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ISSN 1226-2854

The Possible Modification of the Half Life of the ^{133}Cs nucleus in the Finite Space

— 유한한 공간에서 ^{133}Cs 원자핵 반감기의 변화에 대한 연구 —

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— Abstract —

A theoretical investigation has been carried out on how the energy width of the excited state of the nuclei is modulated when the γ-ray source is placed between two gold plates, at the center of the gold cylinder or the sphere. The width of the 81-keV level of ^{133}Cs is shown to become narrower by 3.7% at 4.2 K by reabsorption of γ rays scattered backward from the parallel plates which are made of a 0.05-cm-thick, 3-cm-radius gold plates and separated from each other by 1.0 mm. With a 0.05-cm-thick, 5-cm-long, 1.0-mm-radius gold cylinder, we found that a width became narrower by 6.5%. In addition, when the nuclei is located in a spherical reflector of 1.0 mm in radius made of gold with a thickness of 0.5 mm, the level width is reduced by about 18.2% at a temperature 4.2 K. The results of this study indicates that the life-time of energy level was prolonged.

Key Words: energy width, half life, gamma-ray

Ⅰ. Introduction

Control of nuclear lifetime must be very important in the context of nuclear waste problem, storage of radioactive material and precision measurement of γ-ray spectra. If the widths of nuclear energy levels could artificially be modified, a clue to solve these problems would be provided. For such a reason, several attempts were made in the past. They tried to change the chemical state of radioactive nuclei and apply high pressure or low temperature. However, all these attempts gave only negligible changes of 0.02~0.04%. On the other hand, it is known that spatial structure of the vacuum field can also make change of atomic and nuclear energy levels and widths[1-4)]. Namely, if the space is limited by two perfect conducting plates on the surface of which all wave functions vanish, the vacuum field becomes discrete and, therefore, induces some modification of physical quantities. It extracts only small effects in the millimeter range of the plate separation but observable effects could be obtained in a micrometer order of separation[5-8)].

In the result of the Mössbauer experiment carried out in order to observe the nuclear energy level shifts[8-11)], it was occasionally discovered that its width became narrower and, then, the data was

*접수일(2009년 1월 29일), 1차심사(2009년 2월 25일), 2차심사(2009년 5월 23일), 확정일(2009년 5월 29일)

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carefully reanalyzed. It is very interesting to investigate the mechanism of such a phenomenon which must be different from those effects of chemical environment.

In this situation, a novel idea has been proposed[8] to explain the phenomenon found in the Mössbauer experiment. If the photon emitted from the source could return to be reabsorbed by the original source, the population of nuclei being in the excited state would increase and consequently the nuclear half-life could be prolonged. In a free space, return of the emitted photons to the source is impossible. However, those photons may be forced to return to the source by operating reflectors in the finite space with the metallic parallel plates, a cylinder or a sphere. That the photon returns and is reabsorbed by the source nuclei after back scattering from the plates causes suppression of photon emission. When such a process is repeated many times, the half-life is finally prolonged. Of course, all these processes such as emission, reflection and absorption should be occurred elastically, i.e. without any energy loss. Therefore, the source nucleus should be implanted in a solid.

One step of photon reabsorption would make only a tiny modulation of the lifetime, but iterative processes could make it grow up step by step until the initial amount of nuclei would become a half. One of the challenging problems in low energy nuclear physics is to change nuclear decay constant. It is very important to investigate whether the lifetime of the radio-active nucleus can artificially be changed.

Ⅱ. Theory

The elastic scattering of γ-rays from the metallic surface is coherent and mostly caused by atomic electrons. It is the Thomson cross section

$$\frac{d\sigma}{d\Omega} = Z\left(\frac{e^2}{m_e c^2}\right)^2 \frac{1}{2}\,|f(\theta)|^2 (1+\cos\theta) \quad \cdots\cdots (1)$$

with the scattering angle θ, where e, m_e and c are the electron charge, mass and the speed of light. Z is the atomic number and $f(\theta)$ is the atomic form factor which can be approximated below 100 keV as

$$f(\theta) \simeq [1 + k(1-\cos\theta]^{-1} \quad \cdots\cdots (2)$$

with $k = (E_\gamma / m_e c^2)^{8)}$, where E_γ is the photon energy. The coherent photon scattering by protons in the nucleus may be simultaneously taken into account. It can be described in analogy with the atomic case, i.e. in the form of eq. (1), provided the electron mass is replaced by the proton mass. However, its contribution is negligibly small compared to that of the scattering by the atomic electrons, because the proton mass is much larger than the electron mass and the factor in eq. (1) yields $(e^2/M_p c^{2)^2} \ll (e^2/m_e c^2)^2$. Therefore, it is sufficient to take only the coherent photon scattering by electrons into account. For the backward reflection, $\sigma_\pi = [\,d\sigma(\theta)/d\Omega\,]_{\theta=\pi} = r_0^2 = 7.8\times10^{-26} cm^2$ where $r_0 = 2.8 fm$ is the electron classical radius. The number of electrons, Z, involved in the atom should be multiplied as $Z\sigma_\pi$. It should be replaced by $Z\sigma_\pi$ for the number of electrons, Z, involved in the atom. Since the photon is scattered by gold nuclei associated with the thickness of metallic plates, d, the number of nuclei per cm^2 must be considered. This number can be given by nd where n can be obtained from the density divided by the nuclear mass as $n = \rho/M = \rho N_A/28$ with the Avogadro's number N_A.

For bound nuclei, the effect of lattice vibration should be taken into account by the Debye-Waller factor. The Debye-Waller factor[12] at temperature T is given as

$$f_s = \exp\frac{-3E_\gamma^2}{Mc^2 k_B \theta_D}\left[\frac{1}{4} + \left(\frac{T}{\theta_D}\right)\int_0^{\theta_D/T}\frac{t\,dt}{e^t-1}\right] \quad \cdots\cdots (3)$$

with the Boltzmann constant k_B and the Debye temperature θ_D. The γ−ray absorption cross section is given by

$$\hat{\sigma} = 2\pi\lambda_\gamma \frac{2J_f+1}{2J_i+1}\frac{1}{1+\alpha}\gamma^2 \quad \text{(4)}$$

where $\lambda_\gamma = \hbar c/(81keV)$, $J_f = 5/2$, $J_i = 7/2$ and $\alpha = 1.636$ is the internal conversion coefficient.

The fraction of incident photons which are absorbed by ^{133}Cs is $\zeta = (I_0 - I)/I_0 = 1 - \exp(-f_a n_a x \hat{\sigma})$, where f_a is the DW factor of $BaTiO_3$, n_a is the number of ^{133}Cs per cm^2 in the (source) body and x is the thickness of the source body.

Generally, the decay equation is given with the decay constant λ as

$$dN = -\lambda N dt \quad \text{(5)}$$

the solution of which yields an exponential decay curve. If emitted photons return once after being reflected coherently by the metallic surface, the equation of decay is given by the decay constant λ as

$$dN = -\lambda N dt + \zeta\frac{1}{4\pi}\int (fZn_1\sigma_\pi)\lambda N dt d\Omega$$
$$\equiv -\lambda' N dt \quad \text{(6)}$$

where ζ is the photon absorption probability, and $\lambda' = (1-\Sigma)\lambda$. Σ denotes the integral part and should be carried out over the area of the disk.

$$\Sigma = \zeta\int_0^{\theta_0}\cos^2\theta f\cos\theta\frac{nd}{\cos\theta}\sigma_\pi\frac{\sin\theta}{\cos^3\theta}d\theta$$
$$= \frac{1}{2}\zeta f n d\sigma_\pi \ln[1+(\frac{R}{b})^2] \quad \text{(7)}$$

where R is the disk radius, b is the distance between the source nucleus and the plate shown in Figure 1, and $\theta_0 = \text{arc}\, tg(R/b)$.

If such a process repeats m times during the lifetime, the decay constant is found in the form $\tilde{\lambda} = (1-\Sigma)^m\lambda$. m is actually dependent on the distance between the source nucleus and the point of reflection. Let this distance be u, then the time which the photon takes in a round trip is $t_r = 2u/c$. When the photon made its round trips m times, the amount of radioactive nuclei would become a half, i.e.,

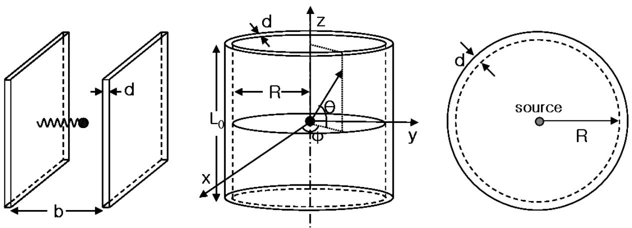

Fig. 1. Geometry of two parallel plates, a cylinder and a sphere

$$\frac{1}{2} = \exp[-\lambda^{(m)} m t_r] = \exp[-\lambda^{(m)} m(\frac{2u}{c})] \quad \text{(8)}$$

By this equation, we can evaluate the value of m as

$$m = \frac{\ln[1-(c\tau_{1/2}/2b)\Sigma]}{\ln(1-\Sigma)} \quad \text{(9)}$$

Since the modified lifetime is $\tilde{\tau}_{1/2} = mt_0$, we find

$$\tilde{\tau}_{1/2} = (\frac{2b}{c})\frac{\ln[1-(c\tau_{1/2}/2b)\Sigma]}{\ln(1-\Sigma)} \quad \text{(10)}$$

The modified decay constant $\tilde{\lambda}$ is actually related to the level width as $\tilde{\Gamma} = \hbar\tilde{\lambda}$ and the half life as $\tilde{\tau}_{1/2} = \ln 2/\tilde{\lambda}$.

By an integral over the cylinder surface Σ is expressed as follows,

$$\Sigma = \zeta\int(\frac{R}{u}n_1 f\sigma_\pi)\hat{\rho}_z dS_z$$
$$= \zeta\int[\cos\psi(\frac{nd}{\cos\psi})f\sigma_\pi](\frac{\cos^2\psi\sin\psi}{4\pi R^2})$$
$$\times\frac{R^2}{\cos^2\psi}d\psi d\theta$$
$$= 2\zeta\frac{2\pi}{4\pi}\int_0^{\psi_L}(ndf\sigma_\pi)\sin\psi d\psi$$
$$= \zeta ndf\sigma_\pi[1-\sqrt{1+(L_0/2R)^2}] \quad \text{(11)}$$

The modified lifetime in the cylinder surface is now found to be

$$\tilde{\tau}_{1/2} = (\frac{2R}{c}) \frac{\ln[1-(c\tau_{1/2}/2R)\Sigma]}{\ln(1-\Sigma)} \quad \cdots\cdots\cdots\cdots (12)$$

Since the surface element on a sphere element of radius R is $dS_R R^2 d\Omega = R^2 \sin\phi d\phi d\phi$, $\rho_R dS_R$ depends only on the solid angle $d\Omega$, and its integral gives the number of photons, which is one on the present case. Thus, the re-excitation coefficient Σ can be obtained as

$$\Sigma = \int (\zeta n d f d_\pi) \rho_R dS_R = \zeta n d f \sigma_\pi \quad \cdots\cdots\cdots\cdots\cdots\cdots (13)$$

with the γ-backward scattering cross section σ_π[11]. Here, we assume that the scatterer is distributed uniformly over the surface of the reflector.

Ⅲ. Numerical Results

Let us examine our theory. As is seen above, the conditions to maximize the effect is how to select material which has large Debye temperature and induces large backward scattering. Furthermore, energy of emitted gamma-ray, E_γ, should be less than $100\,keV$. Otherwise, the Debye-Waller factor becomes very small and, therefore, the effect is greatly reduced. If E_γ is less than $10\,keV$, various noises associated with detectors become large and, therefore, clean data may not be obtained.

Considering these conditions, we try to examine the first excited state $(5/2)^+$ of ^{133}Cs, which is $81\,keV$ level with the lifetime $6.27\,ns$ and is expressed in Figure 2. To eliminate the recoil effect, this nucleus should be implanted in a solid. When a compound $^{133}BaTiO_3$ is taken, ^{133}Ba decays into ^{133}Cs through the electron conversion process because ^{133}Ba is radioactive and, therefore, $^{133}Cs^*$ remains in the compound. Of course, $^{133}Cs^*$ is in the first excited state $(5/2)^+$ and emits the $81\,keV$ gamma ray when it drops into the ground state $(7/2)^+$.

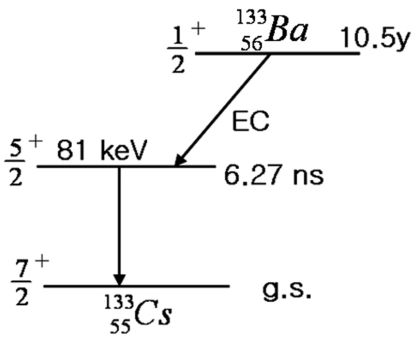

Fig. 2. Decay scheme

The compound $^{133}BaTiO_3$ has a perovskeit structure with a rather high Debye temperature, $\theta_D = 431.8\,K$. Although, the Debye temperature of the perovskeit resulted by the decay of ^{133}Ba into ^{133}Cs is actually not known, it may be assumed to be the same as that of $^{133}BaTiO_3$, because both of them have the perovskeit structure and ^{133}Ba converts merely into ^{133}Cs through the EC process. Therefore, the Debye temperature $\theta_D = 431.8\,K$ is taken for $^{133}CsTiO_3$. Then, the Debye-Waller factor can be found as $f_1 = 0.3434$, 0.3407 and 0.2750 at the temperature $T = 4.2K$, $15K$ and $77K$, respectively.

All cross sections necessary to estimate the gamma absorption probability are obtained by eq. (4) and the relations, $\sigma^N_{pe} = \alpha \sigma^N_\gamma$ and $\sigma^N_{incoh} = (A/Z)^2 \sigma^N_{coh}$, where A and Z denote the nuclear mass number and the atomic number, respectively. σ^N_{coh} has been estimated with eq. (1) where the electron mass was replaced by the proton mass and with the fact that the normalized nuclear form factor is almost unity in the energy region considered here.

The cross sections for the atomic processes should be calculated with the $CsTiO_3$ compound. They can be obtained by using the XCOM Photon Cross Section Database with the modified relativistic form factor.

Table 1. The revised energy level widths and nuclear lifetimes

$T(K)$	parallel plate ($b=0.10\ cm$)			cylinder ($R=0.10\ cm$)			sphere ($R=0.10\ cm$)		
	$\tilde{\Gamma}$ ($10^{-8} eV$)	$\tilde{\tau}_{1/2}$ (ns)	$\Delta\tau/\tau_{1/2}$ (%)	$\tilde{\Gamma}$ ($10^{-8} eV$)	$\tilde{\tau}_{1/2}$ (ns)	$\Delta\tau/\tau_{1/2}$ (%)	$\tilde{\Gamma}$ ($10^{-8} eV$)	$\tilde{\tau}_{1/2}$ (ns)	$\Delta\tau/\tau_{1/2}$ (%)
4.2	7.01	6.50	3.69%	6.80	6.68	6.54%	5.96	7.41	18.2%
15	7.04	6.48	3.35%	6.83	6.66	6.22%	6.02	7.35	17.3%
77	7.26	6.29	0.32%	7.12	6.41	2.23%	6.79	6.69	6.78%

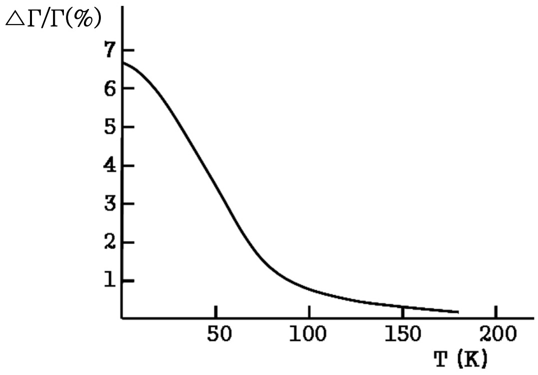

Fig. 3. Temperature dependency of the energy level width for the gold cylinder reflector

The results are suggested in ref. 6, and the absorption probability is found as $\zeta=0.166$, 0.165 and 0.138 at $T=4.2K$, $15K$ and $77K$, respectively.

Gold is also a suitable material as a photon reflector. Its Debye temperature is $\theta_D=165K$, which gives the Debye-Waller factors of 0.150, 0.137 and 0.0102 at $T=4.2K$, $15K$ and $77K$, respectively, for $E_\gamma=81KeV$. Since the modified relativistic form factor is $F(81\ keV,\ \theta=\pi)=3.4057$, the total cross section at $\theta=\pi$ is $\sigma_\pi=0.921\times10^{-24}cm^2$. The density of gold is $18.85(g/cm^{-3})$, and so $n=5.76\times10^{22}cm^{-2}$.

With all the information obtained above, the modified lifetime can be calculated using Eqs. (7), (11) and (13) over the surface of the two parallel plates, an cylinder or sphere, respectively. The metallic sphere has the largest value and it is due to the geometric shape where the emitted γ-rays are backward reflected in all directions and are reabsorbed in the source nucleus. The results are given Table 1. They are within the measurable range.

Temperature dependence of them appears through the Debye-Waller factor of the metallic cylinder and is represented in Figure 3. The energy level width of the 81-keV level of ^{133}Cs is shown to become narrower by 6.5% at 4.2 K. At room temperature, the width and lifetime do not change.

Ⅳ. Conclusion

As is seen above, the decay constant λ is altered step by step at every stage of γ reabsorption. λ is actually related to the level width as $\Gamma=\hbar\lambda$. Therefore, the process definitely changes the level width, and equivalently the halflife. Our results imply that the accuracy in Mössbauer measurements can be improved by setting both the source and the absorber, respectively, between two plates. Spatial structure of vacuum field can also make a change of atomic and nuclear lifetimes[8]. However, in that case, plausible effects appear only for much smaller separation between two plates. Therefore, it is negligible in the present investigation. Other processes, so called "radiation trapping", were also investigated and the prolonged nuclear lifetime was observed. Their interpretation is that the time evolution of gamma-ray emission is modulated as a result of time consuming during photon-exchange between two radioactive nuclei. Namely, the photon is delayed to come out of the system while two nuclei play with the photon. However, it has no relevance to the lifetime unless the energy level width is modified. The lifetime could be changed only when population of the excited state would increase through the mechanism discussed above. The radiation trapping can occur even without modification of energy levels but such a simple “trapping” cannot make any effect on the Mössbauer spectrum connected directly to the energy level width. As is mentioned above, the processes proposed here ought to take place only in recoilless manner and, therefore, the radioactive source nucleus should be implanted in a solid to cut off recoil and measurement is desirable to be carried out at low temperature with specific detectors with good timing performance as well as energy resolution.

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• 국문초록

유한한 공간에서 ^{133}Cs 원자핵 반감기의 변화에 대한 연구

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감마선 원천핵을 두 개의 금속 평행판 사이, 실린더나 구와 같은 유한한 공간 내부에 놓았을 때 에너지 준위폭이 어떻게 변경되는가에 대하여 이론적으로 조사했다. 2개의 금속판에 의해 뒤쪽 산란된 감마선이 원천핵에 재흡수됨으로써 ^{133}Cs핵의 여기상태(81 keV)의 에너지폭은 평행판 간격이 1.0 mm일 때 4.2 K에서 3.7% 정도 감소됨을 볼 수 있었다. 여기서 금속판은 두께 0.5 mm, 반경이 3 cm이며 금으로 만들었다. 반경 1.0 mm, 두께 0.5 mm, 길이 5 cm의 실린더 금판에 의해 뒤쪽 산란되었을 때에는 4.2 K에서 6.5% 에너지 폭이 좁아지는 결과를 얻었다. 또한, 두께 0.5 mm, 반경 1.0 mm의 금으로 된 구 안에 원천핵을 놓았을 때 4.2 K에서 18.2% 에너지준위 폭이 감소하였다. 이러한 에너지 준위 폭의 감소는 그 준위의 반감기가 연장된 것을 의미한다.

중심 단어 : 반감기, 에너지준위, 감마선