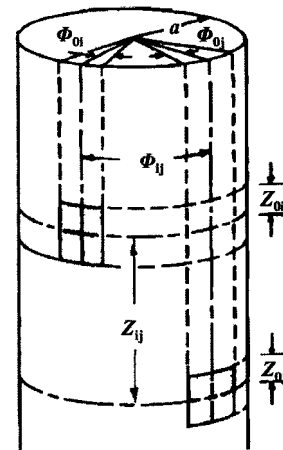


Fig. 1. Rectangular pistons on a cylinder [24].

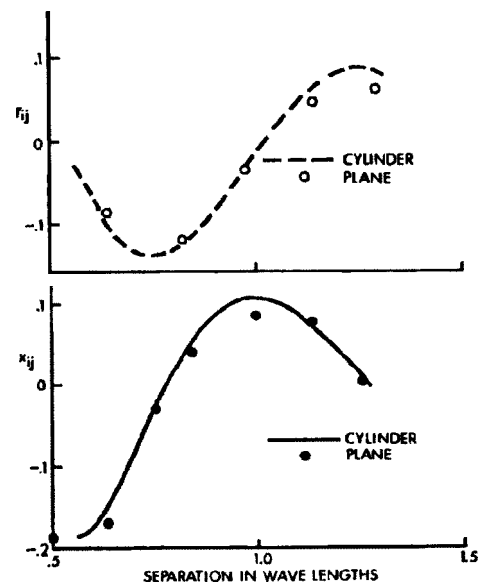


Fig. 2. Comparison of mutual impedance for square pistons ( $kz_{01} = kz_{02} = ka\phi_{01} = ka\phi_{02} = \pi/2$ ) long the generator of a rigid cylinder ( $ka = 36$ ) and circular pistons of the same area ( $kR = \pi^{1/2}$ ) in a rigid plane.  $R$  is the circular piston radius [24].

the cylindrical array and the shape of piston was also considered. The combined Helmholtz integral equation formulation (CHIEF) method in addition to Greenspan's analytical solution [24] was used for calculating the radiation impedance of the rectangular pistons on the cylindrical surface [26]. The mutual radiation impedance was measured when two circumferentially baffled cylindrical shell transducers were electrically uniformly excited for transducer alignments that were coaxial and horizontal [27]. Recently, with these improving computation methods and the consideration of the piston shape, it was investigated whether the total mutual radiation impedance could be changed with the calculation range, the wave number and the size of the baffle [28].

### 2.3. Spherical array

The mutual radiation impedance in the spherical array has been studied because the spherical structure is required for the sonar performance. The first study was to calculate the mutual radiation impedance coefficient in the cases of uniformly vibrating circular and rectangular sources on a rigid sphere (Fig. 3) [2]. This study showed that the interaction between the sources was almost independent of the baffle shape as shown in Fig. 4. As the source separation increased, the baffle shape became important, and sources on a plane were seen to interact more strongly than those on a sphere.

To facilitate the solution of a radiation and scattering problem involving spheres which are not concentrically positioned, it is desirable to express spherical wave functions centered on one sphere. This transformation of the wave functions greatly simplifies the task of satisfying the specified boundary conditions on the various spherical surfaces. The mathematical relationship that accomplishes this transformation is known as the translational addition theorem for spherical wave function [29]. Recently, the characteristics of mutual radiation impedance for piston source on a spherical baffle were analyzed depending on the distance between the sources by algorithm including Finite Element Method (FEM)

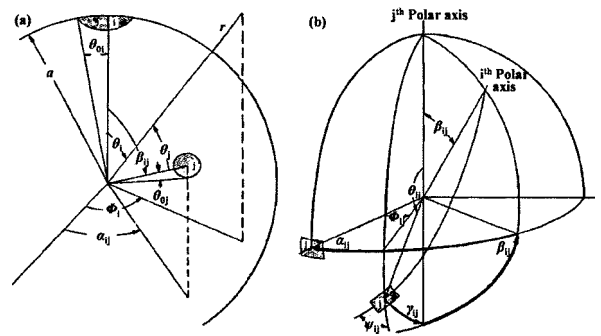


Fig. 3. Coordinates for circular (a) and rectangular (b) sources on a sphere [2].

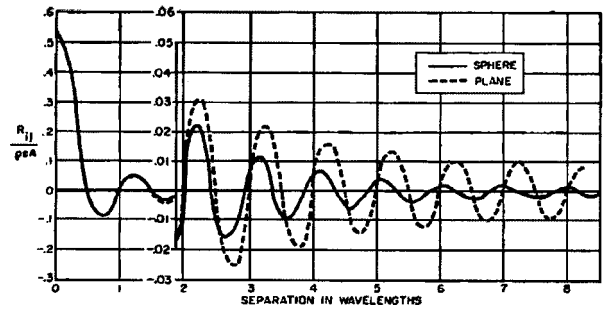


Fig. 4. Comparison of mutual radiation resistance coefficients between circular sources ( $ka = 1.1$ ) on a rigid sphere ( $ka = 34$ ) and on an infinite rigid plane. (Note change of scale at a separation of 1.9 wavelengths) [2].

and Hybrid type Infinite Element Method (HIEM). In addition, the mutual radiation impedance was studied considering the non-rigid spherical baffle [30]. Moreover, the acoustic radiation from a harmonically pulsating piston in radial direction set in the side of a rigid spherical baffle was studied near a hard/soft planar interface by the combination of the image method and the translational addition theorems [31]. This configuration was a practical idealization for a baffled spherical transducer placed near a rigid/free surface.

### 2.4. Conformal array

In order to achieve good system performance for active sonar and autonomous underwater vehicle (AUV), underwater projecting transducers usually pursue low frequency, high power and wideband [32]. Conformal arrays provide benefits of visual unobtrusiveness and non-interference with aerodynamic performance due to the flush panel on which they are mounted. Even for the conformal array, acoustic

interaction effects can seriously degrade the performance of arrays in which the elements are small in size compared to a wavelength. Hence, the acoustic radiated field of a transducer has been investigated in the conformal array [33] [34]. These studies showed the method which could utilize the acoustic radiated field inversion of the transducer conformal array to obtain the required vibration velocity weighting vector using boundary element theory together with the optimization method. To develop the mathematical algorithm in the non-planar conformal array, the vector translation was required to collimate the beam of each element to a common point [35].

## 2.5. The other arrays

The mutual radiation impedance in the spheroid has been studied. First, the mutual radiation impedance in the oblate spheroidal baffle was discussed [36]. The intersection of a hyperbolic cylinder and an oblate spheroid produces a central annular zone (lightly shaded) and two end caps (heavily shaded) as shown in Fig. 5. The formulas and computations of the radiation impedance density were presented for vibrating caps (pistons) and zones (rings) of various sizes and curvatures on rigid oblate spheroidal baffles of various sizes and eccentricities. Fig. 6 shows the effect of changing the surface curvature on the resistance and reactance density by changing the spheroidal coordinate.

Secondly, the prolate spheroidal baffle (Fig. 7) was considered to calculate the mutual radiation impedance with the piston shape, a cap, a ring [37] and a rectangular piston [3]. When the cap becomes large acoustically, the radiation impedance approaches that of a plane wave; the radiation resistance goes to unity (Fig. 8-(a)), and the radiation reactance goes to zero (Fig. 8-(b)). The pattern of the mutual radiation impedance for rings was similar to that for caps (Fig. 8-(c), (d)). Results for the normalized mutual radiation resistance and reactance for square pistons as a function of separation angle  $\phi$  at three locations on the spheroid are shown in Figs. 9 [3]. As  $\eta$  increases, the results become the behavior near

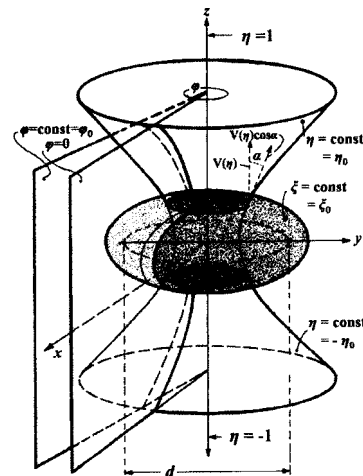


Fig. 5. Geometry of two pistons on an oblate spheroidal baffle [36].

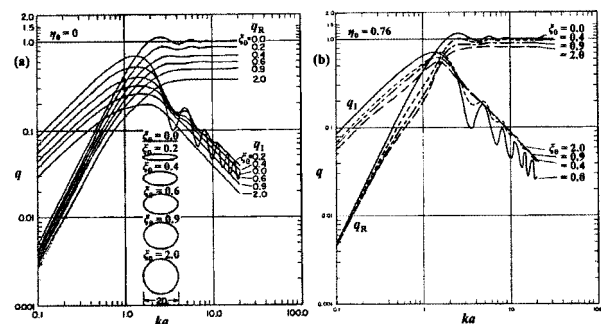


Fig. 6. (a) Radiation impedance density as a function of  $ka$ , where  $a$  is the radius of the resulting disk ( $\eta_0 = 0$ ,  $a = D$ ) obtained by projecting the piston onto the  $xy$  plane for a series of rigid oblate spheroids, illustrated on the graph, vibrating along the  $z$  axis (pulsating spheroid), (b) radiation impedance density as a function of  $ka$ , where  $a$  is the radius of the resulting disk obtained by projecting the piston ( $\eta_0 = 0.76$ ) onto the  $xy$  plane for a series of different curvatures of piston and oblate spheroidal baffles (pulsating piston) [36].

the tip of the spheroid. That means the interaction decreases more rapidly for the case nearest the equator. This is because near the equator, a given separation angle corresponds to a greater separation distance between pistons.

Elliptic cylindrical structures are widely used in aircraft fuselages, marine vehicles, acoustic transducers, and mufflers. The elliptic cylindrical coordinate system in [38] is shown in Fig. 10-(a) and pistons are rectangular in shape, and conformal to the surface of the elliptic cylinder (Fig. 10-(b)). Figure 11 shows the results for the normalized mutual radiation resistance  $r_{ij}$  (Fig. 11-(a)) and reactance

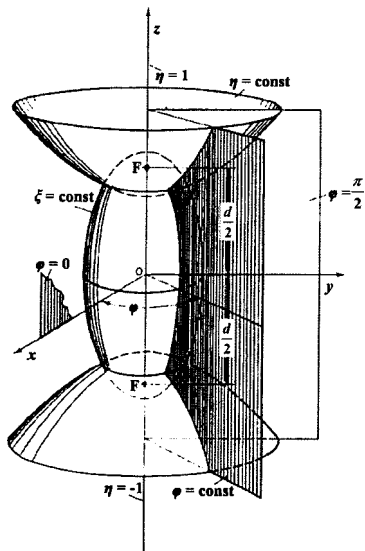


Fig. 7. Prolate spheroidal coordinate system [37].

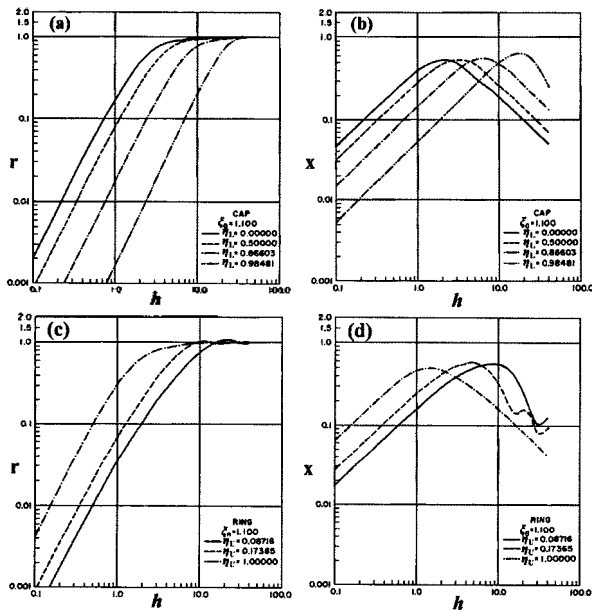


Fig. 8. The normalized radiation resistance (a) and reactance (b) as a function of the acoustic size parameter  $h$  for caps on a rigid prolate spheroidal baffle with shape parameter  $\zeta_0 = 1.100$ . Same results for rings (c), (d) [37].

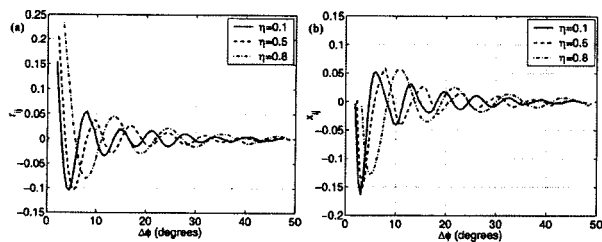


Fig. 9. Normalized mutual radiation resistance (a) and reactance (b) at  $h = 60$  for square pistons with size parameter  $W/((\zeta_0^2 - 1)^{1/2} d) = 0.0168$ , on a spheroid with shape parameter  $\zeta_0 = 1.35$ , as a function of separation angle  $\Delta_\rho$ , for  $\eta = 0.1, 0.5$  and  $0.8$  [3].

$x_{ij}$  (Fig. 11-(b)) for a pair of adjoining square pistons located at the end of the elliptic cylinder, as a function of  $kH$ . The figure also illustrates the normalized mutual resistance (Fig. 11-(c)) and reactance (Fig. 11-(d)) versus the separation angle for pistons of various sizes ( $H/L = 0.02, 0.05, 0.1$ ) on an elliptic cylindrical baffle defined by  $L/D = 10$ .

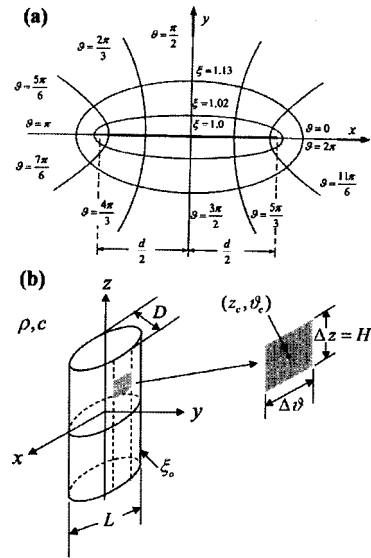


Fig. 10. Elliptic cylindrical coordinate system (a) and rectangular piston conformal to a rigid elliptic cylindrical baffle (b) [38].

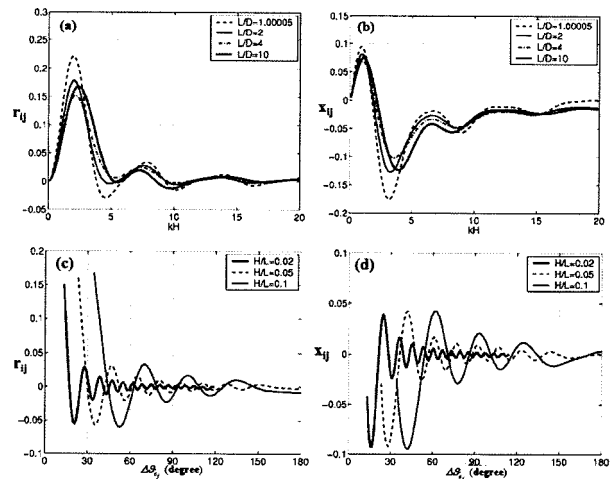


Fig. 11. Normalized mutual radiation resistance (a) and reactance (b) versus  $kH$  of adjoining square pistons of relative size  $H/L = 0.1$  located at  $(\varphi_c = 0^\circ, z_c = 0)$  and  $(\varphi_c = 0^\circ, z_c = 0)$  as a function of the baffle cross-section aspect ratio. Normalized mutual radiation resistance (c) and reactance (d) versus separation angle  $\varphi_c$  for square pistons on an  $L/D = 10$  elliptic cylindrical baffle as a function of piston sizes  $H/L = 0.02, 0.05, 0.1$ , at  $kH = 2.4$  [38].

### III. Improvement of computation methods

#### 3.1. Quadruple integration

Mutual radiation impedances of an ultrasonic transducer array include an inherent quadruple integral by the definition. Because of the difficulties of analytical solution and inefficiency of numerical computation of quadruple integration, it has been an issue to decrease heavy computation of the radiation impedances. The standard numerical algorithms such as Gaussian, Simpson, Newton-Cotes, Romberg, Monte Carlo were not efficient for quadruple integral. Several schemes have been suggested for a fast and accurate computation for practical applications. Thompson [39] used the integration technique of the far field directivity pattern for computation of mutual radiation impedances of uniformly vibrating pistons of different shapes. Finite element approach was introduced for numerical evaluation of integral expressions for the radiation impedance by Burnett and Soroka [40]. The Fourier transform of the impulse response was taken for analytical evaluation of the radiation impedance of a rectangular piston by Stephnisen [41]. Bank and Wright [42] provided the tabulated data of radiation impedance through numerical computation of quadruple integral by using geometric relations for rectangular pistons. Lee and Seo [14] numerically evaluated the impedances of formulated quadruple surface integrals by geometric relations among square pistons in a rigid infinite baffle, and radiation power of the transducers was calculated considering the mutual coupling effect by an equivalent electric circuit scheme [15]. Scandrett et al. [19] borrowed a modal Pritchard approximation for computing the mutual impedance for array elements. Li and Gibeling [18] numerically computed mutual radiation resistance from cross-model coupling for a rectangular plate and their effects on the radiated sound power by recasting the quadruple integrals into several double integrals. Pierce et al. [43] reduced the integration of the radiation impedance to a finite sum of one-dimensional integrals, where the integration is over a finite region, where

the integrand is finite, and where the real and imaginary parts of the integrand have a finite number of maxima and minima, for a special case of rectangular shape when the basis functions must be expressed as a sum of products of exponential complex functions. Kim et al. [16] applied an algorithm to divide a square vibrating surface into small elements in order to eliminate redundant calculations during process of computing mutual effects of elements, and simplified quadruple loops for quadruple integration into a single loop or double loops, confirming short computation time and high accuracy. They extended the method to calculate radiation impedance of a vibrating surface with an arbitrary shape which can be represented by a set of sub-squares [17], and with finite baffle using an imaginary negative acoustic source for the outside of the finite baffle [20].

#### 3.2. Spatial convolution

In order to simplify the numerical integration for a rectangular radiator a lot of efforts have been put forth, while a few investigated the characteristics of the mutual radiation impedance resulting from two flexible rectangular patches. Sha et al. [44] developed an efficient calculation method which is suited to the determination of the radiation impedance in the cases of uniformly and flexibly vibrating rectangular patches. They used the spatial convolution approach to reduce the quadruple integral to a double integral, so that the computational time of calculating the radiation impedance of an arbitrary shaped piston in a rigid infinite baffle could be dramatically reduced. This spatial convolution work was based on their previous works of virtual complex source approach to calculate the primary sound fields radiated by rectangular pistons [45] and the superposition principle where the total radiation impedance of an arbitrarily shaped piston can be calculated as the sum of small incremental rectangular elements which can be evaluated in a closed form with a double integral [46]. Based on this concept, the efficiency of the computation of the

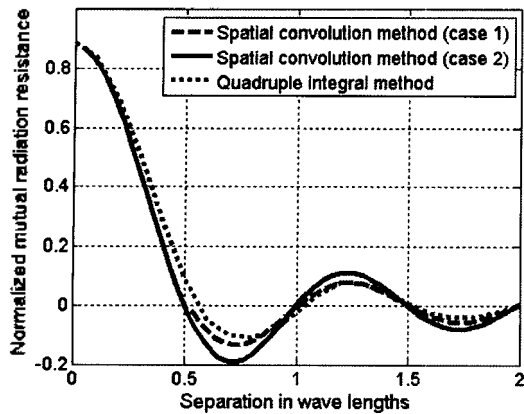


Fig. 12. The comparison between the spatial convolution method and the quadruple integral method. Case 1:  $kz_{0i} = kz_{0j} = kR_a\Phi_{0i} = kR_a\Phi_{0j} = \pi/2$ . Case 2:  $kz_{0i} = kz_{0j} = kR_a\Phi_{0i} = kR_a\Phi_{0j} = \pi/200$  [47].

mutual radiation impedance can be improved in the arbitrary array structure. Recently, the mutual radiation impedance was computed in the cylindrical array using the spatial convolution [47]. As shown in Fig. 12, the differences of computation results between the spatial convolution method and the quadruple integral method were small while the computation time was dramatically reduced.

#### IV. Conclusions

This review paper has briefly surveyed the previous studies about the mutual radiation impedance in the acoustic transducer array over last 60 years. The acoustic radiation impedance of different sources on various baffles was summarized, and the planar, cylindrical, spherical, conformal, the oblate and prolate spheroidal, and elliptic cylindrical arrays were included in this paper. Then the numerical computation schemes of solving quadruple integral shown in the definition of the mutual radiation impedance were reviewed to show efforts to overcome the inefficiency of numerical computation of quadruple integral. A recent approach using spatial convolution to reduce the quadruple integral to a double integral was also dealt in planar and cylindrical array.

#### Acknowledgments

This work was supported by Defense Acquisition Program Administration and Agency for Defense Development under the contract UD070054AD.

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