

# Simplified Machine Diagnosis Techniques Using ARMA Model of Absolute Deterioration Factor with Weight

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**Abstract.** In mass production industries such as steel making that have large equipment, sudden stops of production process due to machine failure can cause severe problems. To prevent such situations, machine diagnosis techniques play important roles. Many methods have been developed focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machine, which is the most common theme in the machine failure detection field. A simplified method of calculating autocorrelation function is introduced and is utilized for ARMA model identification. Furthermore, an absolute deterioration factor such as Bicoherence is introduced. Machine diagnosis can be executed by this simplified calculation method of system parameter distance with weight. Proposed method proved to be a practical index for machine diagnosis by numerical examples.

**Keywords:** ARMA Model, Autocorrelation Function, Cumulants, Absolute Deterioration Factor with Weight

## 1. INTRODUCTION

In mass production industries such as steel making that have large equipment, sudden stops of production process due to machine failure can cause severe problems such as shortage of materials to the later process, delays to the due date and the increasing idling time.

To prevent such situations, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the majority the machine maintenance. TBM performs checks at previously fixed times, but it performs checks at scheduled times without taking into account whether the parts are still keeping in good conditions or not. On the other hand, Condition Based Maintenance (CBM) performs maintenance checks by watching the condition of machines. Therefore, if the parts are still keeping in good condition beyond their expected life span, the cost of maintenance may be lowered because machines can be used longer than planned. Therefore the use of CBM has become dominant, since parts maintenance cost less and also because it has a lower ratio of failure. However, it

is mandatory to catch symptoms of the failure as soon as possible if a transition from TBM to CBM is to be made. Many methods have been developed focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machine, which is the most common theme in the machine failure detection field.

To date, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998; Hoffner, 1991). As for sensitivity indices, Kurtosis, Bicoherence and Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1977; Maekawa *et al.*, 1997; Shao *et al.*, 2001, Song *et al.*, 1998; Takeyasu, 1989). Calculating system parameter distance was also utilized to apply time series data to Autoregressive (AR) model or Autoregressive Moving Average (ARMA) model (Yamazaki, 1977).

In machine diagnosis field, there are two main methods. One is a simple diagnosis and another one is a detailed diagnosis. Watching a waveform is one of the simple diagnosis. Calculating kurtosis or system parameter distance are the method of detailed diagnosis. In

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this paper, we devise a simplified calculation method which enables detailed diagnosis in a simple manner.

In this paper, a simplified method of calculating the autocorrelation function is introduced and utilized for ARMA model identification. Furthermore, an absolute deterioration factor such as Bicoherence is introduced. Machine diagnosis can be executed by this simplified method of calculating system parameter distance with weight.

In section 2, machine diagnosis techniques utilizing system parameter distance are exhibited. The parameter estimation method of ARMA model is stated in section 3. In section 4, moment, Bicoherence and cumulants are introduced for the fundamental preparation. Utilizing 4th cumulants and autocorrelation function, the parameter estimation method of ARMA model is exhibited in section 5. Utilizing these methods, numerical examples are exhibited in section 6. Section 7 summarizes the findings.

## 2. DETECTION OF FAILURE BY SYSTEM PARAMETER DISTANCE

In the analysis of time series data, AR model or ARMA model are adopted frequently. In this paper, we adopt ARMA model, because it is a general technique used in time series analysis.

Consider the  $(p, q)$ th order ARMA model expressed as:

$$x_n + \sum_{i=1}^p a_i x_{n-i} = e_n + \sum_{j=1}^q b_j e_{n-j} \quad (1)$$

Here

$\{x_i\}$ : Sample process of a stationary ergodic Gaussian process  $x(t)$  ( $i = 1, 2, 3, \dots, N, \dots$ )

$\{e_n\}$ : Gaussian white noise with mean 0, and variance  $\sigma_e^2$

$a_i$  ( $i = 1, 2, 3, \dots, p$ ) is a parameter of AR part.

$b_j$  ( $j = 1, 2, 3, \dots, q$ ) is a parameter of MA part.

Assume that (1) satisfies the stationary condition.

Now, we calculate the system parameter distance. Modern control theory shows that  $p \leq q$  when state space representation is used (Tokumaru *et al.*, 1977). In this paper, we set  $p = q$  for simplicity.

In the practical application, it is assumed that the parameter of the last part of AR model may be 0 which leads to a general description in practice.

Set

$$a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{pmatrix} \quad \left. \vphantom{\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix}} \right\}$$

(system parameters under normal condition) (2)

$$\hat{a} = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_p \end{pmatrix} \quad \hat{b} = \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \vdots \\ \hat{b}_p \end{pmatrix} \quad \left. \vphantom{\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_p \end{pmatrix}} \right\}$$

(estimation of system parameters while abnormal condition proceeds) (3)

Then, following  $Z_l$  is utilized as an evaluation function. Set the system parameters under normal condition as  $a_i, b_i$ , and the estimation of system parameters as  $\hat{a}_i, \hat{b}_i$ .

$$Z_l = \frac{1}{W_l + 1} \quad (l = 0, 1, 2, 3, 4, 5, \dots) \quad (4)$$

Where

$$W_l = (2p)^l \times \sum_{i=1}^p \{(\hat{a}_i - a_i)^2 + (\hat{b}_i - b_i)^2\} \quad (5)$$

$$L = \sum_{i=1}^p \{(\hat{a}_i - a_i)^2 + (\hat{b}_i - b_i)^2\}$$

:system parameter distance (6)

Watching the behavior of  $Z_l$ , we can judge the system becomes abnormal when  $Z_l$  falls below the certain value (Figure 1). Then we can make failure detection. This evaluation function is an absolute deterioration factor such as Bicoherence. This is 1.0 when the system is under normal condition, and tends to be 0 when the system becomes abnormal.

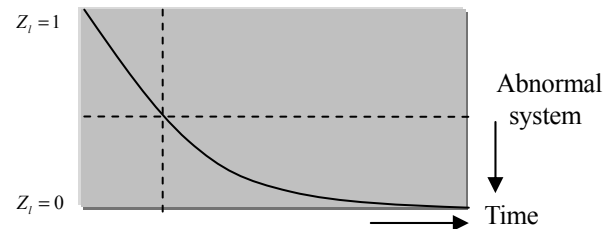


Figure 1. Detecting Abnormal Conditions by the transition of  $Z_l$ .

## 3. VIEWPOINT OF ESTIMATION METHOD OF SYSTEM PARAMETERS

In estimating ARMA model parameters  $\{a_i\} \{b_j\}$ , the right hand side of the equation is itself a colored noise. Therefore, it should be noted that a biased estimation is obtained by using the least squares method. In order to obtain an unbiased estimate, the extended least squares, sequential maximum likelihood, instrumental variable, and pseudo linear regression methods have been developed (Tokumaru *et al.*, 1982; Katayama, 1994). The pseudo linear regression method, utilizes the bootstrap method and makes iterative calculations. However, it has difficulty because there are many cases where these

iterations do not converge (Sagara *et al.*, 1994; Katayama, 1994). We have proposed an improved method for this problem (Takeyasu *et al.*, 2002). By utilizing this method, the total calculation time can be reduced.

Introducing the cross correlation function of output and noise, and also using the theoretical relation, the above improvement was achieved. In this paper, we introduce a simplified method of calculating autocorrelation function in the cases where shock waves occur, and propose simplified machine diagnosis techniques utilizing 4th cumulants and autocorrelation function. Since 4th cumulants are 0 under the normal distribution, cumulants can be calculated in combination using autocorrelation function and utilizing the formulaic relation of 4 variables.

We have previously proposed a bootstrap algorithm before in order to get an unbiased estimate. This time, our aim is to propose a new simplified calculation method for machine diagnosis. Yule-Walker equation for 4th cumulants is introduced. Knowing that we can obtain an unbiased estimate of AR parameters under the condition that MA items are white noise under a normal distribution (Inoue, 1992a, 1992b), the equation for AR parameter estimation is introduced. Furthermore, we introduce a MA parameter estimation equation which can be estimated analytically in the case of 2nd order. This is then applied to a simplified calculation method and numerical calculation is executed. The Proposed method is simple enough to be calculated even on a pocket-size calculator.

## 4. FUNDAMENTAL PREPARATION

### 4.1 About Moment

In cyclic movements such as those of bearings and gears, the vibrations grow larger whenever the deterioration becomes greater. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Assume the vibration signal is a function of time as  $x(t)$ . Also assume that it is a stationary time series with mean  $\bar{x}$ . Denote the probability density function of these time series as  $p(x)$ .

This vibration signal is obtained by attaching sensor to the rotating machine. An acceleration sensor is used for detecting high range frequency of the signal and a velocity sensor is used for detecting middle range frequency of the signal. The former one is often used for detecting the fault of bearings and the latter one is often used for detecting the fault of gears. These are generally calculated by the following methods and the specific features are not required in data handling.

If the system is under normal condition,  $p(x)$  becomes a normal distribution function. This is confirmed in many papers such as Takeyasu (1987, 1989). Under normal distribution, kurtosis is 3.0 theoretically (Yamazaki, 1977). We confirmed this to be 2.961 in the real

data (Takeyasu, 1989; Takeyasu *et al.*, 2008). This is an example data and there are many data around 3.0 for the normal data. Usually, as damages increase, kurtosis value grows up.

The average  $\bar{x}$  is stated as follows:

$$\bar{x} = \int_{-\infty}^{\infty} xp(x)dx \quad (7)$$

Variance  $\sigma^2$  is stated as follows:

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \bar{x})^2 p(x)dx \quad (8)$$

The 3rd moment  $MT(3)$  and the 4th moment  $MT(4)$  are stated as follows respectively (Hino, 1977):

$$MT(3) = \int_{-\infty}^{\infty} (x - \bar{x})^3 p(x)dx \quad (9)$$

$$MT(4) = \int_{-\infty}^{\infty} (x - \bar{x})^4 p(x)dx \quad (10)$$

### 4.2 About Bicoherence

Bicoherence means the relationship of a function at different points in the frequency domain and is expressed as:

$$Bic_{,xxx}(f_1, f_2) = \frac{B_{,xxx}(f_1, f_2)}{\sqrt{S_{,xx}(f_1) \cdot S_{,xx}(f_2) \cdot S_{,xx}(f_1 + f_2)}} \quad (11)$$

Here

$$B_{,xxx}(f_1, f_2) = \frac{X_T(f_1) \cdot X_T(f_2) \cdot X_T^*(f_1 + f_2)}{T^{\frac{3}{2}}} \quad (12)$$

means Bispectrum. Each item of right hand side of Eq. (12) is a Fourier Transform of the original time series. See Appendix in detail. Range of Bicoherence satisfies :

$$0 < Bic_{,xxx}(f_1, f_2) < 1 \quad (13)$$

When there exists a significant relationship between frequencies  $f_1$  and  $f_2$ , Bicoherence is near 1. Otherwise, the value of Bicoherence comes close to 0.

### 4.3 About Cumulants

The characteristic function  $\Psi(u)$  is defined as follows (Hino, 1977):

$$\Psi(u) = \int_{-\infty}^{\infty} e^{iux} p(x)dx \quad (14)$$

The Taylor expansion of the characteristic function is exhibited in (15).  $c(n)$  is the coefficient.

$$\ln\Psi(u) = \sum_{n=1}^{\infty} c(n) \frac{i^n}{n!} u^n \quad (15)$$

$$c(n) = \left[ \frac{(-i)^n d^n \ln\Psi(u)}{du^n} \right]_{u=0} \quad (16)$$

This  $c(n)$  is called cumulants. The following relation holds between cumulants and the moment.

$$c(1) = \bar{x} \quad (17)$$

$$c(2) = \sigma^2 \quad (18)$$

$$c(3) = MT(3) \quad (19)$$

$$c(4) = MT(4) - 3\{MT(2)\}^2 \quad (20)$$

It is well-known that when the probability density function is a normal distribution, cumulants over the 3rd order are 0 (Inoue, 1992).

#### 4.4 2nd Order Autocorrelation Function

Using the following formula concerning the relation of Gaussian 4 variables (Nakamizo, 1988):

$$E[abcd] = E[ab]E[cd] + E[ac]E[bd] + E[ad]E[bc] \quad (21)$$

We can get following equation for  $x(t)$ ,  $x(t + \tau_1)$ ,  $x(t + \tau_2)$ ,  $x(t + \tau_3)$

$$\begin{aligned} E[x(t)x(t + \tau_1)x(t + \tau_2)x(t + \tau_3)] \\ = R(\tau_1)R(\tau_3 - \tau_2) + R(\tau_2)R(\tau_3 - \tau_1) + R(\tau_3)R(\tau_2 - \tau_1) \end{aligned} \quad (22)$$

Where,  $R_{\tau_1}$  ( $R_{\tau_1} = R(\tau_1)$ ) is the autocorrelation function of  $\{x(t)\}$ .

$$R_{\tau_1} = E[x(t)x(t + \tau_1)] \quad (23)$$

Set  $\tau_2 = \tau_1$ ,  $\tau_3 = 0$ , then we can obtain

$$E[x(t)^2x(t + \tau_1)^2] = R_0^2 + 2R_{\tau_1}^2 \quad (24)$$

Set  $\tau_1 = -l$ ,  $\tau_2 = -l$ ,  $\tau_3 = -i$ , then we can obtain

$$E[x(t)x(t-l)^2x(t-i)] = R_0R_l + 2R_lR_{l-i} \quad (25)$$

This is the correlation function of the correlation function. That is to say that this is a 2nd order correlation function.

## 5. SIMPLIFIED ESTIMATION METHOD OF SYSTEM PARAMETERS

First, the AR parameter estimation using Yule-Walker equation concerning 4th cumulants is exhibited. Next, the ARMA parameter estimation algorithm using autocorrelation function is stated.

### 5.1 Estimation of AR Parameter

Under a normal distribution, 4th order cumulants becomes 0.

As  $\{e_n\}$  is white under 4th order cumulants (Inoue, 1992), by multiplying by  $x_n x_{n-l}^2$  on both sides of (1) and averaging, we can get the following equation:

$$E[x_n^2 x_{n-l}^2] + \sum_{i=1}^p a_i E[x_n x_{n-l}^2 x_{n-i}] = 0 \quad (l \geq 1) \quad (26)$$

Rewriting (26), we can get following equation:

$$c_l + \sum_{i=1}^p a_i c_{l-i} = 0 \quad (l \geq 1) \quad (27)$$

where  $c_{l-i}$  is the 4th cumulant.

$$\begin{aligned} c_{l-i} &= c(x_n, x_{n-l}, x_{n-l}, x_{n-i}) \\ &= E[x_n x_{n-l}^2 x_{n-i}] \\ &= R(-l)R(l-i) + R(-l)R(l-i) + R(-i)R(0) \\ &= R_0R_l + 2R_lR_{l-i} \quad (l \geq 1, i = 1, \dots, p) \end{aligned} \quad (28)$$

Varying  $l$  from 1 to  $p$  in (27), following Yule Walker equation about the 4th cumulants can be derived:

$$\begin{pmatrix} c_0 & c_{-1} & \dots & c_{1-p} \\ c_1 & c_0 & \dots & c_{2-p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p-1} & c_{p-2} & \dots & c_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} = - \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{pmatrix} \quad (29)$$

Rewriting this in matrix form, we can get:

$$ca = -c_s \quad (30)$$

Then  $a$  is solved as follows:

$$a = -c^{-1}c_s \quad (31)$$

The estimation of  $c_{l-i}$  ( $l = 1, 2, \dots, i = 1, \dots, p$ ) is expressed as  $\hat{c}_{l-i}$ .  $\hat{c}_{l-i}$  is stated as follows:

$$\hat{c}_{l-i} = \frac{1}{N - (l-i)} \sum_{n=l-i+1}^N x_n x_{n-l}^2 x_{n-i} \quad (32)$$

Substituting  $\hat{c}_i$  for  $c_i$  in each  $c$ ,  $c_s$ ,  $\hat{c}$ ,  $\hat{c}_s$  is obtained. Then the estimate  $\hat{a}$  of  $a$  can be obtained as the unbiased estimate by the following equation.

$$\hat{a} = -\hat{c}^{-1}\hat{c}_s \quad (33)$$

This is the same meaning that an unbiased estimate of AR model parameters can be obtained by solving Yule-Walker equation.

#### 5.1.1 2nd order case

$$\begin{aligned} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} &= - \begin{pmatrix} c_0 & c_{-1} \\ c_1 & c_0 \end{pmatrix}^{-1} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \\ &= - \begin{pmatrix} 3R_0^2 & R_0^2 + 2R_1^2 \\ R_0^2 + 2R_1^2 & 3R_0^2 \end{pmatrix}^{-1} \begin{pmatrix} R_0^2 + 2R_1^2 \\ R_0^2 + 2R_2^2 \end{pmatrix} \\ &= - \frac{1}{4(2 + \rho_1^2)(1 - \rho_1^2)} \begin{pmatrix} (1 + 2\rho_1^2)(1 - \rho_2^2) \\ (1 - 2\rho_1^2 + 3\rho_2^2 - 2\rho_1^4) \end{pmatrix} \end{aligned} \quad (34)$$

## 5.2 Simplified Calculation Method of Autocorrelation Function

In a discrete time system, sampled signal data are expressed as follows:

$$\{x_i\} : i=1, 2, \dots, N$$

$\bar{x}$  which is the mean of  $\{x_i\}$ , is stated as follows:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad (35)$$

For the purpose of generalization,  $\bar{x}$  is not set 0.

Variance  $\sigma^2$  of  $\{x_i\}$  is stated as follows:

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \quad (36)$$

Autocorrelation function is stated as follows:

$$R_l = E[x_i x_{i+l}] = \frac{1}{N-l} \sum_{i=1}^{N-l} x_i x_{i+l} \quad (l=0, 1, 2, \dots) \quad (37)$$

As  $\{x_i\}$  is assumed to be a stationary time series, we can assume  $E[x_i] = 0$  without loss of generality (Tokumaru *et al.*, 1982). Therefore, (37) can be re-stated as follows:

$$\begin{aligned} R_l &= E[(x_i - \bar{x})(x_{i+l} - \bar{x})] \\ &= \frac{1}{N-l} \sum_{i=1}^{N-l} (x_i - \bar{x})(x_{i+l} - \bar{x}) \end{aligned} \quad (38)$$

When  $N$  is sufficiently large, the 0th autocorrelation function is nearly equal to the variance.

The autocorrelation coefficient is expressed as follows:

$$\rho_l = \frac{R_l}{R_0} \quad (39)$$

When the number of failures on bearings or gears arise, the peak value arise cyclically. In the early stage of the defect, the peak signal usually appears clearly. Generally, defects will damage other bearings or gears by contacting the inner covering surface as time passes. Assume that the peak signal which has  $S$  times impacts from the normal signal arises in each  $m$  times of samplings. As for determining the sampling interval, the well-known sampling theorem can be used (Tokumaru *et al.*, 1982), but in this paper, we do not pay much attention to this point in order to focus on our proposal. Assume that the peak signal which has  $S$  times impacts from the normal signal arises in each  $m$  times of samplings, and also assume that the mean and variance are the same except for the case where a special peak signal arises. Expressing  $\sigma^2$  of this case as  $\bar{\sigma}^2$ , we get following equation:

$$\begin{aligned} \bar{\sigma}^2 &= \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \\ &= \frac{N-N}{N-1} \sigma^2 + \frac{N}{N-1} S^2 \sigma^2 \end{aligned}$$

$$\equiv \left(1 + \frac{S^2 - 1}{m}\right) \sigma^2 \quad (40)$$

For the autocorrelation function, we express  $R_l$  in this case as  $\bar{R}_l$  ( $l=1, 2, \dots$ ) in the same way.

The 0th autocorrelation function is nearly equal variance, therefore we get:

$$\bar{R}_0 \equiv \bar{\sigma}^2 \equiv \left(1 + \frac{S^2 - 1}{m}\right) \sigma^2 \quad (41)$$

### 5.2.1 Autocorrelation function of 1st order lag

In the next, we examine the feature of autocorrelation function of 1st order lag. By definition,  $\bar{R}_1$  is stated as follows:

$$\bar{R}_1 = \frac{1}{N-1} \sum_{i=1}^{N-1} x_i x_{i+1} \quad (42)$$

When the peak signal of  $S$  times impacts from the normal signal arises in each  $m$  times of samplings, the following  $\wedge$  parts of  $\{x_i\}$  may be considered to have peak values during the calculation of  $\bar{R}_1$ .

$$1, 2, \dots, m-2, m-1, m, m+1, m+2, \dots, 2m-1, 2m, 2m+1, \dots$$

$\wedge \quad \wedge \quad \wedge \quad \wedge$

From these, the following products arise:

1 product with peak value and  $\{(m-1)-1\}$  products with ordinary values up to  $m$

3 products with peak values and  $\{(2m-1)-3\}$  products with ordinary values up to  $2m$

$\vdots$

$(2k-1)$  products with peak values and  $\{(km-1)-(2k-1)\}$  products with ordinary values up to  $km$

$\vdots$

$2k$  products with peak values and  $\{(km+1-1)-2k\}$  products with ordinary values up to  $km+1$

Therefore when  $N = km$  (case 1)

$(2(N/m)-1)$  products with peak values and  $\{(N-1)-(2(N/m)-1)\}$  products with ordinary values up to  $N$

When  $N = km + \alpha$  ( $1 \leq \alpha \leq m-1$ ) (case 2)

$(2(N-\alpha)/m)$  products with peak values and  $\{(N-1)-(2(N-\alpha)/m)\}$  products with ordinary values up to  $N$

When  $S$  is large, we make the following simplified calculation.

Suppose

$$\begin{aligned} \bar{x} - \varepsilon &< x_i < \bar{x} + \varepsilon \\ \varepsilon &> 0, \quad (i=1, 2, \dots, N) \end{aligned}$$

except for the case when peak signals arise. In that case we get the following relations:

< Case 1 >

When  $\bar{x} > 0$

$$\begin{aligned} \frac{1}{N-1} \left\{ S \left( 2 \frac{N}{m} - 1 \right) + (N-1) - \left( 2 \frac{N}{m} - 1 \right) \right\} (\bar{x} - \varepsilon)^2 &< \bar{R}_1 \\ \bar{R}_1 &< \frac{1}{N-1} \left\{ S \left( 2 \frac{N}{m} - 1 \right) + (N-1) - \left( 2 \frac{N}{m} - 1 \right) \right\} (\bar{x} + \varepsilon)^2 \end{aligned} \quad (43)$$

When  $\bar{x} < 0$

$$\frac{1}{N-1} \left\{ S \left( 2 \frac{N}{m} - 1 \right) + (N-1) - \left( 2 \frac{N}{m} - 1 \right) \right\} (\bar{x} - \varepsilon)^2 > \bar{R}_1$$

$$\bar{R}_1 > \frac{1}{N-1} \left\{ S \left( 2 \frac{N}{m} - 1 \right) + (N-1) - \left( 2 \frac{N}{m} - 1 \right) \right\} (\bar{x} + \varepsilon)^2 \quad (44)$$

Suppose  $\bar{x} \rightarrow 0$  and  $\sigma^2 \equiv \varepsilon^2$ , we get the following equation from (39) and (41):

$$\bar{\rho}_1 = \frac{\bar{R}_1}{\bar{R}_0}$$

$$\equiv \frac{1}{N-1} \left[ S \left( 2 \frac{N}{m} - 1 \right) + (N-1) - \left( 2 \frac{N}{m} - 1 \right) \right] \cdot \frac{1}{1 + \frac{S^2 - 1}{m}}$$

$$= \left\{ \left( \frac{2}{m} - \frac{1}{N-1} + \frac{2}{m(N-1)} \right) (S-1) + 1 \right\} \cdot \frac{m}{m + S^2 - 1} \quad (45)$$

### <Case 2>

When  $\bar{x} > 0$

$$\frac{1}{N-1} \left\{ S \left( 2 \frac{N-\alpha}{m} \right) + (N-1) - \left( 2 \frac{N-\alpha}{m} \right) \right\} (\bar{x} - \varepsilon)^2 < \bar{R}_1$$

$$\bar{R}_1 < \frac{1}{N-1} \left\{ S \left( 2 \frac{N-\alpha}{m} \right) + (N-1) - \left( 2 \frac{N-\alpha}{m} \right) \right\} (\bar{x} + \varepsilon)^2 \quad (46)$$

When  $\bar{x} < 0$

$$\frac{1}{N-1} \left\{ S \left( 2 \frac{N-\alpha}{m} \right) + (N-1) - \left( 2 \frac{N-\alpha}{m} \right) \right\} (\bar{x} - \varepsilon)^2 > \bar{R}_1$$

$$\bar{R}_1 > \frac{1}{N-1} \left\{ S \left( 2 \frac{N-\alpha}{m} \right) + (N-1) - \left( 2 \frac{N-\alpha}{m} \right) \right\} (\bar{x} + \varepsilon)^2 \quad (47)$$

In the same method as <Case 1>, we get the following equation:

$$\bar{\rho}_1 = \frac{\bar{R}_1}{\bar{R}_0}$$

$$\equiv \frac{1}{N-1} \left[ S \left( 2 \frac{N-\alpha}{m} \right) + (N-1) - \left( 2 \frac{N-\alpha}{m} \right) \right] \cdot \frac{1}{1 + \frac{S^2 - 1}{m}}$$

$$= \left\{ \left( \frac{2}{m} \left( \frac{1-\alpha}{N-1} + 1 \right) \right) (S-1) + 1 \right\} \cdot \frac{m}{m + S^2 - 1} \quad (48)$$

(45) and (48) are the simplified calculations of coefficient of autocorrelation function of 1st order lag. Under normal conditions,  $\bar{\rho}_1 = 1$  when  $S=1$ . When  $N \rightarrow \infty$ ,  $S \rightarrow \infty$ ,  $\bar{\rho}_1 \rightarrow 0$ .

Generally,  $\alpha$  is very small compared with  $N$ . Therefore we make simplified calculation using (45). Under the assumption of 5.1, set  $m=12$ . Considering the case  $S=2, 4, 6, 8, 10$  and the case  $N=100$ , we obtain Table 1 from the calculation of (45).

**Table 1.**  $\bar{\rho}_1$  with varying  $S$ .

$S$	2	4	6	8	10
$\bar{\rho}_1$	0.9266	0.6554	0.4573	0.3372	0.2621

$\bar{\rho}_1$ , can be utilized as a deterioration factor. We name this as ‘‘Deterioration Factor of the Autocorrelation Function Type of 1st Order Lag’’.

### 5.2.2 Autocorrelation coefficients of the 2nd order lag through 4th order lag

Then, we can get the autocorrelation function of 2nd order lag through 4th order lag using the same method.

**Table 2.**  $\bar{\rho}_2$  through  $\bar{\rho}_4$  with varying  $S$ .

$S$	2	4	6	8	10
$\bar{\rho}_2$	0.9265	0.6553	0.4572	0.3371	0.2620
$\bar{\rho}_3$	0.9265	0.6552	0.4571	0.3370	0.2619
$\bar{\rho}_4$	0.9264	0.6551	0.4570	0.3369	0.2618

### 5.3 Estimation of MA Parameters about ARMA Model

Generally, the MA part of the ARMA model becomes a non-linear-equation as follows:

Set

$$\tilde{x}_n = x_n + \sum_{i=1}^p a_i x_{n-i} \quad (49)$$

then we can get following equation from (1):

$$\tilde{x}_n = e_n + \sum_{j=1}^q b_j e_{n-j} \quad (50)$$

From (49), (50), we set  $\tilde{r}_k$  as the autocorrelation function of  $\tilde{x}_n$ , and set  $b_0 = 1$ . Then the following relations are well known (Tokumaru *et al.*, 1982).

$$\tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \leq q) \\ 0 & (k \geq q+1) \end{cases}$$

$$\tilde{r}_0 = \sigma_e^2 \sum_{j=0}^q b_j^2 \quad (52)$$

A recursive algorithm has been developed as for this. The only parameters to be estimated are  $b_1$  and  $b_2$ . Therefore they can be solved using the following method.

The case of  $q=2$ ;

$$\tilde{r}_3 = 0 \quad (53)$$

$$\tilde{r}_2 = \sigma_e^2 b_2 \quad (54)$$

$$\tilde{r}_1 = \sigma_e^2 (b_1 + b_1 b_2) \quad (55)$$

$$\tilde{r}_0 = \sigma_e^2 (1 + b_1^2 + b_2^2) \quad (56)$$

When  $\sigma_e^2 \rightarrow 1$ , then we get the following equations:

$$\tilde{r}_0 = 1 + b_1^2 + b_2^2 \quad (57)$$

$$\tilde{r}_1 = b_1(1 + b_2) \quad (58)$$

$$\tilde{r}_2 = b_2 \quad (59)$$

Therefore we can obtain  $b_1$  and  $b_2$  in the following equations.

$$b_1 = \frac{\tilde{r}_1}{1 + \tilde{r}_2} \quad (60)$$

$$b_2 = \tilde{r}_2 \quad (61)$$

From (49), we get the following equations:

$$\begin{aligned} \tilde{r}_1 &= E[\tilde{x}_n \tilde{x}_{n+1}] \\ &= E[(x_n + a_1 x_{n-1} + a_2 x_{n-2})(x_{n+1} + a_1 x_n + a_2 x_{n-1})] \\ &= a_1(1 + a_2)(R_0 + R_2) + a_2(R_1 + R_3) + (1 + a_1^2 + a_2^2)R_1 \quad (62) \end{aligned}$$

$$\begin{aligned} \tilde{r}_2 &= E[\tilde{x}_n \tilde{x}_{n+2}] \\ &= E[(x_n + a_1 x_{n-1} + a_2 x_{n-2})(x_{n+2} + a_1 x_{n+1} + a_2 x_n)] \\ &= a_1(1 + a_2)(R_1 + R_3) + a_2(R_0 + R_4) + (1 + a_1^2 + a_2^2)R_2 \quad (63) \end{aligned}$$

$$\begin{aligned} b_1 &= \frac{\tilde{r}_1}{1 + \tilde{r}_2} \\ &= \frac{a_1(1 + a_2)(R_0 + R_2) + a_2(R_1 + R_3) + (1 + a_1^2 + a_2^2)R_1}{1 + a_1(1 + a_2)(R_1 + R_3) + a_2(R_0 + R_4) + (1 + a_1^2 + a_2^2)R_2} \quad (64) \end{aligned}$$

$$\begin{aligned} b_2 &= \tilde{r}_2 \\ &= a_1(1 + a_2)(R_1 + R_3) + a_2(R_0 + R_4) + (1 + a_1^2 + a_2^2)R_2 \quad (65) \end{aligned}$$

As mentioned above, the (2, 2) of the ARMA model can be solved analytically. We can solve ARMA model only by calculating the autocorrelation function from the 1st through 4th order.

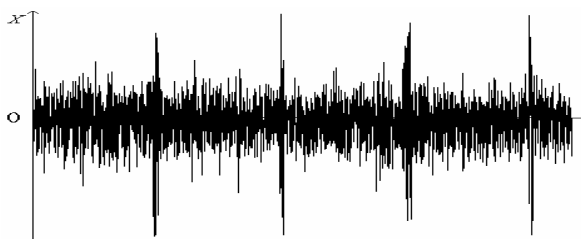
## 6. NUMERICAL EXAMPLES

### 6.1 Estimation of AR Parameters

Under the assumption of 5.2, set  $m=12$ . Considering the case  $S=2, 4, 6, 8, 10$  and the case  $N=100$ , we obtain Table3 from the calculation of (34) using the values of  $\rho_1, \rho_2, \rho_3, \rho_4$  in the Table 1~2 of 5.2.

**Table 3.** Estimation of AR parameters with varying  $S$ .

$S$	2	4	6	8	10
$\hat{a}_1$	-0.2380	-0.1914	-0.1605	-0.1452	-0.1375
$\hat{a}_2$	-0.2373	-0.1913	-0.1605	-0.1452	-0.1375



**Figure 2.** Calculation model data chart.

We get the estimations  $\hat{a}_1, \hat{a}_2$  of the AR parameters  $a_1, a_2$ . Calculation model data chart is exhibited in Figure 2.

### 6.2 Estimation of MA Parameters

Under the same assumption of 5.1, we solve  $\hat{b}_1, \hat{b}_2$ , which are an estimate of  $b_1, b_2$  of MA parameters respectively, using  $\rho_1, \rho_2, \rho_3, \rho_4$ , and  $\hat{a}_1, \hat{a}_2$  as mentioned above.

As  $\{x_i\}$  is assumed to be a stationary time series, we can assume  $\bar{x}=0, \sigma^2=1$  without loss of generality (Tokumaru *et al.*, 1982).

$$R_0 \equiv \sigma^2 = 1$$

Therefore, from (34) we can get following equation:

$$R_l = \rho_l, R_0 \equiv \rho_l \quad (66)$$

Then

$$\rho_0 \equiv 1 \quad (67)$$

Therefore we get the estimates  $\hat{b}_1, \hat{b}_2$  of the MA parameters  $b_1, b_2$  using (64) and (65) (Table 4).

**Table 4.** Estimation of MA parameters with varying  $S$ .

$S$	2	4	6	8	10
$\hat{b}_1$	0.1954	0.1660	0.1226	0.0816	0.0485
$\hat{b}_2$	0.2377	0.1838	0.1237	0.0735	0.0363

### 6.3 System Parameter Distance

#### 6.3.1 AR parameters

When  $S=1$ , the system is considered to be under normal conditions. Therefore we get the following equation by (34).

$$a_1 = -\frac{(1 + 2\rho_1^2)(1 - \rho_2^2)}{4(2 + \rho_1^2)(1 - \rho_1^2)} \quad (68)$$

$$a_2 = -\frac{1 - 2\rho_1^2 + 3\rho_2^2 - 2\rho_1^4}{4(2 + \rho_1^2)(1 - \rho_1^2)} \quad (69)$$

When  $S=1$ , from (45), we get

$$\bar{\rho}_1 = \rho_1 = 1, \bar{\rho}_2 = \rho_2 = 1$$

When

$$\lim_{x \rightarrow 0} f(x) = 0, \lim_{x \rightarrow 0} g(x) = 0 \quad (70)$$

utilizing the following relation:

$$\lim_{x \rightarrow 0} \frac{g(x)}{f(x)} = \lim_{x \rightarrow 0} \frac{g'(x)}{f'(x)} \quad (71)$$

We obtain the following results:

$$a_1 = \lim_{\rho_1 \rightarrow 1, \rho_2 \rightarrow 1} \frac{1 - \rho_2^2}{2 + 4\rho_1^2} = 0 \quad (72)$$

$$a_2 = \lim_{\rho_1 \rightarrow 1, \rho_2 \rightarrow 1} \frac{1 + 2\rho_1^2}{-2 - 4\rho_1^2} = -\frac{1}{2} \quad (73)$$

6.3.2 MA parameters

When  $S=1$ , the system is considered to be under normal conditions. Therefore we can calculate the following equation by substituting  $a_1, a_2$  of (72), (73) into (62), (63).

$$b_1 = \frac{3R_1 - 2R_3}{4 - 2R_0 + 5R_2 - 2R_4} \quad (74)$$

$$b_2 = -\frac{1}{2}R_0 + \frac{5}{4}R_2 - \frac{1}{2}R_4 \quad (75)$$

From (66), (67) and (45) we get

$$R_0 = \rho_0 = 1, R_1 = \rho_1 = 1, R_2 = \rho_2 = 1, \\ R_3 = \rho_3 = 1, R_4 = \rho_4 = 1$$

From (74) and (75), we obtain the following results.

$$b_1 = \frac{1}{5}, b_2 = \frac{1}{4}$$

6.3.3 Calculation of the evaluation function

Calculating  $Z_l$  using (4), we obtain Table 5 and Figure 3 to 6.

Table 5.  $Z_l$  with varying  $S$ .

	$S$	1	2	4	6	8	10
$l=0$	$Z_0$	1.0000	0.8883	0.8791	0.8598	0.8389	0.8204
$l=1$	$Z_1$	1.0000	0.6652	0.6452	0.6053	0.5655	0.5332
$l=2$	$Z_2$	1.0000	0.3319	0.3125	0.2772	0.2455	0.2221
$l=3$	$Z_3$	1.0000	0.1105	0.1020	0.0875	0.07522	0.0666

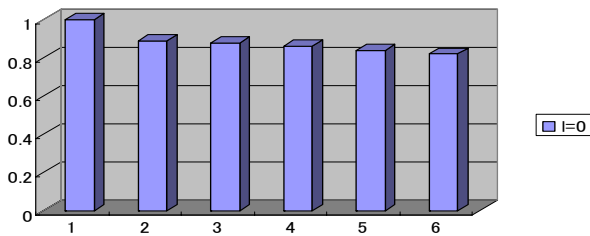


Figure 3. The Transition of Evaluation Function  $Z_0$ .

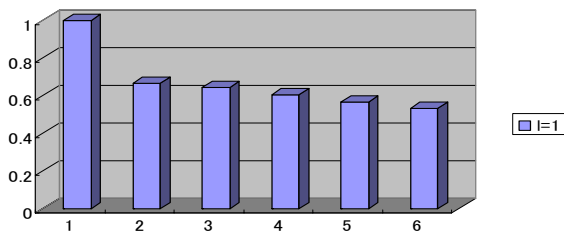


Figure 4. The Transition of Evaluation Function  $Z_1$ .

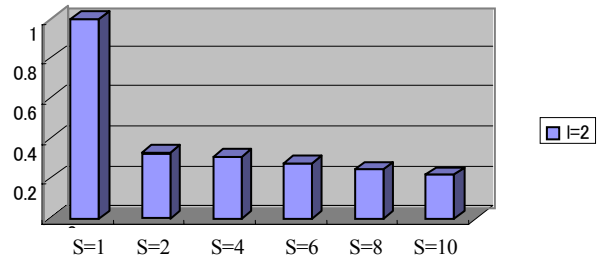


Figure 5. The Transition of Evaluation Function  $Z_2$ .

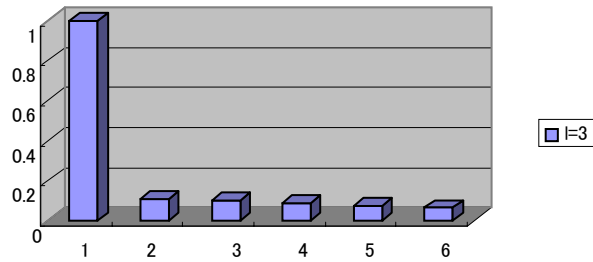


Figure 6. The Transition of Evaluation Function  $Z_3$ .

As  $S$  grows larger, the evaluation function  $Z_l$  becomes smaller. As  $l$  grows larger  $Z_l$  becomes smaller. Therefore we can utilize this index as a machine diagnosis index.  $Z_2$  has relatively suitable value for machine diagnosis which is easy to catch a symptom of machine failure. These results are useful for machine diagnosis in early stage.

Now, we compare this new index with Bicoherence. We made experiment in the past. Summary of the experiment is as follows. Pitching defects are pressed on the gears of small testing machine.

Small defect condition: Pitching defects pressed on 1/3 gears of the total gear.

Middle defect condition: Pitching defects pressed on 2/3 gears of the total gear.

Big defect condition: Pitching defects pressed on whole gears of the total gear.

We examined several cases for the  $f_1, f_2$  in Eq. (12). We got best-fit result in the following case.

$$\begin{cases} f_1: \text{peak frequency of power spectrum} \\ f_2: 2f_1 \end{cases}$$

We obtained the following Bicoherence values in this case (Table 6, Figure 7).

Table 6. Transition of Bicoherence value.

Condition	Normal	Small defect	Middle defect	Big defect
Bicoherence	0.99	0.38	0.09	0.02

Thus, Bicoherence proved to be a very sensitive good index. Bicoherence is an absolute index of which range is 1 to 0. Therefore it can be said that it is a



universal index. In those experiment, small defect condition is generally assumed to be  $S=2$  and big defect condition is generally assumed to be  $S=6$ .

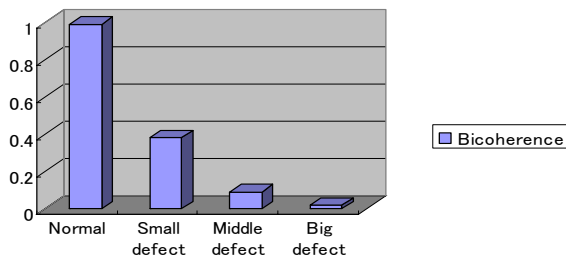


Figure 7. The Transition of Bicoherence.

In the case of  $l=2$  (Figure 5) in Table 5, the value is 0.3319 at small defect condition and 0.3125 at middle defect condition and 0.2772 at big defect condition which show nearly sensitive behavior in the early stage of Bicoherence. It could be said that the case  $l=2$  would be sensitive enough for the practical use. Therefore, the case  $l=2$  would be recommended in this new method. This calculation method is simple enough to be executed even on a pocket-size calculator. Compared with Bicoherence which has to be calculated by Eq. (11), (12), (76) and (77), proposed method is by far a simple one and easy to handle on the field defection.

## 7. CONCLUSION

In order to make machine diagnosis, the method of calculating Kurtosis or Bicoherence was utilized in the past. Calculating system parameter distance was also utilized to apply time series data to Autoregressive (AR) model or Autoregressive Moving Average (ARMA) model.

In this paper, a simplified calculation method of autocorrelation function was introduced and it was utilized for ARMA model identification. Furthermore, an absolute deterioration factor such as Bicoherence is introduced. Compared with the results obtained so far, the results of numerical examples of this paper are reasonable and nearly sensitive of another method such as Bicoherence. Judging from these results, our method is properly considered to be effective for the early stage failure detection especially. This calculation method is very simple and is very practical at the factory of maintenance site. This can be installed in microcomputer chips and utilized as the tool for the early stage detection of the failure. The case  $l=2$  is the typical value of equipment.

Machine diagnosis could be executed by this simplified calculation method of system parameter distance with weight. Proposed method proved to be a practical index for machine diagnosis by numerical examples. The effectiveness of this method should be examined in various cases.

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## APPENDIX

$X_T(f)$  is a Fourier Transform stated as follows.

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (\text{else}) \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi ft} dt \quad (76)$$

$S_{xx}(f)$  is a power spectrum stated as follows.

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \quad (77)$$

\* : conjugate