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Study on Stress Transfer Property for Embedded FBG Strain Sensors in Concrete Monitoring

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Abstract: Fiber Bragg grating (FBG) sensors already have been the focus for structural health monitoring (SHM) due to their distinguishing advantages. However, as bare optical fiber is very fragile, bare FBG strain sensor without encapsulation can not properly be applied in practical infrastructures. Therefore encapsulation techniques for making encapsulated FBG strain sensor show very important in pushing forward the application of FBG strain sensors in SHM. In this paper, a simplified approximate method to analyze the stress transferring rules for embedded FBG strain sensors in concrete monitoring is put forward according to mechanics of composite materials. Shear lag theory is applied to analyze the stress transferring rule of embedded FBG strain sensor in measured host material at the first time. The measured host objects (concrete) and the encapsulated FBG strain sensor are regarded as a composite, and then the stress transfer formula and stress transfer coefficient of encapsulated FBG strain sensor are obtained.

Keywords: fiber Bragg grating strain sensor, stress transfer rule, transfer coefficient, shear lag theory.

1. Introduction

Fiber Bragg grating (FBG) sensors, already have been the focus for structural health monitoring (SHM) due to their distinguishing advantages: electro-magnetic resistance, small size, resistance to corrosion, multiplexing a large number of sensors along a single fiber, etc. At present, FBG strain sensor has been widely investigated and applied in infrastructures,[1-5] since Hill[6] and Meltz[7] developed the FBG fabrication techniques. Aftab[2] even described its wonderful future in the field of innovation infrastructures.

Since bare optical fiber is very fragile, in practical infrastructures bare FBG sensor without encapsulation can not be applied directly as it can not adapt to the rudeness of construction. Therefore, we have to develop special in-situ installation and protection techniques for bare FBG sensors. Unfortunately, the ‘perfect’ installation technique often conflicts with the in-situ construction, or it can not meet the demand of critical schedule of construction. So the encapsulation techniques, which are used to make encapsulated FBG strain sensor, show very important in pushing forward the application of FBG strain sensors in structural health monitoring.

The mechanical theory of encapsulating FBG strain sensors has been discussed in literature,[8] where the exact solution has been obtained, but the result is so complicated that it is unsuitable for encapsulated FBG strain sensor design and in-situ application in civil engineering. In this study, considering the convenience of encapsulated FBG strain sensor design and in-situ application, a simplified approximate method is put forward to analyze the stress transferring rules for FBG strain sensors embedded in concrete based on shear lag theory, according to mechanics of composite materials. Based on the obtained results, the sensitivity of encapsulated FBG strain sensor can be improved by increasing transferring coefficient of coated FBG strain sensors by choosing coating material with appropriate physical parameters.

2. Basic assumptions

Shear lag theory was firstly put forward by Roser[9] to analyze the stress transfer in composite material along longitudinal direction, which assumes that matrix only transfers shear stresses. Hence, stress transfer formula can be derived easily by equilibrium conditions. Though it is not so accurate like elastic theory, it is a simple method to analyze stress transfer rules of FBG strain sensor embedded in concrete.

As shown in Fig. 1, FBG strain sensor is covered by coating materials, and the stress of concrete is transferred to fiber through covering materials. In Fig. 1, $\overline{\sigma}$ is the average equivalent stress of the composite materials composed of concrete, coating layer and fiber, r_a, r_c and r_f are the radius of concrete matrix, coating layer and fiber, respectively. By applying shear lag theory here, some necessary assumptions should be given below:

1) Outer of FBG strain sensor: In the longitudinal direction of

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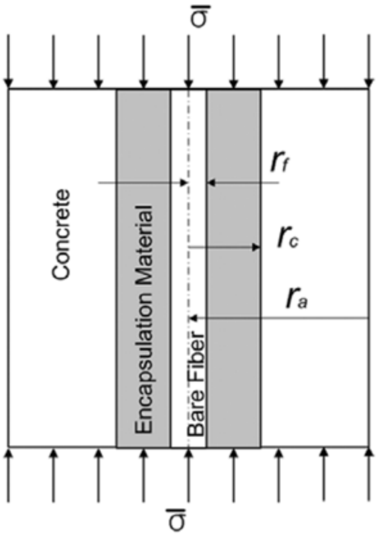


Fig. 1 Schemes of encapsulated FBG sensor embedded in concrete under loading state.

fiber, the axial stress of coating material is transferred by the shear stress of interface between concrete matrix and coating material.

2) Inner of FBG strain sensor: In the longitudinal direction of fiber, the axial stress of FGB sensor is transferred by the shear stress of interface between fiber and coating material.

3) The concrete matrix outside the FBG strain sensor has average performance.

3. Theoretical derivation

At first, the stress that transfers from concrete to coating layer is discussed. Because the radius of fiber is much small, approximately, we think fiber has no influence on the stress transfer between concrete matrix and coatings. And then the composite material is only composed of concrete and coatings without the consideration of fiber.

A unit element of coating material is shown as Fig. 2 cut from Fig. 1, in which, τ_c is the shear stress of the interface between concrete and coatings, σ_c is the axial stress applied on coating material. According to equilibrium conditions, Eq. (1) can be obtained below:

$$(\pi r_c^2)\sigma_c + \tau_c(2\pi r_c \mathrm{d}z) = \pi r_c^2(\sigma_c + \mathrm{d}\sigma_c) \tag{1}$$

And Eq. (1) can be rewritten as:

$$\frac{\mathrm{d}\sigma_c}{\mathrm{d}z} = \frac{2\tau_c}{r_c} \tag{2}$$

Eq. (2) shows the linear relationship between the axial stress on coating materials along z direction and the shear stress of the interface.

According to strength criteria, the below relation exists in composite materials:

$$\bar{\sigma} = (1-\omega)\sigma_a + \omega\sigma_c \tag{3}$$

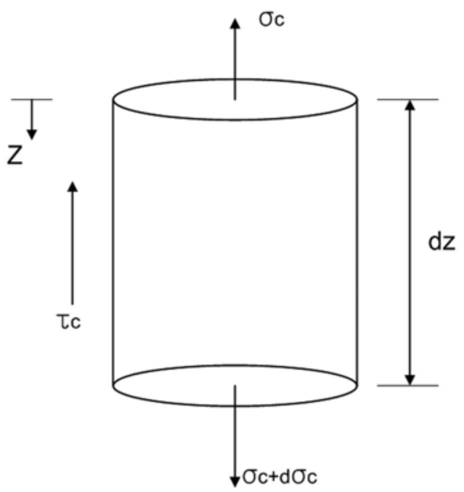


Fig. 2 Unit of coating material without considering fiber.

where $\omega = r_c^2/r_a^2$. σ_a can be obtained by solving Eq. (3):

$$\sigma_a = \frac{r_a^2\bar{\sigma} - r_c^2\sigma_c}{r_a^2 - r_c^2} \tag{4}$$

At the interface of concrete matrix and coating material, there is a shear layer bonded with concrete matrix outside the cylindrical coating material. Its radius is denoted as r_b, which is generally equal to 0.1~0.2 of the radius of coating material. The shear strain γ of concrete matrix can be determined by the relative displacement of matrix and cylindrical coating material following:

$$\gamma = -\frac{\mu_a - \mu_c}{r_b - r_c} \tag{5}$$

where, μ_a is the average displacement of composite matrix, μ_c is the displacement of cylindrical coating material. Stress transfer within cylindrical coating material is through the micro-deformation of concrete matrix. Therefore, coating material and matrix are considered to be in elastic state. Derivating Eq. (5) with respect to *z* by first order, and then substituting into the constitutive relation give:

$$\frac{d\tau_c}{dz} = \frac{G_a}{r_b - r_c}\left(\frac{\sigma_c}{E_c} - \frac{\sigma_a}{E_a}\right) \tag{6}$$

where E_a and G_a are the Young's modulus and the shearing modulus of matrix, respectively and E_c is the Young' modulus of coating material. According to the theory of composite material mechanics, E_a can be derived as below:

$$E_a = \omega E_c + (1-\omega)G_a \tag{7}$$

Derivating Eq. (2) with respect to *z* gives:

$$\frac{d^2\sigma_c}{dz^2} = \frac{2}{r_c}\cdot\frac{d\tau}{dz} \tag{8}$$

Substituting Eqs. (4) and (6) into Eq. (8), then a two-order ordinary differential equation about the stress of coating material is given below:

$$\frac{d^2\sigma_c}{dz^2} - \rho^2\sigma_c + \delta^2\bar{\sigma} = 0 \tag{9}$$

where

$$\rho^2 = \frac{2G_a}{(r_b - r_c)r_c}\left[\frac{1}{E_c} + \frac{r_c^2}{E_a(r_a^2 - r_c^2)}\right] \tag{10}$$

$$\delta^2 = \frac{2G_a}{(r_b - r_c)r_c}\cdot\frac{r_a^2}{E_a(r_a^2 - r_c^2)} \tag{11}$$

with Eqs. (9), (10), and (11) and corresponding initial conditions, one can easily solve the stress distribution along z direction for the coating material.

In what follows, the stress transfer from the coating layer to FBG strain sensor will be analyzed. The rule of stress transfer in composite material composed of coating layer and fiber is same as that in composite made of concrete and coating layer. Therefore a two-order ordinary differential equation about stress of fiber can be obtained similarly:

$$\frac{d^2\sigma_f}{dz^2} - \rho_f^2\sigma_f + \delta_f^2\bar{\sigma}_c = 0 \tag{12}$$

where σ_f is the stress of fiber transferred from the coating layer, $\bar{\sigma}_c$ is the average equivalent stress of composites consisting of fiber and coating layer,

$$\rho_f^2 = \frac{2G_c}{(r_{b1} - r_f)r_f}\left[\frac{1}{E_f} + \frac{r_f^2}{E_c(r_c^2 - r_f^2)}\right] \tag{13}$$

$$\delta_f^2 = \frac{2G_c}{(r_{b1} - r_f)r_f}\cdot\frac{r_c^2}{E_c(r_c^2 - r_f^2)} \tag{14}$$

where G_c is the shearing modulus of the coating material, r_{b1} is the radius of shear layer bonded with coating material outside the fibre. The solution to Eq. (12) is the stress distribution along its longitudinal direction of fiber within composite.

Since concrete and fiber are continuous media, and there is no normal stress at the two ends of the fiber, the equivalent average stress applied on composite composed of coating material and fiber equals the axial stress applied on coating layer.

$$\bar{\sigma}_c = \sigma_c \tag{15}$$

As the FBG sensing part is very short (always within several centimeters), stresses in fiber and concrete membrane within the sensing length can be considered approximately same, this yields:

$$\frac{d^2\sigma_c}{dz^2} = \frac{d^2\sigma_f}{dz^2} = 0 \tag{16}$$

based on Eqs. (9), (12), and (16), we can obtain:

$$\sigma_f = \left(\frac{\delta}{\rho}\right)^2\cdot\left(\frac{\delta_f}{\rho_f}\right)^2\cdot\bar{\sigma} = \alpha\bar{\sigma} \tag{17}$$

where α is defined as transferring coefficient of coated FBG strain sensors, from Eqs. (10), (11), (13), (14), and (17), α can be expressed as:

$$\alpha = \left(\frac{\delta}{\rho}\right)^2\cdot\left(\frac{\delta_f}{\rho_f}\right)^2 = \frac{r_a^2 r_c^2 E_c E_f}{[E_f r_f^2 + E_c(r_c^2 - r_f^2)]\cdot[E_c r_c^2 + E_a(r_a^2 - r_c^2)]} \tag{18}$$

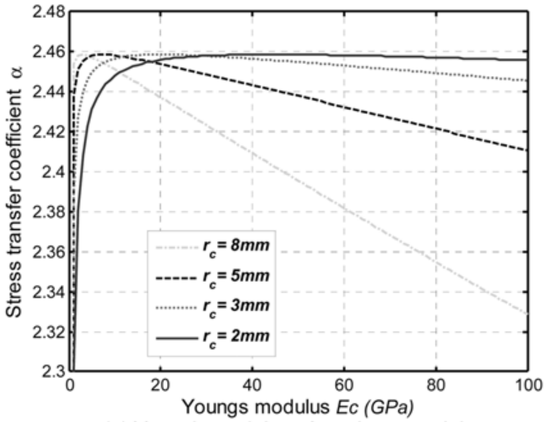
considering $r_c^2 \gg r_f^2, r_a^2 \gg r_c^2$, Eq. (18) can be simplified as:

$$\alpha = \left(\frac{\delta}{\rho}\right)^2\cdot\left(\frac{\delta_f}{\rho_f}\right)^2 = \frac{r_a^2 r_c^2 E_c E_f}{[E_f r_f^2 + E_c r_c^2]\cdot[E_c r_c^2 + E_a r_a^2]} \tag{19}$$

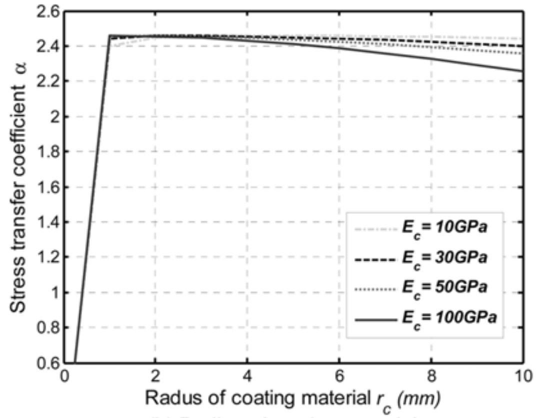
4. Analyzing and discussions

According to Eq. (17), the stress in FBG strain sensor is linear with the average equivalent stress of concrete members, and the more the transferring coefficient (α) is, the higher the stress sensing sensitivity of FBG strain sensor is. Once FBG strain sensor and measured objects are determined, stress transferring coefficient is only related with the size and the Young's modulus of coating material. Appropriately choosing coating material including its size and Young's modulus can improve the sensitivity of FBG strain sensor by increasing the stress transferring coefficient, and get more accurate monitoring data. In order to quantitatively analyze the relationship between stress transfer coefficient and physical parameters of coating material, following calculation and discussion are made, where it assumes that the fiber of FBG strain sensor is single-mode fiber, and its radius $r_f = 62.5$ μm, Young's modulus $E_f = 74$ GPa, the radius of monitored concrete member $r_a = 6$ cm, and concrete modulus $E_a = 30$ GPa, respectively.

When the radius of coating material is determined as 2 mm, 3 mm, 5 mm, and 8 mm respectively, according to Eq. (19), the relation curve between stress transfer coefficient α and the Young's modulus of coating material can be plotted in Fig. 3(a). Seen from Fig. 3(a), even the radius of coating layer changes, the maximal transfer coefficients $\alpha_{\max} = 2.459$ can be reached at different radius. With the decrease of the radius of coating material, transfer coefficient is always high after the initial increasing. Therefore, during the process of FBG strain sensor design, the sensitivity of FBG strain sensor can be improved by increasing stress transferring coefficient via choosing coating material with appropriate Young's modulus according to Eq. (19) once all other parameters of sensors are determined. Fig. 3 (b) shows the relationship between stress transferring coefficient α and radius of coating material when the Young's modulus of coating is determined. For clarity, the detailed part of Fig. 3(b) that the range of radius from 0 to 1 mm and the range from 1 mm to 10 mm are shown in Fig. 4(a) and (b), respectively. Base on Fig. 3 (b), Fig. 4 (a) and (b), for the four different Young's modulus



(a) Young's modulus of coating material



(b) Radius of coating material

Fig. 3 Relations between coefficient α and (a), (b).

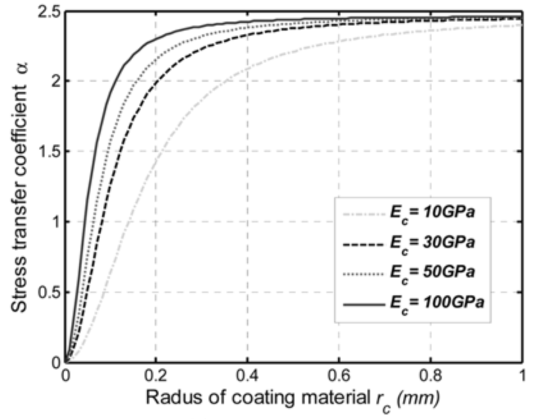
coating material, all of them can reach the maximum coefficient $\alpha_{\max}$ = 2.459. During the process of FBG strain sensor design, the sensitivity of FBG strain sensor can be improved via choosing coating material with appropriate radius based on Eq. (19) once all other parameters of sensors are determined.

According to Figs. (3) and (4), once one of the parameters of coating material (size or Young's modulus) is fixed during packaged FBG strain sensor design, another parameter can be optimally chosen based on Eq. (19) to reach the maximum stress transferring coefficient.

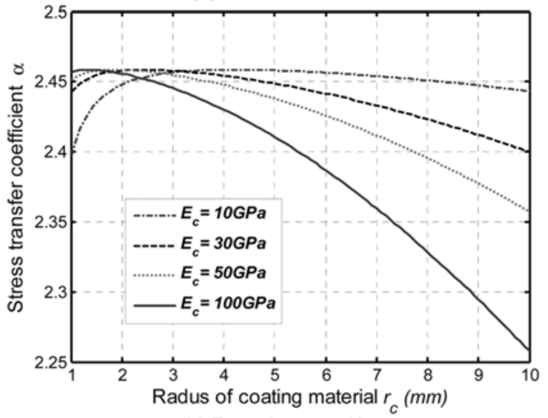
Additionally, one point that should be paid attention is that stress transfer coefficient of sensor has a constant maximum $\alpha_{\max}$ = 2.459 when Young's modulus and size of fiber and concrete are defined. However, the Young's modulus or size of coating material varies. In turn, stress transfer coefficient is related with the product of Young's modulus and radius of coating material. Fig. 5 (a) and (b) give the varying curve of stress transfer coefficient with the varying $E_c r_c^2$. When $E_c r_c^2$ is in the range: 1.56×10^5 N and 2.0×10^5N, $\alpha_{\max}$ is 2.459, which is same with above results.

In brief, once the properties of monitored object and fiber are determined, the relationship between stress transfer coefficient and $E_c r_c^2$ can be obtained by Eq. (19), and then the design of package FBG strain sensor can be optimized to get accurate measuring results.

To demonstrate the results in this paper, in the near future, coated FBG strain sensors with different coating materials and different physical parameters will be fabricated, and then they will be embedded in concrete beam specimens to test and compare their stress transfer coefficient by beam loading test.

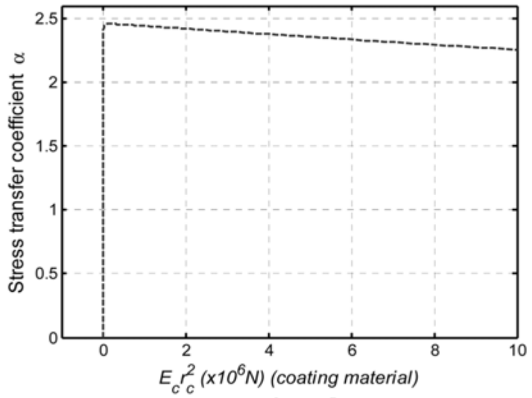


(a) From 0 and 1 mm

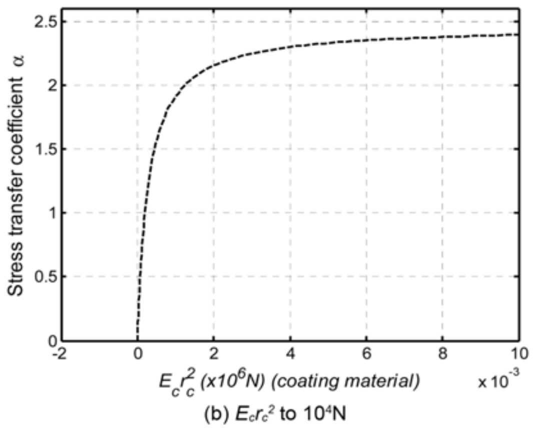


(b) From 1 mm to 10 mm

Fig. 4 The details of Fig. 3 (b) within the radius range.



(a) $E_c r_c^2$ to 10^7N



(b) $E_c r_c^2$ to 10^4N

Fig. 5 Relations between coefficient α and $E_c r_c^2$ of coating material (in details).

5. Conclusions

According to shear lag theory, a simplified approximate method is put forward to analyze the stress transferring rules for embedded encapsulated FBG strain sensors by mechanics of composite materials. Based on the results obtained in this research, during the process of embedded encapsulated FBG strain sensor design, the sensitivity of FBG strain sensor can be improved by increasing stress transfer coefficient among concrete and coated FBG strain sensor via choosing coating material with appropriate physical parameters such as Young's modulus or its size.

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