

광고 효과 확장 코익 모델을 이용한 Aggregated data bias의 재조명

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Re-Considering Aggregated Data Bias by Extending “Koyck Model” of Advertising Effect

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■ Abstract ■

“How does advertising affect sales?” is the fundamental issue of modern advertising research. There is an interesting issue for estimating carryover effects of advertising on sales, and the aggregated data biases exist in the duration of advertising effect.

This research suggests an extended model of Koyck Model which is employed for micro-data (Koyck 1954) to estimate aggregated advertising data, and empirically shows the aggregated data bias.

Our developed model with the aggregated level of actual advertising data is more appropriate than the basic Koyck model for micro-data. The result figures out that it is important to consider the disaggregated data level in the analysis of dynamic effects of advertising such as carryover effects.

Keyword : Koyck Model, Aggregation Bias, Carry-Over Effect, Disaggregated Data Level

1. Introduction

One of the commonly raised questions in busi-

ness world is “how much should we spend on advertising?” and in order to answer this question adequately, we need to consider a funda-

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mental issue like “How does advertising affect sales?” Therefore, the study of relationship between promotional efforts and sales response has been one of the cornerstones of advertising research.

The description of nature of the current and cumulative effects of advertising has long been recognized by both managers and public policy-makers as an important but difficult issue in marketing. There are many theory-based normative models (Sethi, 1979), however, only a few models contain the dynamic process of carryover effect. As early as the 1950s and 1960s, significant amount of research began to focus on the carryover or ‘lagged’ effects of advertising in one period on the sales in subsequent period (Jastram, 1955; Palda, 1964). The magnitude and duration of this carryover effect, as well as its estimations, play an important role in advertising budget setting. However, until recently carryover effect was not precisely specified in most models and the coefficient of determination was not always reported (Assmus, Farley, and Lehmann, 1984).

It is well known in marketing and econometrics that temporal aggregated data bias has estimated in the duration of advertising effect researches (e.g., Clarke, 1976, 1982). The bias is due to the specification error caused by performing the aggregated data (annually, monthly) in the original advertising response model which estimated with real micro-data (real interval of advertising) (Bass and Leone, 1983, Rao, 1986, Weinberg and Weiss, 1982).

In this research, we address this issue by presenting an optimal better solution of the aggregation bias. The purpose of research is 1) to develop a optimal model for aggregated advertis-

ing data by extending the classical ‘Koyck model’ (Koyck, 1954), 2) comparing the fit of the original Koyck model with our developed models and 3) empirically revealing aggregation biases with actual Dairy advertising expenditure data.

2. Literature review

2.1 Advertising-sales effect (Carryover Effect)

Advertising is an important tool that organizations communicate with their both current and potential customers, (Bendixen, 1993). “*This is clearly an important managerial issue given the human and dollar resources spent on advertising*” (Leone, 1995, p. 141), since measuring effectiveness of advertisement expenditures on sales is the biggest concern in practice, many researchers pay attention to the shape of response function of advertisements on sales. There was a variety of econometric models identified how advertising affects sales which estimated parameters of general demand functions from early 1960s (Assmus, Farley and Lehmann, 1984; Bendixen, 1993).

Moriarty (1983) argues that current expenditures on marketing activities often do not have entire impact on sales in the same period in which they are implemented. In other words, the impact of current activities may persist for many future time periods. Kluver and Brodie (1987) indicate that lagged effect of advertising is more significant than other marketing activities. Assmus, Farley and Lehmann (1984) assessed the short-term and long-term effect of advertising on sales through performing a Meta-Analysis of 128 econometric models developed before 1981. They indicated that an appropriate model in these sit-

uations generally should incorporate exogenous factors and carryover effects (or delayed effects).

The delayed response effect arises because of the lapse of time between the sense of a firm's advertisement and the occasion on which a consumer makes a purchase behavior of the firm's product (Moriarty 1983). Some studies regard advertisement's "carry-over effect" as the delayed response effect and customer holdover effect. (Lilien, Kotler, 1983; Simon and Arndt, 1980; Little, 1979, Hahn et al., 1992).

A number of technical issues related to the measurement of these duration intervals have been studied in many previous literatures (Bass and Clarke, 1972, Clarke, 1973, Palda, 1964, Weiss and Windal, 1980) and they note on measuring advertising carryover effects. Many researchers try to measure the cumulative effects of advertising expenditures with some forms of lagged-variable response models e.g., (Weiss, Weinberg, and Windal, 1983). Sales and advertising statistics are typically aggregated for monthly intervals, although the measurement of advertising effectiveness relates to the long term rather than to specific campaigns (Bendixen, 1993, p21). In other words, sales response to advertising focuses on the magnitude and the duration effect of an advertising impulse in future sales.

Therefore, "*How long the carryover effect of advertising on sales persist?*" become a key research issue. Clarke (1976) points out for low priced products, the carryover effect of advertising lasts a matter of months rather than years. Moriarty (1983) provides preliminary evidence that for some durables, advertising effects may have a duration interval that exceeds a year. Givon and Horsky (1990) indicate the carryover effect with several product categories using month-

ly and bimonthly measurement periods. Thereby, the parameter estimation with different level of aggregation (annually, monthly) becomes an important issue in the advertising carryover effect studies. However, most research just focus on the actual interval of aggregated advertising data (annually, monthly) which they collected. As a result, aggregation bias is given rise, and the actual advertising effect could not be estimated precisely.

2.2 Aggregation Bias Estimation

A number of technical issues related to the measurement of these duration intervals have been studied in many previous literatures. Understanding how to adjust the estimation of the duration level is helpful to get a more accurate insight of the true impact of advertising on sales (Leone, 1995). The focus of previous research has primarily been on (a) the influence of disturbance structures and their characteristics on carryover effects estimation (Houston and Weiss, 1975, Clarke and McCann, 1977, Weiss and Windal, 1980), and (b) the testing and evaluation of the various distribution lag functions (Bass and Clarke, 1972, Weiss and Windal, 1980). Less attention has substantially been devoted to the relationship between model specification and the time intervals of the observations, even though there are empirical findings contained in these researches. Researchers have proposed several approaches for retrieving the true parameters from actual aggregated advertising data.

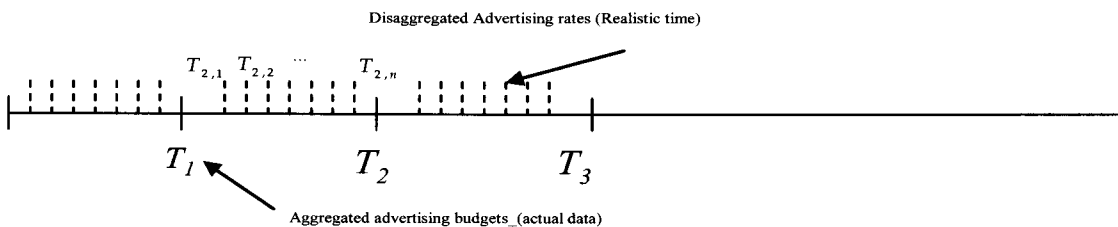
Clarke (1976) firstly observed the systematic relationship between the estimation of the duration interval and the level of temporal aggrega-

tion. He undertook almost 70 studies (weekly, monthly, bimonthly, quarterly, or annual data) to calculate the estimate of duration interval (Leone, 1995). He identified upward bias of the coefficient for the current advertising when omission of carryover effect constituted misspecification. He also documented a serious bias that : “Due to the longer purchase cycle than actual data interval, the weekly results are biased downward” (p.355). A notion he seeded that the purchase interval was the true data interval, however, using this small disaggregated data was not good for measuring carryover effects. Aggregated data (monthly) causes greatly exaggerated carryover effects, though monthly data are appropriate to measure advertisement’s carryover effect (Clarke, 1976). No one since has refuted these suspicions. Rather, one of his suspicions seems to have become part of the axioms in marketing. Seven years later, for example, Weiss et al. (1983, p.279) said, “An emerging convention seems to consider ... [the true micro interval] ... to be the interval between purchases.” Some research have investigated the temporal aggregation bias, their approaches suggested to minimize the data interval bias either through a continuous (blattberg and Jeuland, 1981; Rao, 1986) or discrete time framework (Bass and Leone, 1983, 1986; Weiss et al., 1983; Vanhonacher, 1983, 1987; Russell, 1988;

Givon and Horsky, 1990).

It becomes a marketing science issue on marketing generalizations. Leone (1995) reiterated an earlier recommendation from Bass and Leone that “if one should choose the data interval used in the analysis, one should select the interval corresponding to the inter-purchase time for the product category” (Leone 1995, p. G47). Before 1980s, research had attempted to develop the micro period aggregate specification with partial adjustment model (Theil, 1954; Mundlak, 1961; Moriguchi 1970), but most of these researches focused on the annual data estimation. Weiss, Weinberg and Windal (1983) developed a micro-level brand loyal model to estimate the narrow aggregation (monthly) of carryover effect, and they argued their model dealt well with aggregation bias. Vanhonacker (1983) among others pointed out that neither mathematical proof nor any theory can support this premise. The econometrics literatures have also not provided any answer about the optimal data interval.

Koyck (1954) introduced a distributed lagged effect with this formulation. In a reduced form, the model is : $S_t = a + \lambda S_{t-1} + bA_t + \epsilon_t$. Clark (1976) provided an excellent review with the substantial number of Koyck model application in 1974. Since the development of the Koyck model in 1954, considerable advances have been made in



[Figure 1]

econometric estimation techniques. In this research, we extend the basic Koyck model enable to estimate the advertising carryover with actual advertising expenditure data and reveal aggregated bias empirically. We extend the basic Koyck model in the following ways : First is to transform the basic model enable to deal with the unknown initial advertising budget with predicted sales; Second is to take order with the actual data interval by considering the disaggregated advertising rate and then estimate the carryover effect.

3. Model specification

3.1 Basic Koyck Model

$$S_t = \alpha + \beta_0 \cdot A_t + \beta_1 \cdot A_{t-1} + \dots + \beta_{t-1} \cdot A_1 \quad (1)$$

Equation (1) is the underlying structure of the basic (finite) Koyck Model.

The most important assumption of Koyck Model is Equation (2) which has the constant decay parameter (λ).

$$\beta_{i+1} = \lambda \cdot \beta_i \quad (2)$$

Based on Equation (1) and (2), we can get the original Koyck Equation (3) which is sales response model for the specific advertising budget.

$$S_t = \alpha \cdot (1-\lambda) + \beta \cdot A_t + \lambda S_{t-1} + \varepsilon_t \quad (\beta_i = \beta, \forall t) \quad (3)$$

3.2 Extended Koyck Model

There are some limitations with Equation (3). Firstly, basic Koyck Model represents the carry-over effects of advertising rates, but can't

represent the delayed-response effects. Since delayed-response effects describe the situation where one variable is correlated with the values of another lagging variable at later times, we cannot expect to get the biggest advertising effects at that expected time of periods but only after having time lags.

Secondly, general temporal aggregate data just includes monthly advertising budgets and sales, however, the actual advertising does not always launch at the last day of month, actually, the actual advertising rate is hard to observe. The aggregated bias occurred when carryover effect is estimated by basic Koyck model using actual monthly data. In this reason, we try to resolve this problem with the Extended Koyck Model.

[Figure 1] shows the relationship between advertisement budgets and advertisement rates. $T1, T2, T3 \dots$ represent the time intervals and they contain several advertisement rates in each interval. If this company runs daily advertisements, then T is 30. We emphasize both the aggregated advertising budgets and disaggregated advertising rates.

Suppose each interval has the same number of advertisement. (e.g. daily = 30, weekly = 4). Then, advertisement budget can be aggregated by Equation (4).

$$A_{T_n} = A_{T_n,1} + A_{T_n,2} + \dots + A_{T_n,k} \quad (4)$$

T_n : The n th actual data interval

k : The number of advertising in the T_n

A_{T_n} : Aggregated advertising budgets for T_n

$A_{T_n,k}$: k^{th} Disaggregated advertising in the T_n

Generally, firms usually have record of daily or weekly advertising budgets, and the sales are

reported by month. Based on Equation (4), we extend the Koyck model and try to resolve the bias caused by aggregated advertising budgets. Equation (5) shows the Koyck equation for the unobserved disaggregated sales responses and observed advertising budgets.

$$\begin{aligned}
 S_{T_n,1} &= \alpha + \lambda \cdot S_{T_n-1} + \beta \cdot A_{T_n,1} \\
 S_{T_n,2} &= \alpha + \lambda^2 \cdot S_{T_n-1} + \beta \cdot A_{T_n,2} + \beta \cdot \lambda \cdot A_{T_n,1} \\
 &\vdots \\
 S_{T_n,k} &= \alpha + \lambda^k \cdot S_{T_n-1} + \beta \cdot A_{T_n,k} + \beta \cdot \lambda \cdot A_{T_n,k-1} + \dots \\
 &+ \beta \cdot \lambda^{k-1} \cdot A_{T_n,1} : k^{\text{th}} \text{ Disaggregated sales} \\
 &\text{response in the } T_n \dots
 \end{aligned} \quad (5)$$

Our dependent observation variable is aggregated sales response (S_{T_n}). Therefore, we need to aggregate the disaggregated sales responses in the Equation (5). Similar to Equation (4), we can aggregate them by Equation (6).

$$S_{T_n} = S_{T_n,1} + S_{T_n,2} + \dots + S_{T_n,k} \quad (6)$$

Finally, using Equation (5) and Equation (6), we can achieve the final equation to consider the disaggregated advertising budgets.

$$\begin{aligned}
 S_{T_n} &= (\lambda + \lambda^2 + \dots + \lambda^k) \cdot S_{T_n-1} + \beta \cdot A_{T_n,k} + (1 + \lambda) \\
 &\cdot \beta \cdot A_{T_n,k-1} + \dots + (1 + \lambda + \dots + \lambda^{k-1}) \cdot \beta \cdot A_{T_n,1}
 \end{aligned} \quad (7)$$

Equation (7) is our final proposed model to estimation carryover effect by considering disaggregated level of advertising effect.

4. Data collection

The data for the study was provided by a ma-

ajor dairy product company in Korea. In this research, for simple model estimation we only use the product sales, advertising expenditure and seasonal factors.

<Table 1>

	Content	Period	Analysis Unit
Sales	Product sales	Jan., 05~ Mar., 08	Monthly
Advertising	Advertising expenditure	Jan., 05~ Mar., 08	Monthly Weekly

The data set we using in this study is summarized as <Table 1>, which contains actual monthly sales and monthly and weekly advertising expenditures of one dairy product from January 2005 to March 2008. Dairy products is a frequent purchasing product category, so that this industry is relatively stable. Due to the tough competition, advertising is recognized as an important marketing tool to marketing managers. This is the reason why firms spend a lot of money on advertisement continuously.

The central reason we employ this data set is that there is no price promotion during this dairy product distribution. Thus, we assure that advertising is the main factor to affect sales in our data, and the other factors of marketing mix are controlled appropriately.

5. Analysis and Result

We test 4 models (Model 1-basic Koyck model, Model 2-extended Koyck model, Model 3-basic Koyck model with seasonal variables, Model 4-extended Koyck model with seasonal variables) which are already explained in the Model

specification Section. In this test, we use disaggregated weekly advertising expenditure for extended Koyck Model, but we use aggregated monthly advertising expenditure for basic model. In addition, to consider the decreasing marginal return, we take the log for advertising budget. Carryover effect parameters estimation (λ) and model fits (SSE) are summarized in <Table 2> below :

<Table 2> Model Fit

	λ	SSE	MAD	Adjusted R^2
Model 1	0.517*	51,545	3384	0.439
Model 2	0.327*	44,850	3028	0.466
Model 3	0.517*	29,762	2433	0.646
Model 4	0.316*	25,832	2169	0.661

Note : * : significant at $p < 0.01$.

The result shows that carryover effect parameter λ s of four models are estimated significantly ($P < 0.01$). From the SSE value, we realize that the fit of Model 2 (extended Koyck model) is about 13% (51,545 vs. 44,850) better than Model 1 (basic Koyck model), meanwhile, Model 4 (extended Koyck model with seasonal dummy) is about 13% (29,762 vs. 25,832) better than Model 3 (basic Koyck mode with seasonal dummy). This indicates that extended Koyck model fit better than basic Koyck model, furthermore, model fit gets better by considering seasonal factor, which indicates that seasonal factor is quite important to be considered for advertising effect research of diary product. So we can make a conclusion that analyzing the aggregated data by using the extended model which considered the disaggregated level of actual data is more appropriate than using the basic

Koyck model for micro-data.

In order to make further improvement on our result, week carryover effect (λ_w^k) is compared to month carryover effect (λ_m^k), the result is presented in table 3 as below :

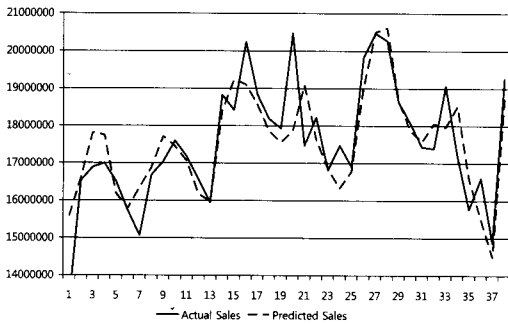
<Table 3> Estimated carry-over effect

	λ_w^k	λ_m^k
1	0.316	0.517
2	0.100	0.267
3	0.032	0.138
4	0.010	0.071
5	0.003	0.037
6	0.001	0.019
7	0.000	0.010

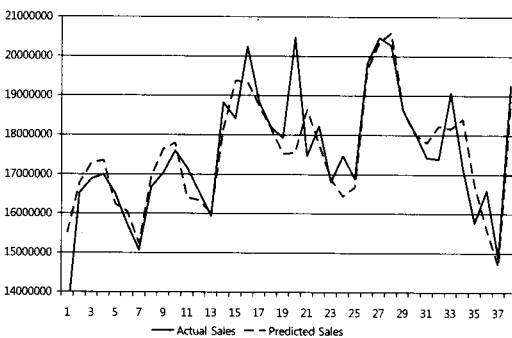
Carryover effect (λ) is estimated by Model 4 with weekly advertising expenditure and monthly advertising expenditure respectively, and the dependent variables are the same monthly sales. In the weekly advertising data estimation, the advertising effect would be continued to seventh week from its exposure, the estimation is make sense in practice, however, the advertising effect would be maintained more than 7 months with monthly advertising data, carryover effect is overestimated apparently. Therefore, this comparison proves our argument of the important of considering disaggregated level of actual data to estimate carryover effect again.

Same result can be observed through the prediction performance figures of Model 3 ([Figure 2]) and Model 4 ([Figure 3]) presented below. The real line represents the actual sales, and the dot line shows the predicted sales. We can observe that [Figure 3] performs slightly better [Figure 2], in other word, the extend Koyck model estimates carryover effect more precisely than

basic Koyck model. As a result, we believe our extended model can capture the effectiveness of advertising on sales better.



[Figure 2] Model 3



[Figure 3] Model 4

6. Conclusion

6.1 Implication

From the previous section, we learn that the extended model which considered the disaggregated level of actual data is more appropriate than the basic model for micro-data. Additionally, we can confirm that aggregated bias is due to the specification error caused by assuming that the true model at the micro-data applies as well to the aggregate data. Furthermore, our model finds the carry-over effect during the inter-ag-

gregated time. This implies that considering data at the disaggregated level is important in the analysis of dynamic effects of advertising such as carry-over effects.

In practice, advertising effect estimation is always the most headache *part*; our model will be helpful to marketing managers to *understand the* reason why advertising be miss estimated, thereby, managers could make more sensible decision on advertising expenditures.

6.2 Limitation

In the final proposed model (Model 2, Model 4), we consider the actual data interval and disaggregated advertising rate, the carryover effect is able to be estimated more precisely, however, there still remain some limitations. First, we employed disaggregated advertising rates in our equation with aggregated sales. It would make the model too complicated if we use disaggregated sales, even though we are able to collect the disaggregated level sales, Thus, this will be the major limitation in our research.

Second, Advertising also has delayed-response effect on sales; however the Koyck model is not proper to capture this effect which is slightly different from carry-over effect. The delayed-response effect means that the highest response rate can be achieved after having marketing activities in previous periods. Our extended model also is not able to test this effect.

Third, the media effect and prints effect of advertising are not considered. Even though we controlled marketing mix such as price cut and change of sales members, we could not control the media (TV, prints, etc), prints effects and content of advertisements in our model. To im-

prove our model, we need to consider these factors in our model in the future.

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